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On the Optimality of Single-Carrier Transmission in Large-Scale Antenna Systems

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Abstract—A single carrier transmission scheme is presented for the frequency selective multi-user (MU) multiple-input single-output (MISO) Gaussian Broadcast Channel (GBC) with a base station (BS) having M antennas and K single antenna users. The proposed transmission scheme has low complexity and for $M \gg K$ it is shown to achieve near optimal sum-rate performance at low transmit power to receiver noise power ratio. Additionally, the proposed transmission scheme results in an equalization-free receiver and does not require any MU resource allocation and associated control signaling overhead. Also, the sum-rate achieved by the proposed transmission scheme is shown to be independent of the channel power delay profile (PDP). In terms of power efficiency, the proposed transmission scheme also exhibits an $O(M)$ array power gain. Simulations are used to confirm analytical observations.

Index Terms—Single-Carrier Transmission, Large MIMO.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems have attracted significant research interest during the last decade due to various advantages they promise, both in single user [1] and multiuser channels [2]. It has been recently shown that the employment of an excess of antennas at the BS (very large MIMO) offers unprecedented array and multiplexing gains both in the uplink and in the downlink [3], [4]. The array gain offered by very large MIMO systems allows for power savings that scale as $1/M$ and $1/\sqrt{M}$ with perfect and imperfect channel state information (CSI) respectively, where M is the number of BS antennas [5]. The multiplexing gains offered by very large MIMO allows tens of users to be allocated the entire system bandwidth simultaneously. This eliminates to a large extent the need for resource allocation and the associated control signaling overhead. Since each user communicates over the whole system bandwidth, even low per user spectral efficiencies can result in very high per user throughput. In a MU-MISO GBC with K users and $M \gg K$ (very large MIMO), a low per user spectral efficiency implies an operating regime where the ratio of the total transmit power to the receiver additive noise power is small. Since MU interference at each receiver is proportional to the total transmit power, the additive noise dominates over MU interference and therefore even suboptimal precoding algorithms (like beamforming with the conjugate transpose of the channel gain matrix) have near optimal performance.

Previous results for very large MIMO systems have only considered frequency flat channels [3], [4], [5]. In this paper we consider a MU-MISO *frequency selective* GBC with $M \gg K$. For this channel OFDM (OFDMA) is an attractive transmission scheme as it facilitates scheduling in the frequency domain and simplifies receiver equalization. However, there is a substantial price to pay for this. OFDM comes at a loss in spectral and power efficiency owing to the insertion of cyclic prefix. Moreover, the signals resulting from OFDM modulation have a very large peak-to-average ratio, requiring the RF power amplifiers to work with a large power backoff and in an operating regime where they have low efficiency. For this reason, single-carrier or single-carrier-like modulation schemes like DFT-precoded OFDM are often used when there are stringent requirements on power efficiency of the RF amplifiers. Single-carrier signals have a much lower peak-to-average ratio and can be shaped to have constant envelope even in multiuser MIMO systems [6].

The contributions made in this paper are summarized as follows. 1) We firstly propose a low complexity single carrier transmission scheme for the frequency selective MU-MISO GBC. 2) At low total transmit power to receiver noise ratio, the proposed transmission scheme is shown to effectively suppress intersymbol interference (ISI) and MU interference at each receiver, thereby achieving near optimal sum-rate performance. 3) Additionally, the proposed scheme does not require any receiver equalization. Also, its simplicity allows for separate, decentralized computation at each BS antenna. 4) An achievable information sum-rate is derived for the proposed scheme. This sum-rate is further shown to be invariant of the channel PDP. 5) In terms of power efficiency, the proposed scheme is shown to exhibit an array power gain proportional to the number of BS antennas.

II. SYSTEM MODEL

A frequency selective MU-MISO downlink channel is considered, with M BS antennas and K single antenna users. The channel between the m-th transmit antenna and the k-th user is modeled as a finite impulse response (FIR) filter with L taps. The l-th channel tap is given by $\sqrt{d_l[k]}h_l^*[m,k]$, where $h_l^*[m,k]$ and $d_l[k]$ model the fast and slow varying components, respectively. In this paper we assume a model where $h_l^*[m,k]$ is fixed during the transmission of a block of N symbols and varies independently from one block to another. However, the slowly varying component (i.e. $\sqrt{d_l[k]}$) is assumed to be fixed throughout the entire communication. We further assume $h_l^*[m,k]$ to be i.i.d. $\mathcal{CN}(0,1)$ distributed. $d_l[k] \geq 0$, $l = 0, \ldots, L-1$ models the PDP of the frequency

