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ON THE NUCLEON-NUCLEON INTERACTION

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Robert Jastrow

August 1, 1950

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ON THE NUCLEON-NUCLEON INTERACTION

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August 2, 1950

ABSTRACT

A charge-independent interaction between nucleons is assumed, which is characterized by a short range repulsion interior to an attractive well. It is shown that it is then possible to account for the qualitative features of currently known n-p and p-p scattering data. Some of the implications for saturation are discussed.

ON THE NUCLEON-NUCLEON INTERACTION

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August 2, 1950

I. INTRODUCTION

Recent experiments on p-p scattering at 340 Mev[1] indicate a cross section roughly isotropic between 20° and 90° with a mean magnitude of approximately 4 millibarns/steradian. This result is in strong contrast to that obtained in n-p experiments at comparable energies where a marked anisotropy is observed, the cross section at 260 Mev rising from approximately 1.2 millibarns at 90° to 10 millibarns at 180°[2]. At low energies, however, complete charge independence is observed in the singlet state, within the limits of error in the determination of the singlet scattering parameters[3,4].

A striking feature of the p-p observations lies in their qualitative disagreement with the results expected from a central attractive potential consistent with the low energy scattering, as shown in figure 3, curve I. This potential predicts strong forward scattering at 340 Mev, with a cross section of 0.2 millibarns at 90° which rises steeply to 11 millibarns at 0°[5].

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1. C. Wiegand and O. Chamberlain, Phys. Rev., (in press), and private communication with E. Segrè, through whose kindness I am able to quote unpublished values on the mean magnitude of the 340 Mev p-p cross section.
2. Kelly, Leith, Segrè, and Wiegand, Phys. Rev. (in press).
3. H. A. Bethe, Phys. Rev. 76, 38 (1949)
4. H. A. Bethe and C. Longmire, Phys. Rev. 77, 647 (1950).
5. R. S. Christian and H. P. Noyes, Phys. Rev. (in press).

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* This work was begun while author was at the Institute for Advanced Study, Princeton, N. J., and completed at the Radiation Laboratory, University of California.

The appearance of strong forward scattering at high energies is indeed a characteristic of the scattering from any central potential which does not change sign, as may be seen from the expressions for the differential cross sections,

$$
\begin{aligned}
\text{(1a)} \quad \sigma_{np}(\theta) &= \frac{1}{4}\left|f_s(\theta)\right|^2 + \frac{3}{4}\left|f_t(\theta)\right|^2 \\
&= \frac{1}{4}\lambda\!\!\!^-{}^2 \sum_{\ell,\ell'}\sum (2\ell+1)(2\ell'+1)\sin\delta_\ell^s \sin\delta_{\ell'}^s \cos\delta_{\ell\ell'}^s P_\ell P_{\ell'} \\
&+ \frac{3\lambda\!\!\!^-{}^2}{4}\sum_{\ell,\ell'}\sum (2\ell+1)(2\ell'+1)\sin\delta_\ell^t \sin\delta_{\ell'}^t \cos\delta_{\ell\ell'}^t P_\ell P_{\ell'} \quad ;
\end{aligned}
$$

$$
\begin{aligned}
\text{(1b)} \quad \sigma_{pp}(\theta) &= \frac{1}{4}\left|f_s(\theta)+f_s(\pi-\theta)\right|^2 + \frac{3}{4}\left|f_t(\theta)-f_t(\pi-\theta)\right|^2 \\
&= \lambda\!\!\!^-{}^2 \sum_{\ell,\ell'}\sum_{\text{even}} (2\ell+1)(2\ell'+1)\sin\delta_\ell^s \sin\delta_{\ell'}^s \cos\delta_{\ell\ell'}^s P_\ell P_{\ell'} \\
&+ 3\lambda\!\!\!^-{}^2 \sum_{\ell,\ell'}\sum_{\text{odd}} (2\ell'+1)(2\ell+1)\sin\delta_\ell^t \sin\delta_{\ell'}^t \cos\delta_{\ell\ell'}^t P_\ell P_{\ell'}
\end{aligned}
$$

Here the superscripts s and t refer to singlet and triplet states, respectively. Successive even Legendre polynomials alternate in sign at 90°, hence the interference terms between states of successive even angular momenta in the p-p cross section, such as the S-D term, for example, are negative at 90° and positive at 0°, provided the potential is everywhere of the same sign and the phases are therefore all of the same sign. The scattering is, consequently, predominantly forward at energies sufficiently high to excite a number of angular momenta. The contribution from states of odd angular momentum is zero at 90° and rises to a maximum at 0°, increasing the forward effect.

We wish to show that it is possible, nonetheless, by means of a charge-independent static potential, to account for the isotropic distribution observed in p-p scattering at high energies while maintaining the anisotropy in n-p scattering at comparable energies.

II. THE INTERACTION

A charge-independent interaction is assumed, the differences in n-p and p-p scattering then being only those determined by the exclusion principle in (1a) and (1b), namely, the domination of σ_{pp} over σ_{np} by a factor of four and the elimination in the former of the odd singlet and even triplet states.

