On the Optimum Power Allocation in the One-Side Interference Channel with Relay

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Abstract—The optimum power allocation of the one-side interference channel with the non-cognitive relay node was studied. Assuming the orthogonal resources were used on the channels between the sources and relay node, we first derived a transmission scheme based on the dirty paper coding and the interference cancellation. Then with this transmission scheme the rates that was achievable in both the weak and strong interference regimes were given. A joint power allocation scheme among the sources and relay node was proposed, which maximized the sum-rate. The performance of the proposed power allocation scheme was proved. More explicit analysis investigated the effects of the non-cognitive feature of the relay node on the power allocation and sum-rate. The relationship between the channel gains and the optimum joint power allocation had also been analyzed.

I. Problem Formulation

In the context of the interference channel [1] [2] [3], the term *One-side* means that only one of the two transmission pairs suffers from the interference from the other one. In the study of the relationship between the coexisted indoor and outdoor transmissions, such as the one between the femto cell and the macro cell, due to the building's blockage, only the interference from the macro cell to the femto cell's user is considered and using the one-side model is straightforward. In the scenarios without the RN, the one-side feature of the interference channel was studied with the Z-interference channel (ZIC) model [4] [5].

When a relay node (RN) is introduced into the interference channel, the nodes form the interference channel with relay (ICR), where one RN assistants two transmission pairs simultaneously [6] [7]. The one-side ICR is a special case of the general ICR.

We focus on the effects of the RN on the achievable rate and the power allocation in the one-side ICR model. [6] [8] discussed the achievable rate and the capacity in the one-side ICR using the message cognitive RN and the signal cognitive RN, where the RN had the message or signal of sources noncausally respectively. Under the same scenario, [9] derived the diversity-multiplexing tradeoff (DMT) and investigated the effect of the channel gains on the DMT. These works brought some meaningful results on the transmission of the

one-side ICR where the source-to-RN channel and the RN-to-sink channel are used to transmit independent messages for the sources.

The use of the cognitive RN simplified the research as the RN's assistance is costless for the two transmission pairs and consequently, is always beneficial [10]. But it is difficult to deploy neither kind of cognitive RNs in the practical scenario and there is the power constrain at the source for sending any signal. These factors limit the significance of the known achievable rate. We analysis the one-side ICR using the non-cognitive RN. Therefore, the causality of the RN forwarded signal is considered in the transmission scheme. We study the transmission power at the source and the RN, then investigate the relationship between the sum-rate and the joint power allocation scheme.

Due to the plight of using the cognitive RN aforesaid, it is important to know whether and how the performance is affected by the change from using the cognitive RN to using the non-cognitive RN. We compare the derived sum-rate with the known best results provided in [8]. In order to have a more explicit understanding on this problem, how the power allocation scheme affects the sum-rate and how the optimum power allocation scheme changes with the different channel conditions are also investigated.

The rest of this paper is organized as follows: the channel model is given in Section II; in Section III the coding scheme and the achievable rate of the one-side ICR are derived, then the optimum joint power allocation scheme that maximizes the sum-rate is given; numerical results and discussion are presented in Section IV; the paper is concluded in Section V.

Throughout this paper, the expectation is denoted by $E\{\cdot\}$; the Frobenius norm is denoted by $\|\cdot\|$; $C(x)$ represents $log(1+x)$; $(x)^{-1}$ is the reciprocal of x.

II. Channel Model

The one-side ICR model is shown in Fig. 1: two transmission pairs, S_1 to D_1 and S_2 to D_2, transmit simultaneously and one RN forwards the signals received from S_1 and S_2. D_1 receives the signals from S_1, S_2 and the RN; D_2 receives the signal from S_2 only. Assuming the orthogonal resources are used on the source-to-RN channels, the RN receives signals from two sources without interference. The rest of
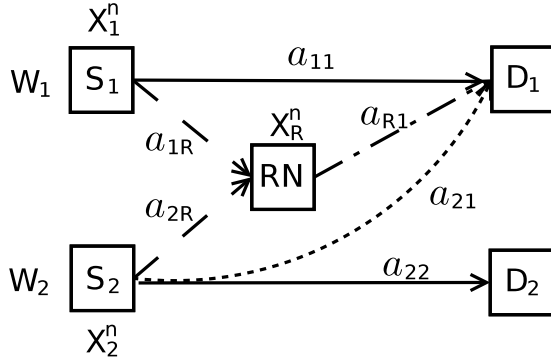
Fig. 1.   One-side ICR model

the transmissions are suffered from mutual interference. The source-to-RN channel and the RN-to-sink channel together are denoted as the relay channels in this paper.