selective channel for the k-th user.[1] Let $x_m[i]$ be the symbol transmitted from transmit antenna m at time i. The received signal at user k at time i is then given by

$$y_k[i] = \sum_{l=0}^{L-1} \sum_{m=1}^{M} \sqrt{d_l[k]} h_l^*[m,k] x_m[i-l] + n_k[i], \qquad (1)$$

where $n_k[i]$ is the $\mathcal{CN}(0,1)$ distributed AWGN at the k-th receiver at time i. Define $\boldsymbol{y}[i] \triangleq [y_1[i], \ldots, y_K[i]]^T \in \mathbb{C}^K$ to be the vector of received user symbols at time i. Similarly, let $\boldsymbol{x}[i] \triangleq [x_1[i], \ldots, x_M[i]]^T \in \mathbb{C}^M$ be the transmitted vector at time i. Let $\boldsymbol{n}[i] \triangleq [n_1[i], \ldots, n_K[i]]^T$, with independent components. The received signal vector at time i is given by $\boldsymbol{y}[i] = \sum_{l=0}^{L-1} \boldsymbol{D}_l^{1/2} \boldsymbol{H}_l^H \boldsymbol{x}[i-l] + \boldsymbol{n}[i]$, where $\boldsymbol{D}_l \triangleq \mathrm{diag}\{d_l[1], \ldots, d_l[K]\}$, and $\boldsymbol{H}_l \in \mathbb{C}^{M \times K}$ is a matrix whose (m,k)-th element is $h_l[m,k]$. Also the channel PDP for each user is normalized such that

$$\sum_{l=0}^{L-1} d_l[k] = 1, \ \forall k = 1, \ldots, K. \qquad (2)$$

The BS is assumed to have full CSI, whereas the users have knowledge of the channel statistics only.[2]

Let $s_k[i]$ denote the information symbol to be communicated to the k-th user at time i. The information symbol vector $\boldsymbol{s}[i] = [s_1[i], \ldots, s_K[i]]^T$ is considered to have i.i.d. $\mathcal{CN}(0,1)$ components, i.e. $\mathbb{E}\left[\boldsymbol{s}[i]\boldsymbol{s}^H[i+j]\right] = \boldsymbol{I}_K \delta_j$, $\mathbb{E}\left[\boldsymbol{s}[i]\boldsymbol{s}^T[i+j]\right] = \boldsymbol{0}$. In this paper we propose a precoding scheme, where the transmitted vector at time i is given by

$$\boldsymbol{x}[i] = \sqrt{\frac{\rho_f}{MK}} \sum_{l=0}^{L-1} \boldsymbol{H}_l \boldsymbol{D}_l^{1/2} \boldsymbol{s}[i+l], \qquad (3)$$

where $\rho_f \triangleq \mathbb{E}\left[\|\boldsymbol{x}[i]\|^2\right]$ is the long-term average total power radiated by the BS antennas. In the following, we derive an achievable sum-rate for the proposed precoder in (3).

III. ACHIEVABLE SUM-RATE

The bounding technique of [7], [8] is used here to obtain an achievable rate. In the following, a set of achievable rates is presented. For notational brevity we define $\boldsymbol{v}_l[k] \triangleq \boldsymbol{H}_l \boldsymbol{D}_l^{1/2} \boldsymbol{e}_k$, where $\boldsymbol{e}_k$ is the all-zero vector except for the k-th component which is equal to 1. Using (1) and (3) the signal received by user k at time i is given by

$$y_k[i] = \underbrace{\left(\sqrt{\frac{\rho_f}{MK}} \sum_{l=0}^{L-1} \mathbb{E}\left[\boldsymbol{v}_l^H[k]\boldsymbol{v}_l[k]\right]\right) s_k[i]}_{\text{Desired Signal Term}} + \underbrace{n_k'[i],}_{\text{Effective Noise Term}} \qquad (4)$$

where[3]

$$n_k'[i] \triangleq \underbrace{\sqrt{\frac{\rho_f}{MK}} \left(\sum_{l=0}^{L-1} \boldsymbol{v}_l^H[k]\boldsymbol{v}_l[k] - \sum_{l=0}^{L-1} \mathbb{E}\left[\boldsymbol{v}_l^H[k]\boldsymbol{v}_l[k]\right]\right) s_k[i]}_{\text{Additional Interference Term (IF)}}$$

---

[1]PDP determines the distribution of the received power across different channel taps.