We adopt an interaction characterized in the singlet states by a short-range repulsion and a surrounding attractive well, the parameters in the combination being chosen for agreement with low energy scattering constants[6]. The attractive part of the field is perhaps to be associated with the π-meson and the short-range repulsion with a heavier particle.

The small magnitude of the triplet effective range (1.7×10^{-13} cm.) precludes the possibility of a triplet repulsion larger than 0.2×10^{-13}cm. Since the effects of a core of this size are unimportant at the energies considered, we may simplify the interaction by taking the triplet radius of repulsion to be zero.

The assumption of a spin-dependent core radius is somewhat artificial. However, the absence of the core in the triplet states may also be considered as arising from the superposition of an attractive well of spin-dependent depth on a repulsive interaction constant in all states. (Figure 1).

The triplet interaction is assumed to be the same as that fitted by Christian and Hart[7] to the deuteron constants and low energy n-p scattering parameters, except for the addition by us of a weak tensor force in the odd states.

The repulsive field is represented by a hard sphere for convenience in calculation. An exponential radial dependence is chosen for the attractive well since the S phase may then be expressed analytically[8]. The interaction

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[6] A similar type of interaction has been considered by N. M. Kroll in connection with p-p scattering. P. O. Olsson has examined the possibility of introducing a repulsion into the n-p interaction, as have also G. Parzen and L. Schiff, Phys. Rev. 74, 1564 (1948).

[7] R. S. Christian and E. W. Hart, Phys. Rev. 77, 441 (1950).

[8] The exponential well possesses this advantage over the Yukawa well. With regard to the possibility of other radial forms, the only important requirement is that the tail of the well be approximately as long as that of the exponential or Yukawa wells.

then takes the form:

$$\text{(2)} \quad \text{Singlet:} \quad V = \infty, \; r < r_o$$

$$V = \left(\frac{1+P_x}{2}\right) V_{os} \exp\left(-\frac{r-r_o}{r_s}\right), \; r > r_o \, .$$

$$\text{Triplet:} \quad V = \left[a+(1-a)P_x + \left[b+(1-b)P_x\right] \gamma \, S_{12} \right] V_{ot} \exp\left(-\frac{r}{r_t}\right).$$

If one chooses $r_o = 0.60 \times 10^{-13}$ cm., the remaining parameters are then fixed at the following values by the deuteron constants and by n-p and p-p scattering at various energies:

$$\begin{array}{ll} r_o = 0.60 \times 10^{-13}\,\text{cm.} & V_{os} = 375\ \text{Mev}^{9} \\ r_s = 0.40 \times 10^{-13}\,\text{cm.} & V_{ot} = 69\ \text{Mev} \\ r_t = 0.75 \times 10^{-13}\,\text{cm.} & \gamma = 1.84 \\ a = 0.50 & b = 0.30 \end{array}$$

III. P-P SCATTERING

Qualitative Effect of the Core

The introduction of a short-range singlet repulsion has the following effect on p-p scattering: When energies are reached comparable with or greater than the depth of the surrounding attractive well, the S wave will be affected less by the well than by the inner core, and the sign of the S phase shift will change from positive to negative in this energy region. States of higher angular momentum are, however, affected more by the outer or attractive region of the potential, and the corresponding phase shifts will remain positive until energies are reached which are greater than that at which the S phase changes sign. Thus, there will always exist an energy region in which the sign of the S phase is opposite to that of states of higher angular momentum. In this region the S-D interference term in (1b)

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9. The magnitude of the singlet well depth arises from the narrow range employed. Although large in comparison with the customarily quoted well depths for square wells without repulsion, the figure of 375 Mev appears to be more reasonable when compared with the singlet and triplet depths of 100 and 160 Mev, respectively, which occur when the exponential well without core is used.

will be positive at 90° and negative at 0°, tending to destroy the forward scattering and build up the scattering at 90°[10].

These effects are illustrated in Figure 2, where the singlet p-p cross sections are plotted for three sets of parameters in the neighborhood of those given in (2).

The dip in the singlet cross section at intermediate angles is a consequence of the vanishing of the Legendre polynomial, P_2, at 55°. Tensor scattering fills in this gap, resulting in the 340 Mev p-p cross section shown in Figure 3, curve II, for $r_o = 0.60 \times 10^{-13}$cm. Between 90° and 30° curve II agrees with the experimental values of 4.0 ± 0.6 millibarns[1]. At 20°, however, it has risen to 6.2 millibarns in disagreement with observation. An increase of this order of magnitude at small angles is characteristic of the interaction. It is possible that the rise may be diminished by the assumption of a more complicated potential of the same general type. However, in view of the neglect of non-static forces and relativistic effects an emphasis on precision of fit is probably unjustified in comparison with the importance of simplicity in the potential.

