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ON THE ORIGIN OF SOLAR FLARES

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Summary

Two major problems in the interpretation of solar flares are discussed: firstly, by what means high concentrations of energy can be stored in the chromosphere; and secondly, what process can account for the suddenness with which this energy is released during a flare? It is shown that the energy storage can be accounted for only by a particular class of magnetic fields whose lines of force have the general shape of twisted loops protruding above the photosphere. In active regions with strong magnetic fields it is likely that chromospheric motions are entirely dictated by the flow of the much more massive photospheric material. Forces arising from pressure, weight or acceleration of chromospheric gas are most of the time small, so that the motions there are closely those that retain at each moment a force-free field compatible with the magnetic boundary conditions at the photosphere and, as the chromosphere is a good conductor, with the past history of the motion. A sudden release of energy in such a system can result only in a case where the magnetic forces act so as to drive the system away from the force-free configuration, and the energy associated with chromospheric currents can then be dissipated into motion or heat. This will occur when twisted magnetic loops of opposite sense and opposite twist meet. Such loops attract each other, and the annihilation of the longitudinal component of the field where they meet leads to a sudden constriction of the current and through this to a dissipation of the energy associated with that current.

1. *Introduction*.—Of the phenomena that happen in the solar atmosphere, flares are the ones connected with the greatest amounts of energy and with the greatest range of associated observable effects. An understanding of the basic physical processes underlying the flare phenomenon is therefore required for the interpretation of the great range of solar observations.

Attempts have been made to explain the phenomenon of solar flares in terms of electromagnetic processes. Early work by Giovanelli, by Hoyle, and by Dungey recognized the importance of neutral points and neutral lines of the magnetic field in this connection. Recent observational and theoretical work by Severny has gone far towards demonstrating the correctness of the magnetic interpretation in general and of the importance of the neutral points and lines in particular (1, 2, 3, 4).

Quite apart from the cogency of the direct observations reported by Severny, there is a compelling reason why a magnetic interpretation of flares is necessary. No other way has been found that can explain the concentration of energy in the flare region, or for its sudden transport there during the flare phenomenon itself.

The optical emission during an intense flare can be estimated to amount to more than 5×10^7 ergs/cm²/sec. (The chief uncertainty here is the degree of concentration of the source into filaments, which may be seen with a resolution that is possibly inadequate.) Thus for a flare with a duration of approximately 1000 seconds the total optical emission is of the order of 5×10^{10} ergs/cm^2. For an intense flare occupying an area of, say, 1 per cent of the solar disk, the total integrated emission amounts to 10^{31} ergs.

The total energy stored in the responsible regions of the solar atmosphere must thus amount to at least 5×10^{10} ergs for each cm^2 of the solar surface. If, further, we were to assume that flare emission takes place uniformly over a total interval of height as great as the entire height above the photosphere at which the phenomenon is known to occur, a height of the order of 10^8 cm, the required energy storage per cm^3 would amount to approximately 5×10^2 ergs. Although such an estimate is presumably not accurate in a strict sense, it suffices to show that the energy density in flares is much greater than the normal thermal energy density, which amounts to only approximately 10 ergs per cm^3 for material of density 10^{13} atoms per cm^3 at a temperature of 7000 °K.

The kinetic energy content of the gas in the chromosphere before a flare is also quite inadequate. This is most easily seen from the fact that Hα lines of very great width are seen in intense flares which, when interpreted as arising from mass motions of material, correspond to speeds of 80–250 km/sec. This is about 10 times greater than the speeds that are normally found in such regions of the chromosphere. The dynamical energy per unit mass released in flares therefore appears to be about 10^2 times the normal value. It may also be noted that the energy density, as estimated from optical considerations, in material with particle density of the order of 10^{13} atoms per cm^3, is similar to the kinetic energy density of such material moving at 100 km/sec, namely approximately 10^3 ergs per cm^3.