The one-side ICR model is used to analysis the transmission in the cellular system where the two adjacent base stations(BS) suffer mutual interference and share one RN. And the signal from one BS to the user in the other cell and the signal from the RN to the same user aforementioned are both blocked by the building or severely attenuated by the large distance. This scene can be found between the macro cell and the femto cell, where the BS of the macro cell is built in the outdoor and the BS of the femto cell and the RN are both located in the building. Moreover, this model can also be used to study the cooperation between the multiple radio interfaces or standards (for example, between the Wi-Fi and the 3G cellular network) in the future networks, where the devices are powered with the access to the orthogonal resources.

In the one-side ICR model, W_i for $i = 1, 2$ is the message of S_i. The signal corresponding to W_i can be divided into two parts: $X_{ii}^n(W_i)$ is sent to D_i on the source-to-sink channel; $X_{iR}^n(W_i)$ is sent to the RN on the source-to-RN channel. The RN itself has no message to transmit and it works in a decode-and-forward manner. The message decoded from $X_{iR}^n(W_i)$ is W_{Ri}, which is then mapped into signal X_{Ri}^n. The RN forwarded signal is X_R^n, $X_R^n = X_{R1}^n + X_{R2}^n$ These signals satisfy the power constrains at sources and the RN respectively: $\frac{1}{n}E\{X_{ii}^2\} \leq P_{ii}$, $\frac{1}{n}E\{X_{iR}^2\} \leq P_{iR}$, $P_{ii} + P_{iR} = P_i$ and $\frac{1}{n}E(\|X_R^n\|^2) = \frac{1}{n}(E(\|X_{R1}^n\|^2) + E(\|X_{R2}^n\|^2)) \leq P_{R1} + P_{R2} = P_R$.

The channel parameters are assumed to be block fading, i.e. the channel parameters are random variables, but their values are fixed during the transmission of the n-length symbol period. a_{ij} for $ij = \{11, 21, 22, R1\}$ is the channel parameter and known to the sources and the RN.

At the time slot $t, t = 1, 2 \cdots$, the received signals are:

$$y_{1,t}^n = a_{11}x_{11,t}^n + a_{R1}x_{R,t}^n + a_{21}x_{22,t}^n + z_{1,t} \quad (1)$$
$$y_{1R,t}^n = a_{1R}x_{1R,t}^n + z_{R,t} \quad (2)$$
$$y_{2R,t}^n = a_{2R}x_{2R,t}^n + z_{R,t} \quad (3)$$
$$y_{2,t}^n = a_{22}x_{22,t}^n + z_{2,t} \quad (4)$$

where $Y_i^n = \{y_{i,t}^n\}$ and $Y_{iR}^n = \{y_{iR,t}^n\}$ for $i = 1, 2$ are the signals received at D_i and the RN respectively. $X_{11}^n = \{x_{11,t}^n\}$ and $X_{1R}^n = \{x_{1R,t}^n\}$ are the signals sent by S_1, $X_{22}^n = \{x_{22,t}^n\}$ and $X_{2R}^n = \{x_{2R,t}^n\}$ are the signals sent by S_2, $X_R^n = \{x_{R,t}^n\}$ is the signal forwarded by the RN. $z_{1,t}, z_{R,t}, z_{2,t}$ are the independent Gaussian noises, each is with variance N.

III. The Optimum Power Allocation

Since the RN has the orthogonal access to both two sources and S_2 is the only interferer, the signals from S_1 and S_2 can be transmitted with useful signal forwarding and the interference processing respectively. With this scheme, the achievable rates and the optimum joint power allocation are studied.