The central phases entering into the p-p cross sections were computed by numerical integration with the exception of $L = 2$ at 30 Mev and $L = 4$ at higher energies, where the Born approximation was permissible. Tensor scattering was computed in Born approximation with the exception of 30 Mev where tensor-coulomb interference was included[11].

The Scattering at 30 Mev

The comparison of the scattering experiments at 30 Mev with the

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[10] Only the S-D interference term is mentioned because singlet states with L greater than 2 do not make important contributions at the energies with which we are concerned.

[11] I am indebted to R. S. Christian and H. P. Noyes for communication of their results on the calculation of this term.

scattering expected from attractive well consistent with the low energy data has been made by Christian and Noyes[5], and we mention here only the most important points. The experiments indicate a cross section with a mean magnitude of approximately 15 millibarns on which is superposed a drop of 2 millibarns between 90° and 25°. On the other hand, the scattering from a Yukawa well consistent with the low energy data consists of S scattering of mean magnitude 15 millibarns on which is superposed a rise of approximately 2 millibarns having its origin in a large D phase shift. The magnitude of the D phase results from the long-tailed character of the Yukawa well[12].

An important effect of the hard core at 32 Mev lies in the diminution of the D phase shift. The origin of this decrease may be seen as follows: in general, the introduction of the hard sphere interaction increases the effective range of the potential[13], and one has, in fact, the approximate relation valid for large scattering length,

$$r_{eff} \sim 2(r_o + 2r_1) \tag{3}$$

where r_{eff} is the effective range, r_o the core radius and r_1 the range of the surrounding well. In order to keep the effective range at the value determined by low energy scattering it is necessary to decrease the range of the surrounding attractive well below its value without the core. The consequent contraction of the tail of the well reduces the phase shifts of all states with $L > 0$. With $r_o = 0.60 \times 10^{-13}$cm the range of the

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[12] The long-tailed exponential and Yukawa wells lead to D phase shifts of 1.2° and 1.4°, respectively, at 32 Mev. The square well is superior in this respect, predicting a D phase shift of only 0.6°.

[13] This may be seen from the useful formula of H. A. Bethe for the effective range (reference 3),

$$\tfrac{1}{2}r_o = \int_0^\infty (v_o^2 - u_o^2)\, r^2 dr$$

where u_o and v_o are the zero-energy wave function in the presence and absence, respectively, of the nuclear potential. It is clear that the introduction of a short-range repulsion, keeping the volume of the surrounding well constant, will decrease u_o in the region occupied by the repulsion, thereby increasing r_o.

exponential well must be decreased from 0.72 to 0.40 x 10^{-13}cm, the magnitude of the D phase shift being reduced at the same time from 1.2° to 0.6°.

The core also causes a reduction from 47° to 42° in the S phase shift at 30 Mev. The resultant drop in the mean magnitude of the 30 Mev cross section is made up by the tensor contribution.

The 30 Mev p-p cross section assuming a core radius of 0.60 x 10^{-13}cm is compared in Figure 4 with the data at 29.4 Mev[14].

Possible Variations in the Interaction Parameters

An initial freedom exists as to the choice of core radius. When the core radius has been selected the singlet well parameters in even states are fixed by the singlet scattering length and effective range, as determined from low energy n-p or p-p scattering. The triplet interaction in even states is determined by the deuteron constants and by the n-p triplet scattering parameters and must be identical with the Christian-Hart interaction in these states, once the possibility of a triplet repulsion of appreciable radius has been excluded. N-p scattering at high energies indicates that there is no appreciable central force in odd states, thus determining the amount of space exchange in the singlet and triplet wells[7]. The core is considered as a phenomenological manifestation of nucleonic structure and probably not subject to ordinary exchange effects. The strength of the tensor force in odd states is fixed by the requirement that at 340 Mev and 90° the sum of tensor and singlet contributions to σ_{pp} must equal the observed value of 4 millibarns. The form of radial dependence is somewhat arbitrary, but high energy n-p scattering apparently requires a long-tailed well[7], and it seems reasonable to require that this be the same for the central and tensor interactions.

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[14] W. K. H. Panofsky and F. Fillmore, UCRL Report 481.

Thus, on the assumption of a given radial dependence, the only important free parameter is the core radius. The effect of variations in the core radius on the singlet cross section at 340 Mev is shown in figure 2. The choice of $r_o = 0.60$ (curve II) represents the best compromise between the demand for a flat cross section at 340 Mev and for the correct mean magnitude at 30 Mev.

An increased core radius (curve I) results in an increase in the singlet cross section at 90° and below 40°. In order that the 90° cross section shall remain 4 millibarns, it is necessary to decrease the tensor strength. Consequently, the singlet dip in the neighborhood of 55° is less completely filled, while at the same time the singlet scattering below 40° has been increased, the net effect being the destruction of the desired isotropy.