[2]In a time division duplex (TDD) system, CSI at the BS can be acquired through uplink training and exploiting the uplink-downlink channel reciprocity.

[3]Following [7], [8], we have split the coefficient of the term $\sqrt{\frac{\rho_f}{MK}} \sum_{l=0}^{L-1} \boldsymbol{v}_l[k]^H \boldsymbol{v}_l[k] s_k[i]$ into a sum of its mean value (which is known to the receiver) and the deviation around its mean.

$$+ \underbrace{\sqrt{\frac{\rho_f}{MK}} \sum_{\substack{a=1-L \\ a \neq 0}}^{L-1} \sum_{l=\max(a,0)}^{\min(L-1+a,L-1)} \boldsymbol{v}_l^H[k]\boldsymbol{v}_{l-a}[k] s_k[i-a]}_{\text{Intersymbol Interference (ISI)}}$$

$$+ \underbrace{\sqrt{\frac{\rho_f}{MK}} \sum_{\substack{q=1 \\ q \neq k}}^{K} \sum_{a=1-L}^{L-1} \sum_{l=\max(a,0)}^{\min(L-1+a,L-1)} \boldsymbol{v}_l^H[k]\boldsymbol{v}_{l-a}[q] s_q[i-a]}_{\text{Multiuser Interference (MUI)}}$$

$$+ \underbrace{n_k[i]}_{\text{AWGN}} \qquad (5)$$

is the effective noise term. This term includes (i) the IF term which represents the variation of the desired signal around its mean, (ii) the ISI term between the current symbol of user k, i.e. $s_k[i]$, and the symbols intended to the same user at other time instances (i.e. $s_k[i+j]$, $j \neq 0$), (iii) the MUI term due to the information symbols intended for other users and, (iv) the AWGN term. In the proposed precoder, each user's codeword is long enough such that it spans across multiple coherence intervals. With long codewords, the effective variance of $n_k'[i]$ is no longer dependent on a particular channel realization but only depends on the channel statistics. From this it follows that the desired signal $s_k[i]$ is uncorrelated with the effective noise $n_k'[i]$, i.e. $\mathbb{E}\left[s_k[i]n_k'[i]\right] = 0$, where the expectation is taken over the channel realizations, the information symbols and additive noise. Therefore, with long codewords the channel is effectively an additive noise channel with the noise $n_k'[i]$ being non-Gaussian and uncorrelated to the information symbol $s_k[i]$. Further, the user has perfect knowledge of its channel statistic and therefore it knows the scaling factor $\sum_{l=0}^{L-1} \mathbb{E}\left[\boldsymbol{v}_l^H[k]\boldsymbol{v}_l[k]\right]$. Hence, an achievable information rate for the channel in (4) is given by considering the worst case uncorrelated additive noise having the same variance as $n_k'[i]$. Given that the data signal $\boldsymbol{s}[i]$ is Gaussian, the worst uncorrelated additive noise is circularly symmetric Gaussian distributed with the same variance as $n_k'[i]$. Therefore, the following information rate is achievable for the k-th user

$$R_k = \log_2\left(1 + S_k / \mathrm{Var}\left(n_k'[i]\right)\right) \qquad (6)$$

where $S_k = \mathbb{E}_{s_k[i]}\left[\left|\sqrt{\frac{\rho_f}{MK}} \sum_{l=0}^{L-1} \mathbb{E}\left[\boldsymbol{v}_l^H[k]\boldsymbol{v}_l[k]\right] s_k[i]\right|^2\right]$ is the average power of the desired signal term in (4) and $\mathrm{Var}\left(n_k'[i]\right) \triangleq \mathbb{E}\left[|n_k'[i] - \mathbb{E}\left[n_k'[i]\right]|^2\right]$.

Proposition 1: The variance of $n_k'[i]$ is invariant of any PDP that satisfies (2), and is given by

$$\mathrm{Var}\left(n_k'[i]\right) = \rho_f + 1. \qquad (7)$$

Proof: Using (5), the effective noise variance is given by

$$\mathrm{Var}\left(n_k'[i]\right) = \frac{\rho_f}{K} \sum_{q=1}^{K} \sum_{a=1}^{L-1} \sum_{l=a}^{L-1} \left(d_{l-a}[k]d_l[q] + d_l[k]d_{l-a}[q]\right)$$

$$+ \frac{\rho_f}{K} \sum_{q=1}^{K} \sum_{l=0}^{L-1} d_l[k]d_l[q] + 1, \qquad (8)$$

where the expectation is taken over the statistics of $\boldsymbol{H}_l$, $l = \{0, \ldots, L-1\}$, $\boldsymbol{s}[i+a]$, $a = \{1-L, \ldots, L-1\}$ and $n_k[i]$. Define $\boldsymbol{\Delta} \in \mathbb{R}^{K \times L}$ such that $[\boldsymbol{\Delta}]_{i,j} = d_{j-1}[i]$ and let $\boldsymbol{1} \in$