A. Useful Signal Forwarding

The message W_1 is divided into two parts. One part is sent to the RN by the signal $x_{1R,t}^n$ and then forwarded to D_1 by the signal $x_{R1,t+1}^n$. The other part is directly sent to D_1 by the signal $x_{11,t}^n$. $x_{R1,t+1}^n$ and $x_{11,t}^n$ are decoded at D_1 separately.

The message in $x_{1R,t-1}^n$ is forwarded by $x_{R1,t}^n$. Due to the causality, S_1 knows this part of the message in the RN forwarded signal. Therefore, S_1 can use the dirty paper coding (DPC) [11] to get $x_{11,t}^n$ decoded correctly without affecting the decoding of $x_{R1,t}^n$ at D_1.

B. Interference Processing

For $k = 1, 2, \cdots$, S_2 sends a message to the RN in the odd time slot $t_1 = 2k - 1$ and then S_2 sends the identical message to D_1 in the following even time slot $t_2 = 2k$. As a result, the RN can predict the interference signal received by D_1. RN uses this knowledge to generate its forwarding signal, which arrives at D_1 with the interference signal simultaneously and eliminates the interference.

C. The Achievable Rate with Interference Cancellation

Theorem 1: With orthogonal resources used on the source-to-RN channels, the following rates are achievable:

$$R_1 = R_{11} + R_{1R}$$
$$R_{11} \leq C\left(\frac{P_{11}\|a_{11}\|}{N}\right) \quad (5)$$
$$R_{1R} \leq \min\left\{C\left(\frac{P_{1R}\|a_{1R}\|}{N}\right), C\left(\frac{P_{R1}\|a_{R1}\|}{N}\right)\right\} \quad (6)$$
$$R_2 \leq \min\left\{C\left(\frac{P_{2R}\|a_{2R}\|}{N}\right), C\left(\frac{P_{22}\|a_{22}\|}{N}\right)\right\} \quad (7)$$

where $P_{11} = \alpha_1 P_1$, $P_{1R} = \alpha_2 P_1$, $P_{22} = \beta_1 P_2$, $P_{2R} = \beta_2 P_2$, $P_{R1} = \gamma_1 P_R$, $P_{R2} = \gamma_2 P_R$. For $j = 1, 2$, α_j, β_j and γ_j are the power allocation ratios at S_1, S_2 and the RN respectively. P_{22} and P_{R2} satisfy $P_{22}\|a_{21}\| = P_{R2}\|a_{R1}\|$.

Proof: Coding Scheme at the Sources: W_1 is divided into (W_{1R}, W_{11}), which is with rate pair (nR_{1R}, nR_{11}). W_{1R} and W_{11} are sent to D_1 on the relay channels and the source-to-sink channel respectively. W_{1R} is mapped into the n-dimensional codeword $X_{1R}^n(W_{1R})$. The $X_{1R}^n(W_{1R})$ is iid and obeys the distribution of $\mathcal{N}(0, P_{1R})$, i.e. the n elements of

$X_{1R}^n(W_{1R})$ are *iid* and each of them is Gaussian distributed with variance P_{1R}. The state information is denoted as

$$Q = a_{R1} x_{R1,t}^n \tag{8}$$

Since S_1 knows the message in the forwarded signal X_{R1}^n, Q is known at S_1. With this knowledge, the DPC scheme described in [11] is used: S_1 maps W_{11} into the codeword $X_{11}^n(W_{11})$ and $X_{11}^n(W_{11})$ is jointly typical with Q.

W_2 is mapped into $X_{2R}^n(W_2)$, which is *iid* and obeys the distribution of $\mathcal{N}(0, \min\{P_{22}, P_{2R}\})$.

At the end of the time slot t, S_1 sends $x_{11,t}^n$ to D_1, sends $x_{1R,t}^n$ to the RN. S_2 sends $x_{2R,t}^n$ to the RN and it also sends the copy of $x_{2R,t-1}^n$ as $x_{22,t}^n$ to D_2.