For the same reason, a decrease in the core radius (curve III), and consequently in the 90° singlet cross section, requires an increase in tensor strength. The effect of the singlet dip at intermediate angles is thereby removed while at the same time the singlet forward scattering is reduced, resulting in a materially greater degree of isotropy. However, the necessary increase in tensor strength is accompanied by an excessive tensor contribution at lower energies, destroying the agreement with observation at 30 Mev. With $r_o = 0.50 \times 10^{-13}$cm the 30 Mev cross section is 17 millibarns at 90°, decreasing to 11.5 millibarns at 25°. (Compare with Figure 4.)

Energy Dependence of the 90° Cross Section

In the neighborhood of 150 Mev the S phase shift changes sign in consequence of the interference between positive and negative regions of the potential. The small magnitude of δ_o^s in this region results in the appearance of a minimum in the singlet cross section between 100 and 200 Mev, which is partially compensated by a rise in the tensor cross section. The variation of the resultant 90° cross section with energy is shown in Figure 5 for core radii of 0.50 and 0.60 x 10^{-13}cm. It is seen that the effect of the dip in the singlet cross section becomes more pronounced with increasing core radius.

The angular distributions at 100 Mev and 250 Mev are shown in Figure 4, with the effect of the Coulomb field included at the lower energy. One sees that the variation of the cross section with energy and with angle is surprisingly small between 100 Mev and 350 Mev.

IV. N-P SCATTERING

The principal contribution to the n-p cross section comes from the triplet states because of their statistical weight[15], hence the n-p cross section predicted by (2) may be expected to have the same general characteristics as the Christian-Hart cross section. Figure 6 compares the n-p cross sections calculated from (2) with the observed distributions[2].

The major effect on the n-p cross section of the changes introduced into the Christian-Hart interaction appears in the total cross sections, which are seen to be relatively larger at high energies (Table I). The experimental value at 40 Mev is rather uncertain. The angular distribution at 40 Mev in Figure 6 has been normalized to 170 millibarns as determined by Segrè, et al.[2], as well as to the value of 203 millibarns obtained by Leith, et al.[16b].

The effect of odd tensor forces has not been included in Figure 6. Calculations by Christian[17] indicate that the major effect of these forces on the angular distribution is a decrease of the order of 10° in the position

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[15] This is not true for σ_{pp} because of the exclusion of triplet even states by the Pauli principle.

[17] Private communication.

of the minimum in the cross section. Their contribution to the total cross section varies from 10 percent at 40 Mev to 25 percent at 260 Mev and has been included in the figures of Table I.

Figure 6 indicates that at 40 Mev the theoretical value of $\sigma(\pi)/\sigma(\pi/2)$ is 1.3 as compared to the experimental 1.55. The discrepancy can be removed by assuming a triplet well with a longer tail, ($\sigma(\pi)/\sigma(\pi/2)$ = 1.4 for the Yukawa well), but only at the expense of a further increase in the already excessive figure for the predicted total cross section, as may be seen from a comparison of columns B_1 and B_2 in Table I. It appears difficult to introduce a reasonable modification of the Christian-Hart triplet interaction which will at the same time lower the total cross section and increase the $\sigma(\pi)/\sigma(\pi/2)$ ratio. The disagreement is the more serious in that it is most pronounced at 40 Mev where little help may be expected from non-static forces.

V. IMPLICATIONS FOR SATURATION

On the assumption of a short-range repulsion it is clear that saturation will result, of the type existing in classical liquids. However, since the radius of repulsion assumed is smaller by a factor of four than the observed nucleonic spacing in the heavy nuclei, it appears to be questionable whether reasonable values of nuclear density and binding energy will result from (2). In this connection it is, however, important to note that the effect of the impenetrability interaction contained in (2) is greater than the classical liquid model leads one to expect, because of the quantum-mechanical zero-point energy resulting from the exclusion by the hard sphere interactions of the part of the volume available in the nucleus for each nucleon[18]. An analogy may be drawn with liquid helium,

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[18] The magnitude of the zero-point energy is reduced by the fact that the wave function of a system of nucleons is symmetric with respect to the interchange of only 3/8 of the relative coordinates.

where the particles are coupled by an interaction similar to (2) except for scale. The radius of repulsion in liquid helium is approximately 2 Å. and the mean spacing 3.8 Å., the difference in these figures having its origin in the zero point energy associated with the repulsion. Moreover, the radius of repulsion may be expected to be greater for the thermal collisions occurring in nuclei than for collisions at energies of several hundred Mev which were made the basis for setting the repulsive range at 0.60. In a potential of more realistic form than that used in these calculations the hard sphere may be replaced by a potential which crosses the axis with finite slope, as in Figure 1, and at a distance greater than the range of repulsion effective at higher energies.