$\{1\}^{L \times L}$ denote the matrix with all entries equal to one. Then, (8) can be expressed as

$$
\begin{aligned}
\text{Var}\left(n_k'[i]\right) = & \frac{\rho_f}{K} \sum_{q=1}^{K} \sum_{a=1}^{L-1} \sum_{l=a}^{L-1} \boldsymbol{e}_k^T \boldsymbol{\Delta} \left(\boldsymbol{e}_{l-a+1} \boldsymbol{e}_{l+1}^T \right. \\
& \left. + \boldsymbol{e}_{l+1} \boldsymbol{e}_{l-a+1}^T \right) \boldsymbol{\Delta}^T \boldsymbol{e}_q + \frac{\rho_f}{K} \sum_{q=1}^{K} \boldsymbol{e}_k^T \boldsymbol{\Delta} \boldsymbol{\Delta}^T \boldsymbol{e}_q + 1 \\
= & \frac{\rho_f}{K} \sum_{q=1}^{K} \boldsymbol{e}_k^T \boldsymbol{\Delta} \boldsymbol{1} \boldsymbol{\Delta}^T \boldsymbol{e}_q + 1. \quad (9)
\end{aligned}
$$

From (2) it follows that $\boldsymbol{e}_k^T \boldsymbol{\Delta} \boldsymbol{1} = [1 \ \ldots \ 1]$. Using this fact in (9) completes the proof. ∎

It is apparent from (7) that the variance of the effective noise consists of the variance of the white noise term (which is 1) and the variance of the sum of interference terms (which is ρ_f). In the following we provide an explanation as to why the variance of the effective noise term is invariant of the PDP. Note that the precoder in (3) is like a matching pre-filter whose impulse response is a time reversed and complex-conjugated image of the channel impulse response (CIR). Due to this special structure of the proposed precoder, $n_k'[i]$ is composed of terms which consist of all non-zero auto-correlation lags of the CIR for the k-th user (ISI term in (5)), as well as all cross-correlation lags between the CIR of user k and the CIR of the remaining $(K-1)$ users (MUI term in (5)). The effective MUI in $y_k[i]$ from the symbols intended for the q-th user, depends only upon the total power in all channel correlation lags between the CIR's of the k-th and the q-th user. Due to the same channel and information symbol statistics for all users, the effective MUI in $y_k[i]$ from each of the remaining $(K-1)$ users is identical, and is independent of the individual PDPs (the total power in the cross-correlation lags depends only upon the total power in the CIR for each user, which is independent of k due to (2)).

Further, the useful signal term in $y_k[i]$ is proportional to the zero-lag auto-correlation of the CIR for the k-th user. This zero-lag auto-correlation (i.e. maximum gain combining of the lags) is proportional to the total channel power gain (combining all taps) from the M BS antennas to the k-th user and is therefore $O(M)$. The average power of the desired signal term in (4) is given by $\mathbb{E}_{s_k[i]}\left[\left|\sqrt{\frac{\rho_f}{MK}} \sum_{l=0}^{L-1} \mathbb{E}\left[\boldsymbol{v}_l^H[k]\boldsymbol{v}_l[k]\right] s_k[i]\right|^2\right] = \rho_f M/K.$ Using this fact and (7) in (6), the achievable rate R_k for user k is given by $R_k = \log_2\left(1 + \rho_f M/(K\rho_f + K)\right)$. The achievable sum-rate is therefore given by

$$
R_{\text{sum}}(\rho_f, M, K) = \sum_{k=1}^{K} R_k = K \log_2\left(1 + \frac{\rho_f M}{K\rho_f + K}\right). \quad (10)
$$

For the sake of comparison, we also consider a co-operative upper bound on the sum-capacity of the frequency selective GBC.[4] Essentially, we get an upper bound by considering the users to be co-operative, which reduces the MU channel to a single user MIMO channel, with perfect CSI at both the transmitter and the receiver. We further consider transmission

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[4]The sum-capacity of the MIMO GBC is known. However, for the results reported in this paper, it suffices to consider only the co-operative upper bound on the sum-capacity.

in time with large blocks (block size $\gg L$), where in each block the last few transmit vectors are zeros so as to avoid any inter-block interference. The sum-capacity for this single user MIMO block channel is given by beamforming along the right singular vectors of the effective channel matrix, thus transforming the channel into a set of parallel channels. Gaussian symbols are communicated over the parallel channels and power allocation is given by the waterfilling scheme.