Forwarding and Decoding: In the time slot t, when $x_{1R,t}^n$ and $x_{2R,t}^n$ satisfy (6) and (7) respectively, the RN can get the messages $w_{1R,t}$ and $w_{2,t}$ correctly, i.e. the RN can decode the received signals with average error rate goes to zero, as $n \to \infty$.

Using the same codebooks used at S_1 and S_2, the RN maps the messages: $w_{1R,t-1}$ and $w_{2,t-1}$, into $x_{R1,t}^n$ and $\hat{x}_{R2,t}^n$ respectively. Hence, $x_{R1,t}^n = x_{1R,t-1}^n$ and $\hat{x}_{R2,t}^n = x_{2R,t-1}^n$. Using the knowledge of the channel parameters a_{R1} and a_{21}, the RN generates $x_{R2,t}^n = -a_{R1} a_{21}^{-1} \hat{x}_{R2,t}^n$ to cancel the interference from S_2 at D_1..

The signal $x_{R,t}^n = x_{R1,t}^n + x_{R2,t}^n$ and the interference are sent to D_1 at the end of the time slot t. When (7) holds, $x_{R2,t}^n$ is eliminated alone with interference . Hereafter, the received signal at D_1 is

$$y_{1,t}^n = a_{11} x_{11,t}^n + a_{R1} x_{R1,t}^n + z_t \tag{9}$$

Sequential decoding is applied at D_1. The process starts at decoding $x_{11,t}^n$. Since X_{R1}^n obeys the same distribution as X_{1R}^n, i.e. X_{R1}^n is *iid* and Gaussian distributed, Y_1^n in (9) and Q in (8) are both *iid* and Gaussian distributed. Following the jointly decoding procedure in [11], R_{11} satisfying (5) and R_{1R} satisfying (6) are achievable. ∎

Noticed that with the transmission scheme introduced above, the interference at D_1 can always be canceled by the RN forwarded signal and this precess is not affected by the strong or weak interference condition.

D. Optimum Joint Power Allocation Scheme

Using the transmission scheme introduced above, the optimum joint power allocation scheme that maximizes the sum-rate is given in the following proposition.

Proposition 1: the optimum joint power allocation scheme

that maximizes the sum-rate is

$$\begin{cases} \alpha_1 = \begin{cases} 0 & P_1 \leq \frac{\|a_{1R}\| - \|a_{11}\|}{\|a_{11}\|\|a_{1R}\|} \\ 1 & P_1 \leq \frac{\|a_{11}\| - \|a_{1R}\|}{\|a_{11}\|\|a_{1R}\|} \\ \frac{\|a_{11}\| - \|a_{1R}\| + P_1\|a_{11}\|\|a_{1R}\|}{2P_1\|a_{11}\|\|a_{1R}\|} & \text{else} \end{cases} \\ \alpha_2 = \begin{cases} 1 & P_1 \leq \frac{\|a_{1R}\| - \|a_{11}\|}{\|a_{11}\|\|a_{1R}\|} \\ 0 & P_1 \leq \frac{\|a_{11}\| - \|a_{1R}\|}{\|a_{11}\|\|a_{1R}\|} \\ \frac{\|a_{1R}\| - \|a_{11}\| + P_1\|a_{11}\|\|a_{1R}\|}{2P_1\|a_{11}\|\|a_{1R}\|} & \text{else} \end{cases} \\ \beta_1 = \frac{\|a_{2R}\|}{\|a_{2R}\| + \|a_{22}\|}, \quad \beta_2 = \frac{\|a_{22}\|}{\|a_{2R}\| + \|a_{22}\|} \\ \gamma_1 = \frac{\alpha_2 P_1\|a_{1R}\|}{P_R\|a_{R1}\|}, \quad \gamma_2 = \frac{P_2\|a_{21}\|\|a_{2R}\|}{P_R\|a_{R1}\|(\|a_{2R}\| + \|a_{22}\|)} \end{cases} \tag{10}$$