VI. CONCLUSIONS

It is seen thus that the introduction of a short-range repulsive force permits one to reconcile the preservation of charge independence with the qualitative features of n-p and p-p scattering over the energy range thus far explored. This type of interaction may have, furthermore, desirable characteristics with respect to the saturation properties of the heavy nuclei. Several points must, however, be emphasized in considering the significance of these results:

i. The omission of non-static forces is not to be taken as implying that such forces are negligible, but only, as the results of this calculation indicate, that a rough description of the scattering in terms of static forces cannot be excluded by present data. Since there may be appreciable non-static contributions at 350 Mev, the quantitative implications of a static potential which has been fitted, in part, to a cross section at that energy must not be given undue weight.

ii. Other, basically different, types of forces may be used for the charge-independent representation of the n-p and p-p interactions. In addition to the singular spin-orbit coupling of Case and Pais[19], Christian and Noyes[5] have been able to account for p-p scattering by means of a singular tensor interaction which may be possibly incorporated into a charge-independent potential[20]. Each of these interactions possesses, in common with the short-range repulsive force, the characteristic of a strongly singular behavior at short distances. It may be said that the one general conclusion to be drawn from the high energy scattering experiments is that a strong singularity exists in the nucleon-nucleon interaction at small distances, and that on the assumption of charge independence one may show that the consequences of this singularity are partially masked in n-p scattering by the effect of the Pauli principle. It does not seem possible to obtain more definite information on the nature of this short-range force until non-static and relativistic effects are well enough understood to permit a quantitative comparison of the calculations with experiment.

VII. ACKNOWLEDGMENTS

I should like to express my thanks to Drs. K. M. Case, J. R. Oppenheimer and A. Pais for stimulating discussions and comments throughout the course of this work. The criticism of Mr. R. S. Christian and of Professors N. M. Kroll and R. Serber concerning the analysis of the calculations was extremely valuable. I am in addition grateful to Drs. E. M. Hart and P. Noyes and, in particular, to Mr. Christian, for their

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[19] The velocity-dependent spin-orbit interaction has been investigated by K. M. Case and A. Pais, Phys. Rev. (in press), who conclude that it is possible by means of a singular interaction of this type to represent qualitatively the n-p and p-p cross sections on a charge-independent basis. Some consequences of spin-orbit coupling have also been considered by C.H. Blanchard, R. Avery, and R.G. Sachs, Phys. Rev. 78, 292 (1950)

[20] N. M. Kroll, private communication.

cooperation in supplying information and results relating to tensor scattering which made possible a considerable abbreviation of the computations. I am indebted to the A.E.C. for a grant to the Institute for Advanced Study in support of this research.

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TABLE I

TOTAL N-P CROSS SECTIONS ($10^{-27}cm^2$)

	THEORETICAL			EXPERIMENTAL
Energy	A	B_1	B_2	
40	222	217	231	170 ± 15[16a] 203 ± 7[16b]
90	95	87	102	76 ± 10[16a] 83 ± 4[16c] 73 ± 2[16d] (95 Mev)
260	41	31	37	35 ± 9 [2]

A. Hard Core Potential (2)

B. Christian-Hart Potential:

1. Exponential
2. Yukawa

[16a] Hadley, Kelly, Leith, Segrè, Wiegand and York, Phys. Rev. 75, 351 (1949)

[16b] Hildebrand and Leith, Phys. Rev., (in press)

[16c] Cook, McMillan, Peterson and Sewell, Phys. Rev. 75, 7 (1949)

[16d] DeJuren and Knable, Phys. Rev. 77, 606 (1950)

FIGURE CAPTIONS

Figure 1. Composition of the potential from a repulsion constant in all states and a spin-dependent attractive well.

V_o = repulsion potential

V_t = triplet attraction

V^s = singlet attraction.

Figure 2. Singlet p-p cross section at 340 Mev, calculated from potential (2) for several core radii:

I. $r_o = 0.70$

II. $r_o = 0.60$

III. $r_o = 0.50$ (units of 10^{-13} cm).

Figure 3. Differential p-p cross section at 340 Mev calculated from potential (2) assuming

I. $r_o = 0.60 \times 10^{-13}$

II. $r_o = 0$.

Figure 4. Differential p-p cross sections at intermediate energies calculated from (2) with $r_o = 0.60 \times 10^{-13}$cm.

Figure 5. P-p cross section at 90° vs. energy, calculated from (2) for core radii of 0.50 and 0.60×10^{-13}cm.

Figure 6. Differential N-p cross sections calculated from (2) for a core radius of 0.60×10^{-13}cm. The experimental distributions are taken from reference 2.

△ 40 Mev distribution normalized to the total cross section measurement of Segrè, et al[16a)].

▲ 40 Mev distribution normalized to the total cross section measurement of Hildebrand and Leith[16b)].

× 90 Mev [16a)].

○ 260 Mev [16e)].