We are therefore forced to the conclusion that energy must be stored in the solar atmosphere in some way not involving thermal or mass motions, and that this energy storage must be built up before the flare can take place. The buildup must occur over prolonged periods of time, since any rapid method of transferring and concentrating energy would itself lead to visible effects.

If the energy is stored in the form of a magnetic field, then $H^2/8\pi$ must be of the order of 10^3 ergs per cm^3, and the magnetic field intensity must hence be a few hundred gauss.

In addition to solving the energy storage problem, a theory of flares must account for the extreme suddenness of the event. There are no other processes on a comparable scale occurring on the Sun that take place so quickly, and some form of catastrophic instability must therefore be involved. The requirements of the theory can therefore be stated quite definitely. Magnetic field configurations must be found that are capable of storing energy densities hundreds of times greater than occur in any other form, and that are stable most of the time. A situation that occurs only a small fraction of the time must be able to lead to instability in which this energy can rapidly be dissipated into heat and mass motion.

There are two possibilities concerning the suddenness of the phenomenon. Either it is in the nature of a breakdown, occurring when the stored energy exceeds a certain value, so that the process of charging up the region with more energy is itself the triggering mechanism. Or, alternatively, the onset of the instability is triggered by some circumstance which occurs only very rarely, so that the energy-storing process can usually proceed without hindrance. In this case it would be possible for the storage process to be a very common one, and a fertile ground for flares might hence exist over most active solar regions—but with the initiation of the discharge a sufficiently improbable process to limit flares to the observed rate of occurrence. Seeing that a great variety of complicated magnetohydrodynamical motions are known to occur over active regions, as observed by Hα cinematographic techniques, it is clear that in this second case a very special condition must be postulated as the requirement for the initiation of the discharge.

2. *Force-free fields.*—Strong fields that are to be present for prolonged periods in the chromosphere must be largely force-free, otherwise rapid motions with velocities of the order of $H(4\pi\rho)^{-1/2}$ where ρ is the mass density, would immediately be developed. This requirement would of course be met by fields that are due entirely to currents flowing in the massive photosphere below, but such fields are not of interest to us here, for there is no way in which this field energy can be released except in the regions where the currents responsible for it are flowing. We will thus have to be concerned here with force-free fields in which at least some of the currents that are flowing are situated in the chromosphere and therefore in the force-free part. Forces arising from the term $\mathbf{j}\times\mathbf{H}$ must be small in the chromosphere, not because $\mathbf{j}$ is absent, but because it does not have a significant component normal to $\mathbf{H}$.

The ultimate energy source for any large manifestations of free energy in the solar atmosphere must no doubt be sought in the hydrodynamics of the convective motion in and below the photosphere. We therefore wish to find a component of this turbulent motion, and a certain initial field configuration, which can result in a gradual buildup of chromospheric currents along magnetic lines of force. Whilst the observational evidence is not very definite in defining the length of time for which such a buildup may be taking place, it seems certain that a time much longer than the observed flare process is involved. No sudden processes are seen in the photosphere, and no mechanism could be suggested that would supply the energy from below the photosphere at the rate at which it is observed to be released. Also it is certain that flares tend to occur along arcs that are closely related to chromospheric patterns, and again for this reason a sudden photospheric supply, concurrent with the flare radiation, seems ruled out.

We are led to think then of a flare process in two separate stages. Firstly, a buildup of energy storage in the chromosphere by means of currents directed along magnetic lines of force. These currents must be generated through the interaction of the magnetic field with a component of photospheric turbulent motion. The second process is an explosive release of the stored energy into heat, bulk motion, emission line and possibly also continuum light, as well as high energy particles.