Proof: Theorem 1 implies that the strength of the received signal on the S_1-to-RN channel and the one on the RN-to-D_1 channel are the same; the strength of the received signal on the S_2-to-D_2 channel is as same as the one on the S_2-to-RN channel. As a result, the power allocation ratio γ_1, γ_2 and β_2 can be rewrote by α_1, α_2 and β_1 as follows:

$$\gamma_1 = \frac{\alpha_2 P_1\|a_{1R}\|}{P_R\|a_{R1}\|} \quad \gamma_2 = \frac{\beta_1 P_2\|a_{21}\|}{P_R\|a_{R1}\|} \quad \beta_2 = \frac{\beta_1\|a_{22}\|}{\|a_{2R}\|} \tag{11}$$

The optimization problem of finding the joint power allocation scheme that maximizes the sum-rate is:

$$\{\alpha_1, \alpha_2, \beta_1\} = \underset{\alpha_1, \alpha_2, \beta_1}{\arg\max}(R_1 + R_{22}) \tag{12}$$

$$\text{s.t. } \alpha_i, \beta_i, \gamma_i \in [0, 1] \qquad i = 1, 2$$

$$\alpha_1 + \alpha_2 \leq 1, \quad \beta_1 + \beta_2 = \beta_1 + \frac{\beta_1\|a_{22}\|}{\|a_{2R}\|} \leq 1$$

$$\gamma_1 + \gamma_2 = \frac{\alpha_2 P_1\|a_{1R}\| + \beta_1 P_2\|a_{21}\|}{P_R\|a_{R1}\|} \leq 1$$

where

$$R_1 = C\left(\frac{\alpha_1 P_1\|a_{11}\|}{N}\right)$$
$$+ \min\left\{C\left(\frac{\alpha_2 P_1\|a_{1R}\|}{N}\right), C\left(\frac{\gamma_1 P_R\|a_{R1}\|}{N}\right)\right\} \tag{13}$$

$$= C\left(\frac{\alpha_1 P_1\|a_{11}\|}{N}\right) + C\left(\frac{\alpha_2 P_1\|a_{1R}\|}{N}\right) \tag{14}$$

$$R_{22} = \min\left\{C\left(\frac{\beta_2 P_2\|a_{2R}\|}{N}\right), C\left(\frac{\beta_1 P_2\|a_{22}\|}{N}\right)\right\} \tag{15}$$

$$= C\left(\frac{\beta_1 P_2\|a_{22}\|}{N}\right) \tag{16}$$

Since R_1 and R_{22} are independent to each other, the maximization of the sum-rate can be achieved when each of them is maximized.

Because the transmission pair S_2 to D_2 is not interfered by the signals from S_1 and the RN, in order to achieve the higher R_1, it is intuitive to make S_1 use all of its power to transmit, i.e. $\alpha_1 + \alpha_2 = 1$. Then, let the derivative of (14) with respect to α_1 equals to zero, the optimum α_1, α_2 maximizing R_1 can be derived.

The transmission between S_2 and D_2 will cast interference at D_1. However, since all the interference will be canceled
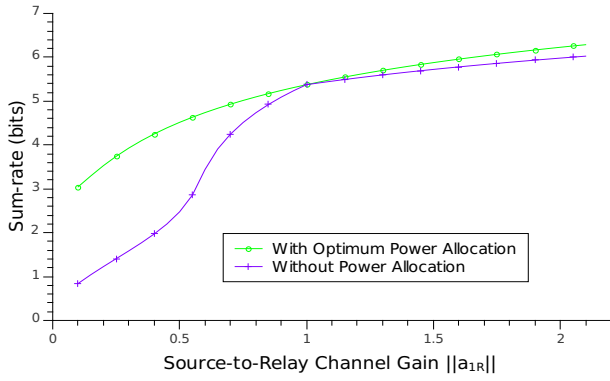
Fig. 2. Sum-rate with and without the optimum power allocation under different S_1-to-RN channel parameters.
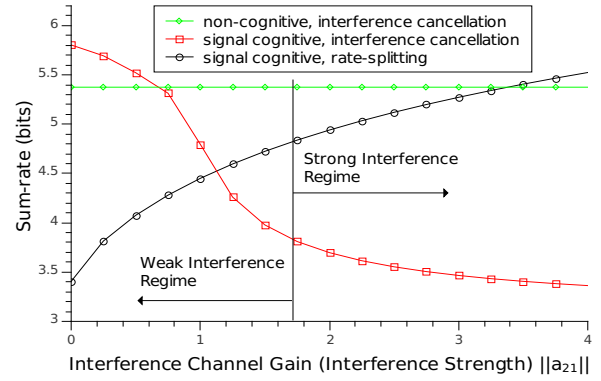