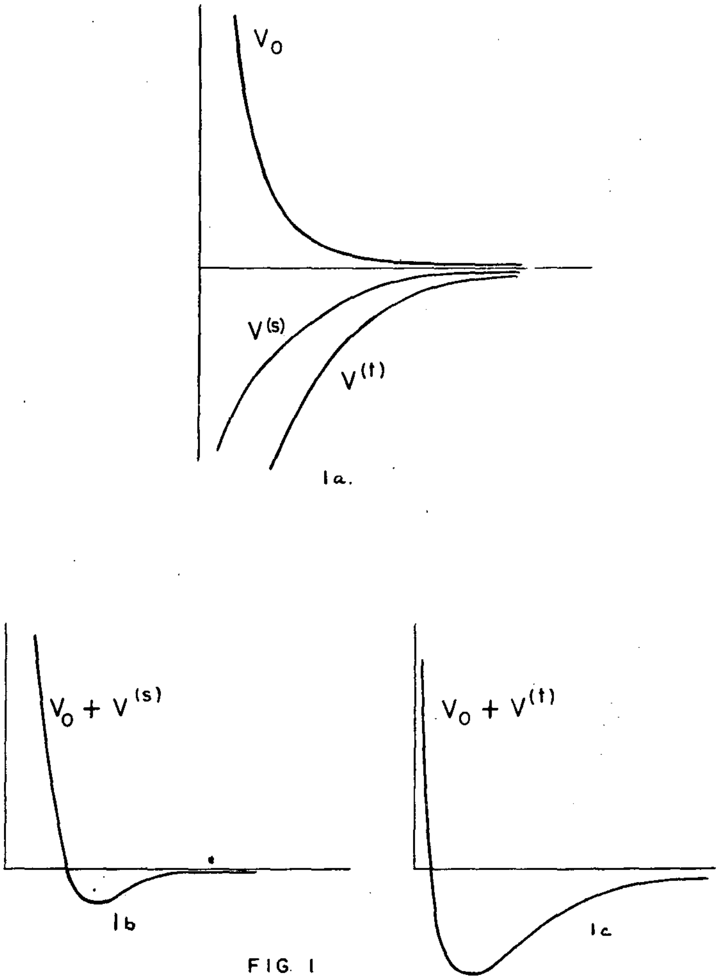
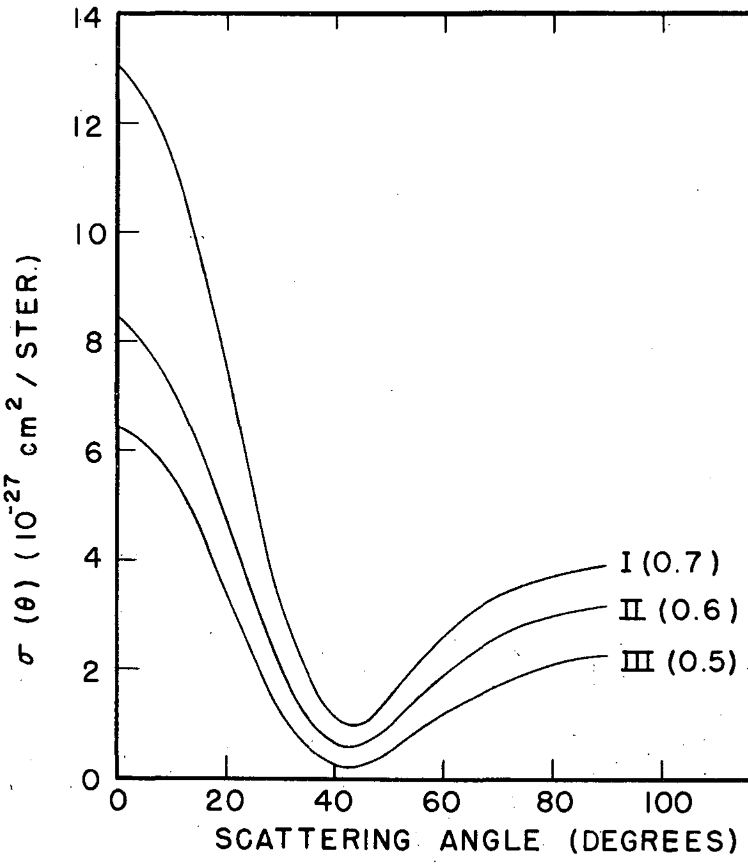


FIG. 1

MU 574


14
12
10
8
6
4
2
0
0
20
40
60
80
100
σ (θ) (10^{-27} cm^2 / STER.)
SCATTERING ANGLE (DEGREES)
I (0.7)
II (0.6)
III (0.5)