With i.i.d complex normal entries in $\boldsymbol{H}_l$, it is clear that for fixed K, $\frac{\boldsymbol{H}_l^H \boldsymbol{H}_l}{M} \to \boldsymbol{I}_K$ as $M \to \infty$. Therefore for $M \gg K$, the K singular values of $\boldsymbol{H}_l$ are all roughly equal to $\sqrt{M}$ (i.e., the power gain for each parallel channel is $\approx M$). With a uniform power allocation of ρ_f/K across parallel channels, the co-operative upper bound on the ergodic sum-capacity of the GBC is given by

$$
C_{\text{coop}}(\rho_f, M, K) \approx K \log_2\left(1 + \rho_f M/K\right). \quad (11)
$$

We conclude our analysis with two propositions on the near-optimality and the array gain of the proposed precoder.

Proposition 2: When $\rho_f \ll 1$ and $M \gg K$, $R_{\text{sum}} \approx C_{\text{coop}}$ and the proposed precoder is near-optimal.

Proof: Observe that when $\rho_f \ll 1$, the effective noise variance, $\text{Var}(n_k') = \rho_f + 1 \approx 1$ (essentially the additive white noise dominates over the interference terms in (5)). It follows that, $K\rho_f + K \approx K$ and therefore the expressions in (10) and (11) are approximately equal. ∎

Proposition 3: The proposed precoder exhibits an $O(M)$ array power gain.

Proof: For the proposed precoder, using (10) the minimum transmit power ρ_f required to achieve a fixed desired sum-rate R_{sum} with K users and M BS antennas is given by $\rho_f(M) = \frac{K\left(2^{R_{\text{sum}}/K} - 1\right)}{M + K\left(2^{R_{\text{sum}}/K} - 1\right)}$. Since $\lim_{M \to \infty} \frac{1}{M} \frac{\rho_f(1)}{\rho_f(M)} = \frac{1}{1 + K\left(2^{R_{\text{sum}}/K} - 1\right)} > 0$ from [9] it follows that the proposed precoder achieves an $O(M)$ array power gain. ∎

This implies that for a sufficiently large M, $\rho_f(M)/\rho_f(1) \propto 1/M$ (i.e. the total transmitted power can be reduced linearly by increasing the number of BS antennas). A similar analysis of the co-operative sum-capacity (see (11)) reveals that the array power gain achieved by a sum-capacity achieving scheme is also $O(M)$.

IV. SIMULATION RESULTS

In the following, representative simulation results are presented, where the performance of proposed precoder is compared to the co-operative upper bound. Throughout the conducted simulations, the PDP is exponential with $L = 4$ and $d_l[k] = \frac{e^{-\theta_k l}}{\sum_{i=0}^{3} e^{-\theta_k i}}$, $l = \{0, \ldots, 3\}$, where $\theta_k = (k-1)/5$, $k = \{1, \ldots, K\}$. As proved in Proposition 1, the achievable sum-rate is invariant of the PDP. Hence, any other PDP which satisfies (2), would also yield the same results. Proposition 2 is supported by Fig. 1, where the sum-rate is plotted as a function of ρ_f, for $M = 50$ and $K = 10$. The sum-rate performance of the proposed precoder is given both by the theoretical expression (10) and via simulations. Similarly, the co-operative sum-capacity upper bound is calcu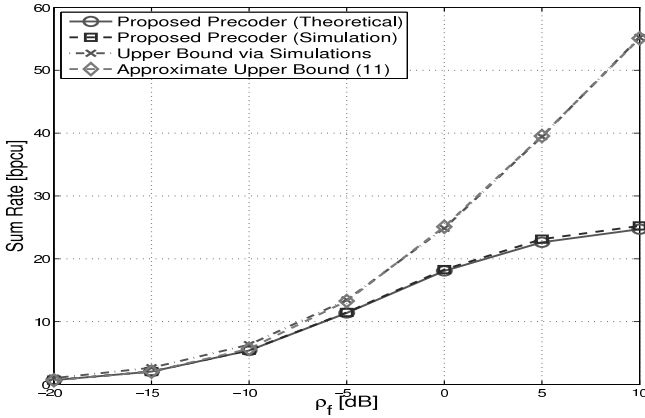lated via simulations and by the approximation in (11). For