Fig. 3. sum-rate of one-side ICR with the non-cognitive RN and the signal cognitive RN when $\|a_{11}\| = \|a_{22}\| = 1$, $\|a_{1r}\| = \|a_{2r}\| = 1$, $\|a_{r1}\| = 0.5$, $P_1 = P_2 = 10$ and P_R changes as (18).

by the RN forwarded signal, S_2 can use all the transmission power, i.e. $\beta_1 + \beta_2 = 1$, without affecting the decoding at D_1. Moreover, since R_{22} monotonously increases with β_1 in (16), the optimum power allocation at S_2 is the solution of

$$\begin{cases} \beta_1 + \beta_2 = 1 \\ \beta_2 = \frac{\beta_1 \|a_{22}\|}{\|a_{2R}\|} \end{cases} \quad (17)$$

Thus: $\beta_1 = \frac{\|a_{2R}\|}{\|a_{2R}\| + \|a_{22}\|}$, $\beta_2 = \frac{\|a_{22}\|}{\|a_{2R}\| + \|a_{22}\|}$.

With this result, the optimum power allocation at the RN can be obtained from (11). ∎

Notice that in practice, the RN does not reserve any power during the transmission, so $\gamma_1 + \gamma_2 = 1$ always holds. Therefore, the supremum of the RN transmission power in the optimum joint power allocation scheme is derived as follows:

$$P_R = \frac{\alpha_2 P_1 \|a_{1R}\| (\|a_{2R}\| + \|a_{22}\|) + P_2 \|a_{21}\| \|a_{2R}\|}{\|a_{R1}\| (\|a_{2R}\| + \|a_{22}\|)} \quad (18)$$

IV. NUMERICAL RESULTS

First, the sum-rates with and without optimum joint power allocation are compared. The results show the advantages of using the derived joint power allocation scheme in maximizing the sum-rate. Furthermore, the derived sum-rate is compared with the maximum sum-rates of the one-side IC with the signal cognitive RN using interference cancellation and rate-splitting schemes respectively. The results indicate that the source-to-RN channel gain affects the joint power allocation scheme. Following this lead, the optimum joint power allocation with the different source-to-RN channel gains is investigated.

A. Sum-rate with/without the Optimum Joint Power Allocation

The sum-rate derived with the optimum joint power allocation scheme is compared with the sum-rate derived with the equal power allocation scheme. In the equal power allocation scheme, each source allocates its transmission power equally, the RN allocates its power adaptively to cancel the interference and uses the rest of the power to forward the signal. Let P_R satisfying (18) be the maximum power can be used by the RN in both cases. Since the change on the interference strength can be eliminated by the interference cancellation, the comparison

is shown with the respect of the source-to-RN channel gains $\|a_{1R}\|$ and $\|a_{2R}\|$.

The result is shown in Fig. 2 where: $\|a_{2R}\| = \|a_{1R}\|$, $\|a_{11}\| = \|a_{22}\| = 1$, $\|a_{21}\| = 1$, $\|a_{R1}\| = 1$. For simplicity, the transmission powers of the two sources are assumed to be $P_1 = P_2 = 10$, and the noise is assumed to be with unit variance, i.e. $N = 1$. The result shows that the proposed joint power allocation scheme maximizes the sum-rate and the maximum sum-rate varies with different channel parameters.

