Exploration

On the Origin of Nuclear Charge Radius & Planck's Constant

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Abstract

In this paper the authors made an attempt to fit and understand the origin of the nuclear charge radius, the Planck's constant and the strong coupling constant by introducing the unified mass unit $M_C \cong \sqrt{e^2/4\pi\varepsilon_0 G}$ in a unified approach. This approach connects gravity and strong interaction via the electromagnetic and gravitational force ratio of proton and electron. Finally, by considering the proton rest energy the authors made an attempt to fit the semi-empirical mass formula energy coefficients in a very simple way.

Key Words: gravity, strong interaction, unification, nuclear charge radius, Planck's constant, strong coupling constant, semi-empirical mass formula.

1. Introduction

Even though Quantum mechanics and General theory of relativity both are having independent existence, strong mathematical back ground and good physical beauty, combining them is beyond the scope of current physics standards and demands sound knowledge on unknown and hidden things of atom and the universe. Even though 'dark energy' holds 70% of the unseen matter content of the universe, its role in understanding the basic concepts of unification is very insignificant. Even though SUSY is having excellent theoretical support and in-depth mathematical back ground, based on SUSY concepts so far no single SUSY boson could be detected in the Large Hadron Collider. This puzzling issue casts doubt on the continued existence of SUSY. In a nutshell, it is very clear that something is missing from our 'unification' knowledge net!

Missing knowledge can be obtained only through intellectual thinking, mathematical modeling, probing the atomic nucleus and universe to the possible extent, constructing semi empirical relations among physical constants of various interdisciplinary branches of physics with all possible interpretations and so on. Which way/method is the best - will be decided by future experiments, observations and interpretations. As it is interconnected with all branches of physics, 'semi empirical approach' seems be the easiest and shortcut way. It sharpens and guides human thinking ability in understanding the reality of unification. For any theoretical concept or mathematical model or semi empirical relation, 'workability' is more important than its inner beauty and 'workability' is the base of any semi empirical approach.

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By introducing a unified mass unit $M_C \cong \sqrt{e^2/4\pi\varepsilon_0 G}$, the authors made an attempt in this paper to fit and understand the origin of the nuclear charge radius and strong coupling constant in a unified approach that connects gravity and strong interaction via the electromagnetic and gravitational force ratio of proton and electron. Finally by considering the proton rest energy the authors made an attempt to fit the semi empirical mass formula energy coefficients in a very simple way.

2. Fitting the nuclear charge radius, the Planck's constant and the strong coupling constant

It is well known that *e*, *c* and *G* play a vital role in fundamental physics. With these 3 constants space-time curvature concepts at a charged particle surface can be studied. Similar to the Planck mass an interesting unified mass unit can be constructed in the following way.

$$\left(M_C\right)^{\pm} \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \cong 1.859272\times10^{-9}\text{ Kg} \cong 1.042975\times10^{18}\text{ GeV/c}^2 \tag{1}$$

Note that $M_C \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$ plays a crucial role microscopic physics as well cosmology [1]. With this mass unit in unification program with a suitable proportionality it may be possible to represent the characteristic mass of any elementary particle. It can be considered as the seed of galactic matter or galactic central black hole. It can also be considered as the seed of any cosmic structure. If 2 such oppositely charged particles annihilate, a large amount of energy can be released. It is well assumed that free space is a reservoir for pair particles creation. If so under certain extreme conditions at the vicinity of massive stars or black holes, a very high energy radiation can be seen to be emitted by the virtue of pair annihilation of M_C . Note that the basic concept of unification is to understand the origin of 'mass' of any particle. Mass is the basic property in 'gravitation' and 'charge' is the basic property in 'atomicity'. So far no model established a cohesive relation in between 'electric charge' and 'mass' of any 'elementary particle'. From astrophysics point of view the fundamental questions to be answered are: 1) Without charge, is there any independent existence to 'mass' of any star? 2) Is black hole – a neutral body or electrically a neutralized body? To understand these questions the authors made an attempt to construct the above unified mass unit.

The subject of final unification is having a long history. After the nucleus was discovered in 1908, it was clear that a new force was needed to overcome the electrostatic repulsion of the positively charged protons. Otherwise the nucleus could not exist. Moreover, the force had to be strong enough to squeeze the protons into a volume of size 10^{-15} meter. In general the word 'strong' is used since the strong interaction is the "strongest" of the four fundamental forces. Its observed strength is around 10^2 times that of the electromagnetic force, some 10^5 times as great as that of the weak force, and about 10^{39} times that of gravitation. The aim of unification is to understand the relation that connects 'gravity', 'mass', 'charge' and the 'microscopic space-time curvature'. Many scientists addressed this problem in different ways [2-4]. The authors also made many attempts in their previously published papers [5-10]. Experimentally observed nuclear charge radius [11-14] can be fitted with the following strange and simple unified relation.