3. *Dissipative decay of currents.*—Before proceeding with the discussion of the flare mechanism, it is necessary to establish the circumstances in which dissipative decay of electric currents may be significant. It can be seen that force-free chromospheric currents, running parallel to the magnetic field, will generally decay in time scales that are very long compared with those arising from other considerations. In the force-free case the time scale for decay is of the order $\sigma L^2/c^2$, where σ is the conductivity parallel to $\mathbf{H}$, L is the characteristic dimension of the system under consideration, and c is the velocity of light. The value of

7*

σ at temperatures in the range from 5 000 °K to 10 000 °K can be taken as 10^{13} sec^{-1} independent of the density (5). Thus for $L \sim 10^8$ cm, the time scale for decay is 10^8 sec, which is long compared with the times that are normally of importance in chromospheric processes.

The situation is more complicated for a component of **j** normal to **H**, especially in the lower chromosphere where the conductivity is much decreased by the presence of neutral hydrogen, as was first pointed out by Piddington (6). An expression for the conductivity in directions normal to **H** has been given by Cowling (7) in a form that is convenient for the present discussion. When the great majority of the atoms are neutral, as is the case before the onset of a flare, the conductivity is less in directions normal to **H** than it is in the direction parallel to **H** by the factor,

$$1 + \frac{e^2H^2}{mc^2}[M\nu_i(\nu+\nu_e)+m\nu\nu_e]^{-1} \tag{1}$$

where the symbols have the following meanings:

e, m are the electron charge and mass respectively,
H is the magnetic intensity,
c is the velocity of light,
M is the proton mass,
ν is the collision frequency between electrons and protons,
ν_e is the collision frequency between electrons and neutral hydrogen atoms,
ν_i is the collision frequency between protons and neutral hydrogen atoms.

Formula (1) assumes that the ionization of hydrogen is more important than the ionization of other atoms that are present in the gas, such as the metals. If another atom, magnesium for example, were the most abundant positive ion, then it would be approximately correct to replace protons by magnesium ions in the above definitions.

Since the cross-sections for collisions of electrons and protons with neutral atoms are not greatly different, we have $\nu_e \cong (M/m)^{1/2}\nu_i$ (the factor M/m arises simply from the greater thermal speeds of the electrons), and hence

$$m\nu_e \cong (Mm)^{1/2}\nu_i \ll M\nu_i.$$

It follows therefore that (1) can be written to sufficient accuracy as

$$1 + \frac{e^2H^2(\nu+\nu_e)^{-1}}{mMc^2\nu_i} \tag{2}$$

The quantities ν, ν_e, ν_i can be determined from the following equations:

$$\begin{aligned} \nu &\cong \sigma n_i u_e, \\ \nu_e &\cong \sigma_e n u_e, \\ \nu_i &\cong \sigma_i n u_i, \end{aligned} \tag{3}$$

where

σ is the collision cross-section between electrons and protons,
σ_e is the collision cross-section between electrons and neutral atoms,
σ_i is the collision cross-section between protons and neutral atoms,
n_i is the ion-density,
n is the density of neutral atoms,
u_e is the mean thermal speed of the electrons,
u_i is the mean thermal speed of the protons.

It is a satisfactory approximation, both in the lower chromosphere and at the photosphere, to put

$$\begin{aligned} \sigma &\cong 3\times 10^{-12}\,\text{cm}^{-2}, \\ \sigma_e &\cong \sigma_i \cong 10^{-15}\,\text{cm}^{-2}, \\ u_e &\cong 5\times 10^{7}\,\text{cm sec}^{-1}, \\ u_i &\cong 10^{6}\,\text{cm sec}^{-1}, \end{aligned} \tag{4}$$

from which ν, ν_e, ν_i can immediately be estimated as soon as n, n_i are specified.

At the photosphere we may put $n\cong 10^{17}$ atoms cm^{-3}, $n_i\cong 10^{13}$ ions cm^{-3}. These values, together with (4), enable (2) to be expressed as $1+\sim 3\times 10^{-7}H^2$. This factor of reduction for the conductivity normal to **H** is evidently of order unity, even for field intensities as high as 10^3 gauss, and hence is of little importance at the photosphere.