B. Sum-rate with Non-cognitive RN and Signal Cognitive RN

The effects of the RN's signal cognitive feature on the sum-rate is shown in Fig. 3. With the signal cognitive feature, the RN knows the signals of both sources non-causally and forwards them with different transmission power. After all the possible power allocation combinations have been enumerated, the one with the maximum sum-rate is chosen.

Consider the situation that in both the cases with signal cognitive RN and with non-cognitive RN, the interference cancellation are used. Knowing the signals on the sources non-causally, the signal cognitive RN cannot get the message more than the one sent to the sink directly on the source-to-sink channel. Consequently, the signal cognitive feature actually reduces the gain getting form the transmission on the source-to-RN channels and therefore, the maximum sum-rate is affected. When the interference becomes stronger, the source transmits more bits using optimum joint power allocation with non-cognitive RN (as shown in Fig. 3).

Then, consider the situation that in the case with signal cognitive RN, the rate-splitting scheme is used. The sum-rate achieved using rate-splitting scheme in [8] is shown in Fig. 3.

The sum-rate derived in this paper outperforms the one in [8] in the weak interference regime. This is because, with interference cancellation, D_1 does not need to decode the interference from S_2. So S_2 can send more bits to D_2 in the weak interference regime(as shown in Fig. 4). This can explain the gap between the two curves in the weak interference regime in Fig. 3.

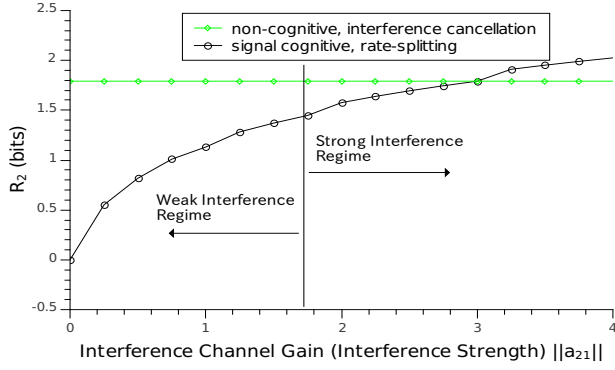Still with the rate-splitting case, Fig. 5 shows the derived

Fig. 4. R_2 achieved using rate-splitting with the signal cognitive RN and interference cancellation with the non-cognitive RN.
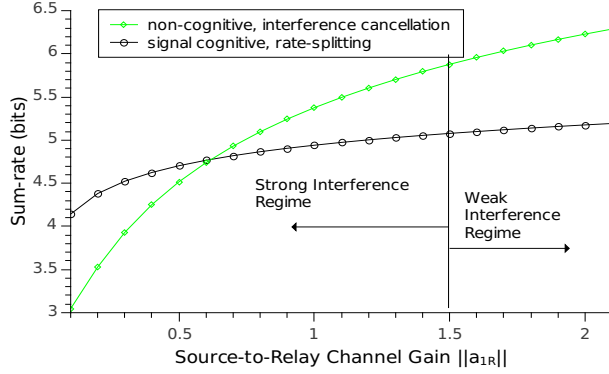

Fig. 5. Sum-rate changes with a_{1R} using interference cancellation scheme with signal cognitive RN and non-cognitive RN, when $\|a_{2R}\| = \|a_{1R}\|$, $\|a_{11}\| = \|a_{22}\| = 1$, $\|a_{21}\| = 2$, $P_1 = P_2 = 10$, P_R satisfies (18).

sum-rate and the sum-rate of the one-side IC with the signal cognitive RN using rate-splitting scheme with the respect of $\|a_{1R}\|$. The derived sum-rate increases when the source-to-RN channel gain, i.e. $\|a_{1R}\|$, becomes larger. And for the weak interference regime and some places in the strong interference regime, the derived sum-rate is outperformed. Notice that the best sum-rate result in the strong interference regime was achieved using the rate-splitting scheme [8], the result in Fig. 5 indicates that with the joint power allocation scheme and the non-cognitive RN, the sum rate benefits more from the transmission gain on the source-to-RN channel.

C. The Optimum Joint Power Allocation with Different $\|a_{1R}\|$

Extending the analysis above further, the change of optimum joint power allocation with the source-to-RN channel gains, i.e. $\|a_{1R}\|$ and $\|a_{2R}\|$, is studied in follows.

