

## ON THE OHLIN LEMMA FOR HERMITE–HADAMARD–FEJÉR TYPE INEQUALITIES

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*Abstract.* Using Ohlin's Lemma [21] on convex stochastic ordering, we get a simple proof of known Hermite-Hadamard-Fejér type inequalities. We also prove new inequalities. Using  $s$ -convex stochastic ordering [12], we also give some Hermite-Hadamard-Fejér type inequalities in the case of higher order convex functions. The obtained results are useful in proving some inequalities between the quadrature operators [31], [32].

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