$$R_c \cong \sqrt{\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)\cdot\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)}\cdot\left(\frac{2GM_C}{c^2}\right) \cong 1.252 \text{ fermi} \tag{2}$$

Here $\frac{2GM_C}{c^2}$ can be considered as the Schwarzschild radius [15,16] of the proposed of mass unit M_C.

$$\frac{R_c c^2}{2GM_C} \cong \sqrt{\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)\cdot\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)} \tag{3}$$

Let $$X = \frac{e^2}{4\pi\varepsilon_0 G m_p m_e} \quad \text{and} \quad \ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right) = \ln(X) \tag{4}$$

Whether the expression $\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right) \cong \ln(X)$ playing a 'key unified role' or 'only a fitting role' to be confirmed from the following relation. With a great accuracy the famous Planck's constant can be fitted with the following relation.

$$\begin{aligned} h &\cong \frac{1}{2}\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)\cdot\left(\sqrt{m_p m_e}\cdot c\cdot R_c\right) \cong \frac{1}{2}\ln(X)\cdot\left(\sqrt{m_p m_e}\cdot c\cdot R_c\right) \\ &\cong 6.63862\times 10^{-34} \text{ j.sec} \end{aligned} \tag{5}$$

Recommended value of h is $6.6260695729\times 10^{-34}$ j.sec and the error is 0.189%.

$$\begin{aligned} h &\cong \left[\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)\right]^{3/2}\left\{\sqrt{\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}}\left(\frac{GM_C\sqrt{m_p m_e}}{c}\right)\right\} \\ &\cong \left[\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)\right]^{3/2}\left(\frac{e^2}{4\pi\varepsilon_0 c}\right) \end{aligned} \tag{6}$$

Comparing this with the standard definition,

$$\left(\frac{2\pi}{\alpha}\right) \cong \frac{4\pi\varepsilon_0 hc}{e^2} \cong \left[\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)\right]^{3/2} \cong \left[\ln(X)\right]^{3/2} \tag{7}$$

where α is the fine structure ratio. Proceeding further qualitatively and quantitatively currently believed strong coupling constant [17] can be fitted with the following relation.

$$\alpha_s \cong \left[\sqrt{\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right)}-1\right]^{-1} \cong \left[\sqrt{\ln(X)}-1\right]^{-1} \cong 0.11738 \tag{8}$$

Its recommended value is 0.11847 and uncertainty is 5.9×10^6 ppb. Now the characteristic nuclear force can be expressed as follows.

$$\frac{e^2}{4\pi\varepsilon_0 R_c^2} \cong \frac{1}{4X\ln(X)}\left(\frac{c^4}{G}\right) \cong \frac{1}{4\ln(X)}\cdot\frac{4\pi\varepsilon_0\left(m_p c^2\right)\left(m_e c^2\right)}{e^2} \tag{9}$$

Note that $\left(c^4/G\right)$ can be considered as the limiting magnitude of any kind of force. Similarly $\left(c^5/G\right)$ can be considered as the limiting magnitude of any kind of power [18,19,20].

3. Fitting & correlating the semi empirical mass formula energy coefficients

In nuclear physics, the semi-empirical mass formula is used to approximate the mass and various other properties of an atomic nucleus. As the name suggests, it is based partly on theory and partly on empirical measurements [21-23]. The theory is based on the liquid drop model proposed by George Gamow, which can account for most of the terms in the formula and gives rough estimates for the values of the coefficients. It was first formulated in 1935 by German physicist Carl Friedrich von Weizsacker, and although refinements have been made to the coefficients over the years, the structure of the formula remains the same today. In the following formulae, let A be the total number of nucleons, Z the number of protons, and N the number of neutrons. The mass of an atomic nucleus is given by

$$m = Zm_p + Nm_n - \left(B/c^2\right) \tag{10}$$

where m_p and m_n are the rest mass of a proton and a neutron, respectively, and B is the binding energy of the nucleus. The semi-empirical mass formula states that the binding energy will take the following form.

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + \delta(A,Z) \tag{11}$$

Its modern representation is

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \tag{12}$$

Here a_v is the volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. By maximizing $B(A,Z)$ with respect to Z, one can find the number of protons Z of the stable nucleus of atomic weight A as,

$$Z \cong \frac{A}{2+(a_c/2a_a)A^{2/3}}. \tag{13}$$

This is roughly $A/2$ for light nuclei, but for heavy nuclei there is an even better agreement with nature. Now with the proposed number, $\ln\left(\frac{e^2}{4\pi\varepsilon_0 G m_p m_e}\right) \cong \ln(X)$ the SMEF nuclear binding energy coefficients can be fitted in the following way.

a) The coulombic energy coefficient can be expressed in the following way.

$$a_c \cong \frac{\alpha}{\sqrt{\ln(X)}}\left(m_p c^2\right) \cong 0.72 \text{ MeV} \tag{14}$$

where α is the fine structure ratio and $m_p c^2$ is the rest energy of proton.