MU 573

FIG. 2

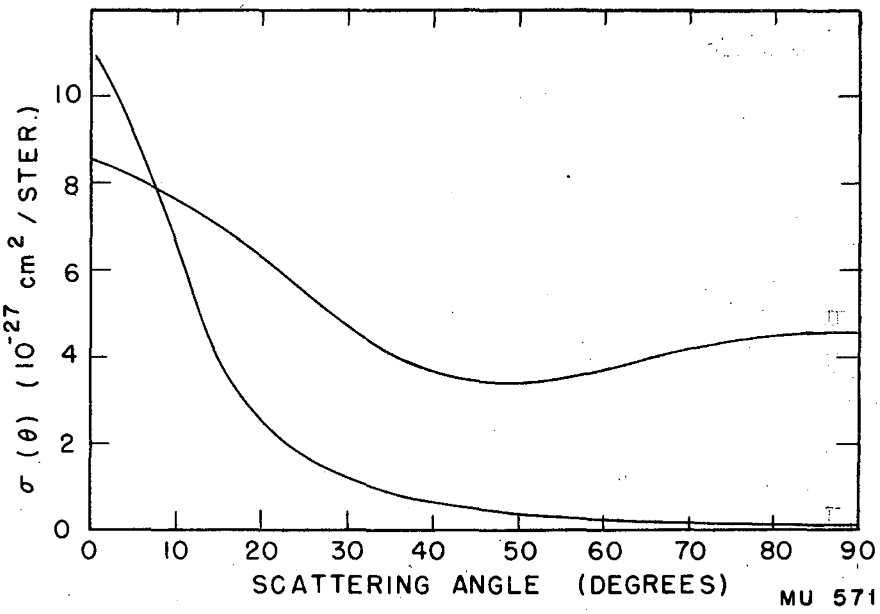
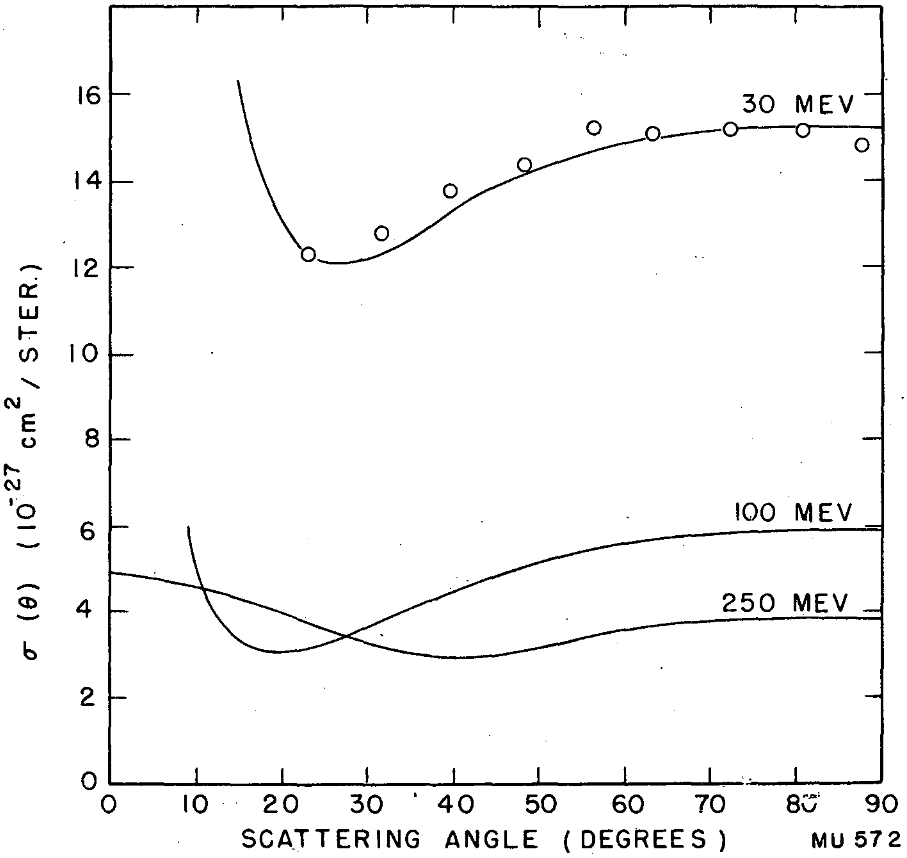
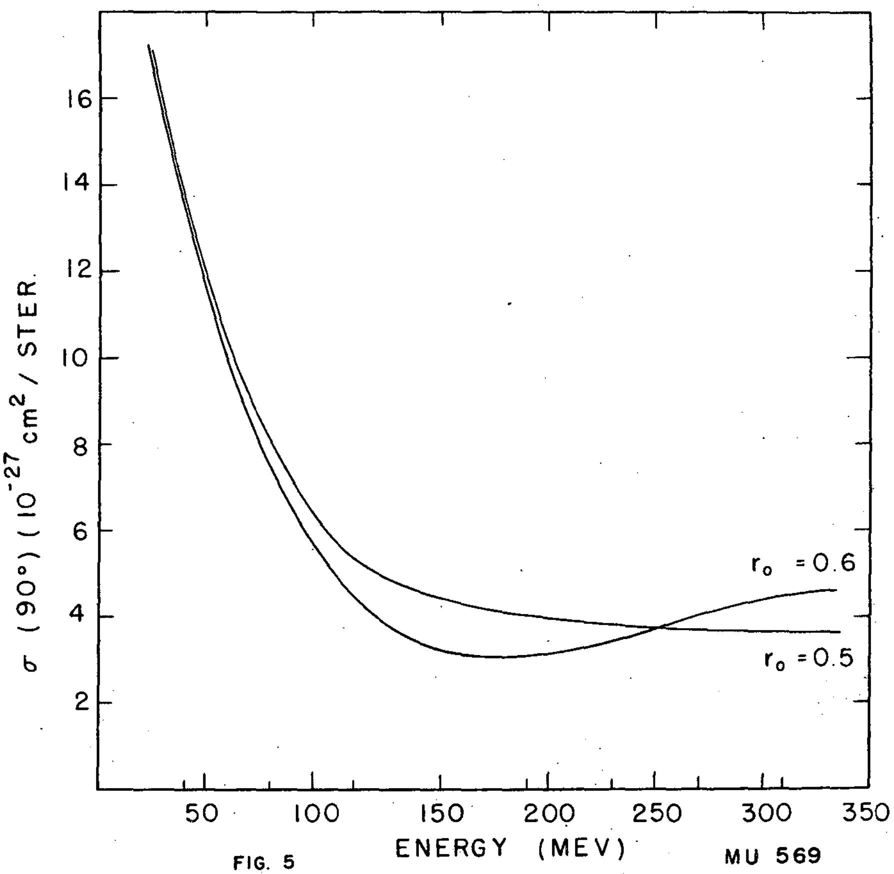

