

## ON THE NORMS OF $r$ -CIRCULANT MATRICES WITH THE HYPER-FIBONACCI AND LUCAS NUMBERS

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**Abstract.** In this paper, we study norms of circulant matrices  $F = \text{Circ}(F_0^{(k)}, F_1^{(k)}, \dots, F_{n-1}^{(k)})$ ,  $L = \text{Circ}(L_0^{(k)}, L_1^{(k)}, \dots, L_{n-1}^{(k)})$  and  $r$ -circulant matrices  $F_r = \text{Circ}_r(F_0^{(k)}, F_1^{(k)}, \dots, F_{n-1}^{(k)})$ ,  $L_r = \text{Circ}_r(L_0^{(k)}, L_1^{(k)}, \dots, L_{n-1}^{(k)})$ , where  $F_n^{(k)}$  and  $L_n^{(k)}$  denote the hyper-Fibonacci and hyper-Lucas numbers, respectively.

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