b) The pairing energy coefficient can be expressed as

$$a_p \cong \left[\frac{\alpha}{\sqrt{\ln(X)}} + \frac{1}{\ln(X)}\right] m_p c^2 \cong 11.1 \text{ MeV} \tag{15}$$

c) The asymmetry energy coefficient can be expressed in the following way.

$$a_a \cong 2a_p \cong 22.2 \text{ MeV} \tag{16}$$

d) The surface energy coefficient can be expressed as

$$a_s \cong \left(a_a^2 a_p\right)^{1/3} \cong 17.62 \text{ MeV} \tag{17}$$

e) The volume energy coefficient can be expressed as

$$a_v \cong \left(a_a a_p\right)^{1/2} \cong 15.7 \text{ MeV} \tag{18}$$

$$\text{Thus} \quad a_v + a_s \cong a_p + a_a \cong 3a_p \tag{19}$$

See table-1 for the comparison of the semi empirical mass formula energy coefficients.
See table-2 for the calculated semi empirical mass formula nuclear binding energy.

Table-1: Existing and proposed SEMF binding energy coefficients

Existing energy coefficients	$a_v \cong 15.78$ MeV	$a_s \cong 18.34$ MeV	$a_c \cong 0.71$ MeV	$a_a \cong 23.21$ MeV	$a_p \cong 12.0$ MeV
Proposed energy coefficients	$a_v \cong 15.70$ MeV	$a_s \cong 17.62$ MeV	$a_c \cong 0.72$ MeV	$a_a \cong 22.2$ MeV	$a_p \cong 11.1$ MeV

Table-2: To fit the SEMF binding energy with the proposed energy coefficients

Z	A	$(BE)_{cal}$ **in MeV**	$(BE)_{meas}$ **in MeV**
26	56	494.2	492.254
34	84	730.1	727.341
50	118	1009.3	1004.950
60	142	1186.7	1185.145
79	197	1562.7	1559.40
82	208	1634.3	1636.44
92	238	1816.3	1801.693

Accuracy can be increased by considering Z^2 in the coulombic term as expressed below.

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \tag{20}$$

For light and heavy atoms (including super heavy stable isotopes), proton-nucleon stability relation can be expressed with the following semi empirical relation. Clearly speaking by considering Z its corresponding stable mass number can be estimated directly.

$$\frac{A_S}{2Z} \cong 1 + \left[2Z \left(\frac{a_c}{a_s + a_c} \right)^2 \right] \tag{21}$$

where A_S can be considered as the stable mass number of Z.

$$A_S \cong 2Z + \left(Z^2 * 0.00617 \right) \tag{22}$$

4. Discussion and Conclusions

The main object of unification is to understand the origin of elementary particles rest mass, magnetic moments and their forces. Right now and till today 'string theory' with 4 + 6 extra dimensions not in a position to explain the unification of gravitational and non-gravitational forces. More clearly speaking it is not in a position to bring down the Planck scale to the nuclear size. Note that general relativity does not throw any light on the 'mass generation' of charged particles. It only suggests that space-time is curved near the massive celestial objects. More overit couples the cosmic (dust) matter with geometry. But how matter/dust is created? Why and how elementary particle possesses both charge and mass? Such types of questions are not being discussed in the frame work of general relativity. The first step in unification is to understand the origin of the rest mass of a charged elementary particle. Second step is to understand the combined effects of its electromagnetic (or charged) and gravitational interactions. Third step is to understand its behavior with surroundings when it is created. Fourth step is to understand its behavior with cosmic space-time or other particles. Right from its birth to death, in all these steps the underlying fact is that whether it is a strongly interacting particle or weakly interacting particle, it is having some rest mass. To understand the first two steps somehow one can implement the gravitational constant in sub atomic physics. In this regard $M_C \cong \sqrt{e^2/4\pi\varepsilon_0 G}$ can be considered as the nature's given unified mass unit. To bring down the Planck mass scale to the observed elementary particles mass scale certainly a large scale factor is required. In this regard, the electromagnetic and gravitational force ratio of proton and electron can be considered as the nature's given universal scale factor.

Unification means: finding the similarities, finding the limiting physical constants, finding the key numbers, coupling the key physical constants, coupling the key physical concepts, coupling the key physical properties, minimizing the number of dimensions, minimizing the number of inputs and implementing the key physical constant or key number in different branches of physics. This is a very lengthy process. In all these cases observations, interpretations, experiments and imagination play a key role. The main difficulty is with interpretations and observations. At fundamental level understanding the observed new coincidences and confirming the observed coincidences seem to be a very tough job. Thinking positively the proposed relations for fitting the nuclear charge radius, the Planck's constant and strong the coupling constant can be considered for further analysis positively.

Acknowledgements: The first author is indebted to professor K. V. Krishna Murthy, Chairman, Institute of Scientific Research on Vedas (I-SERVE), Hyderabad, India and Shri K. V. R. S. Murthy, former scientist IICT (CSIR) Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

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