σ (θ) (10^-27 cm^2 / STER.)
0
2
4
6
8
10
0
10
20
30
40
50
60
70
80
90
SCATTERING ANGLE (DEGREES)
π
T
MU 571

FIG. 3



30 MEV
100 MEV
250 MEV
σ (θ) (10^{-27} cm^2 / STER.)
0
2
4
6
8
10
12
14
16
0
10
20
30
40
50
60
70
80
90
SCATTERING ANGLE (DEGREES)
MU 572

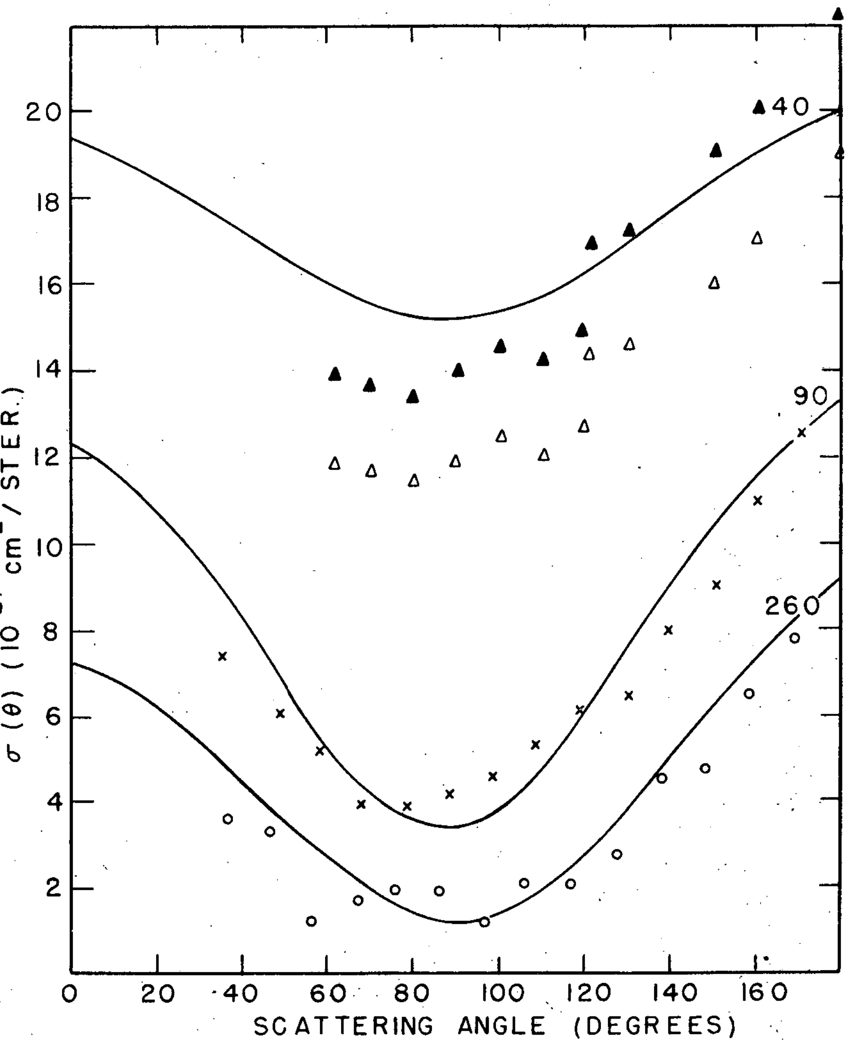
FIG. 4



σ (90°) (10⁻²⁷ cm² / STER.
16
14
12
10
8
6
4
2
50
100
150
200
250
300
350
ENERGY (MEV)
r₀ = 0.6
r₀ = 0.5

FIG. 5

MU 569



σ (θ) (10^-27 cm^2 / STER.)
20
18
16
14
12
10
8
6
4
2
0
20
40
60
80
100
120
140
160
SCATTERING ANGLE (DEGREES)
40
90
260

FIG. 6 MU 570