The situation is very different in the regions of the chromosphere that are relevant to the flare problem, however. Here the ratio n_i/n is probably of order 10^{-2} under normal conditions ($n\cong 10^{13}$ atoms cm^{-3}, $n_i\cong 10^{11}$ atoms cm^{-3}), and must become considerably larger during flare conditions. This implies that ν always much exceeds ν_e, so that ν_e can be neglected in (2), which accordingly reduces to

$$1+\frac{e^2H^2}{mMc^2\nu\nu_i}. \tag{5}$$

With x defined by

$$n_i = xn$$

(5) becomes

$$1+\frac{10^{24}H^2}{n^2x}. \tag{6}$$

With $n\cong 10^{13}$ atoms cm^{-3}, $x\cong 10^{-2}$, $H^2\cong 10^4$ ergs cm^{-3}, the value of (6) is $\cong 10^4$. Currents normal to **H** decay faster than currents parallel to **H** by this factor. Remembering that we have already shown that the characteristic time scale for the decay of currents parallel to **H** is of order 10^8 sec (for a system with linear dimensions $\sim 10^8$ cm), it follows that the time scale for the decay of the current normal to **H** in such a system is $\sim 10^4$ sec.

The time scale for decay becomes still shorter for smaller values of n. At $n\cong 10^{12}$ atoms cm^{-3}, $x\cong 10^{-1}$, $H^2\cong 10^4$ ergs cm^{-3}, the time scale $\sim 10^3$ sec; and at $n\cong 10^{11}$ atoms cm^{-3}, $x\cong 1$, $H^2\cong 10^4$ ergs cm^{-3}, the time scale is $\sim 10^2$ sec. These values are evidently of an appropriate order to be of importance in the flare problem.

While a density of 10^{13} atoms/cm^3 may be a good estimate for the emitting material of a flare, such material has probably experienced some degree of compression (4). Hence we should think of the initial particle density as being somewhat lower during the phase that precedes the flare. A value of $n=10^{12}$ atoms/cm^3 might well be more appropriate in the initial situation.

4. *The energy storage mechanism.*—Observations of flares seen in the light of Hα against the solar disk show them frequently clearly aligned along a pre-existing filamentary pattern. Such filaments, when seen against the solar limb, are thought to be the commonly occurring arch structures. (Although these markings may sometimes represent high-level coronal prominences—which are known not to be directly related to flares—most of the patterns visible in Hα are undoubtedly related to arch structures in the chromosphere and lower corona.) Such filaments

should most probably be thought of as bundles of lines of force of a magnetic field that emerge from sub-photospheric regions at one root and re-enter at another root.

Some such firm anchoring in the massive photospheric material seems to be essential if field strengths of the order of 100 gauss are to be maintained in the chromosphere for long periods of time. This anchoring then makes possible a class of field configurations that are force-free above the photosphere, but not below it. Despite their high energy density, such fields will not cause an explosion above the photosphere, while below the photosphere the energy density will in no case exceed that of the local material.

In fact the energy of mass motion of the highly convective sub-photospheric material is of the order of 10^5 ergs/cm^3, which is greater than that required in chromospheric filaments by a factor of the order of 100, if these are to be the seat of the flare phenomenon. No energy difficulty will therefore be encountered if a process can be found whereby magnetic energy can be fed into the chromosphere as a consequence of events that take place at and below the photosphere. If this energy is to be dissipated in the chromosphere, it must be associated with currents at that level. Since the strong fields must be force-free in the chromosphere, these currents must flow along the lines of force. We may therefore ask: what are the photospheric motions that generate electric currents along the lines of force, in those cases where the lines of force stretch up into the chromosphere in the form of an arch?

In general an electric field will be established between the two ends of a magnetic arch by horizontal motions in the photospheric material. The integral of $\mathbf{E}.ds$ along a line of force joining the two bases of an arch evidently must not vanish. This condition will in general be satisfied by any non-uniform motion in the photosphere.