The optimum α_1 under different channel conditions are shown in Fig. 6. For convenience, the channel gains shown are given in logarithm. Assuming $\|a_{1R}\| = \|a_{2R}\| = \|a_{R1}\|$, when $\|a_{11}\|$ and $\|a_{1R}\|$ are both with low values, i.e. both of the source-to-sink channel between S_1 and D_1 and the RN-to-sink channel are in poor conditions, the amount of power allocated for the transmission on the source-to-sink channel and the transmission on the RN-to-sink channel are corresponded to
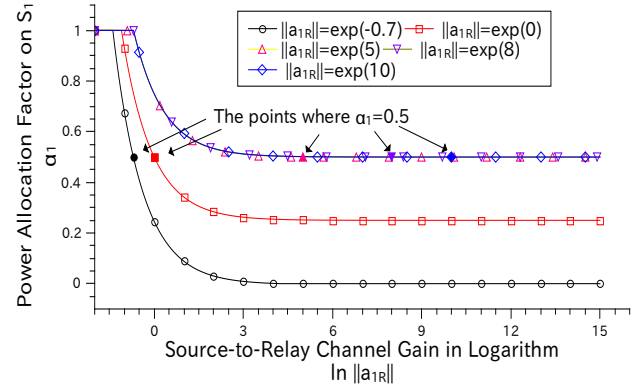

Fig. 6. Optimum α_1 with different $\|a_{11}\|$, $\|a_{1R}\|$, when $\|a_{22}\| = \|a_{11}\|$, $\|a_{2R}\| = \|a_{1R}\|$, $\|a_{21}\| = 0.3$, $\|a_{R1}\| = 0.5$, $P_1 = P_2 = 1$. P_R satisfies (18).

the values of $\|a_{11}\|$ and $\|a_{1R}\|$.

The points where $\alpha_1 = 0.5$ are marked in Fig. 6. When a_{11} and a_{1R} are both low, the ratio of power allocated for transmitting signal to the RN changes in the way described above. However, when both a_{11} and a_{1R} are large, the source only uses at most half of its power to transmit new messages through the RN-to-sink channel.

The explanation can be found in (10), where

$$\lim_{\|a_{11}\|, \|a_{1R}\| \to \infty} \frac{\|a_{11}\| - \|a_{1R}\| + P_1\|a_{11}\|\|a_{1R}\|}{2P_1\|a_{11}\|\|a_{1R}\|} = 0.5$$

Hence, when $\|a_{11}\|$ and $\|a_{1r}\|$ are large enough, for any $\|a_{1R}\| \geq \|a_{11}\|$, $\alpha_1 \to \frac{1}{2}$.

On the source-to-sink channel and the relay channels, the ability of improving the sum-rate with the transmission power can be measured by the power allocation at source [10]. So a conclusion can be drawn that when both the source-to-sink channel and the RN-to-sink channel of the one-side ICR have high transmission gain, the abilities of using the transmission power to improve the sum-rate on the two channels are the same. Consequently, on the source, the power allocation with no bias performs better.

V. CONCLUSION

The optimum power allocation in the one-side interference channel with the non-cognitive RN have been studied. Based on the analysis of the achievable rate region and transmission scheme, a joint power allocation scheme that can maximize the sum-rate in both weak and strong interference regimes has been proposed. We have proved that the performance of the proposed power allocation scheme is not affected by the interference condition. The results also indicate that the non-cognitive feature of the RN can help the joint power allocation scheme improve the sum-rate and promise a higher sum-rate than the best know result of the case with signal-cognitive RN. And when the source-to-sink channel gain and the source-to-RN channel gain are both very large, the optimum power allocation scheme for the source is to allocate at most half of its power to the transmission through the relay channels.

The studies of the joint power allocation scheme and the way that different factors affect the optimum results in this paper shed light in the research of the ICR and its practical applications. More effective transmission schemes as well as the outer bound of the rate region will be investigated with the consideration of power consumption in future works.

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