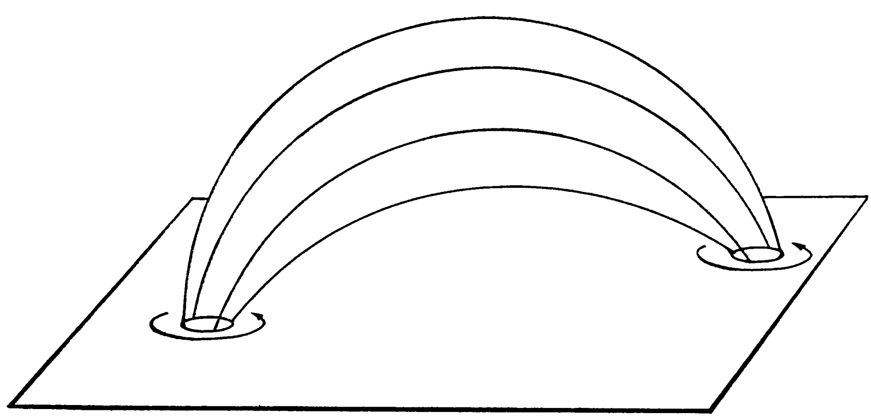

FIG. 1.—*Bundle of lines of force protruding from the photosphere into the chromosphere, with a twisting motion at the base.*

It is perhaps easiest to discuss this question in terms of the concept of the convection of lines of force with the flow (cf. Figs. 1 and 2). The generation of a current along the lines of force of an arch is equivalent to generating a field with the structure of a twisted arch. The non-uniform motion in the photosphere required to send the current along the arch is therefore the motion that will twist up the initial arch. A horizontal rotation in the photosphere around each base in the same clock-sense is therefore the basic type of motion that generates force-free, but not current-free, fields at a higher level.

It may be thought that the motion may not be exactly the one required for a force-free field, but of course it should be realized that, for fields of the strengths that we are concerned with, the chromospheric material will always immediately

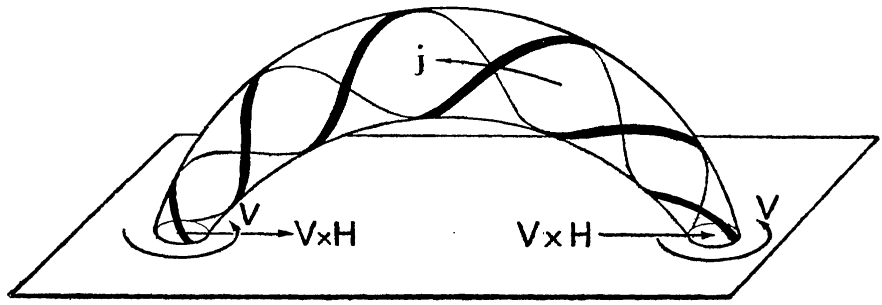

FIG. 2.—*Bundle of lines of force after a twisting motion from the base has generated a helical shape. This is equivalent to considering the establishment of a current, j, along the lines of force, due to the $V \times H$ terms arising at the base.*

move in a direction in which any force is applied. As photospheric motions cause an initially untwisted arch to become twisted, the arch moves at all stages into a force-free configuration.

This in turn means that we are concerned with currents flowing in the chromosphere along the magnetic lines of force. Hence the currents do not suffer the greatly enhanced rate of decay that would occur for chromospheric currents flowing in other directions. For currents along the lines of force the decay time constants are long enough to make a discussion in terms of the convection of lines of force quite applicable for the speeds and dimensions that are in fact observed.

A torque acts across planes normal to the axis of such a twisted filament. It is important to realize that this is the case not only at the photosphere, but also over the force-free chromospheric portions of the arch (the force-free condition does not require the individual components of the stress tensor p_{ik} to vanish, only its divergence

$$\left. \sum_{k=1}^{3} \frac{\partial p_{ik}}{\partial x_k} \right) .$$

For the purpose of estimating this torque we anticipate a result obtained in the next section; namely that the number of turns of the lines of force about the filament axis, N say, is related to the length of the filament, l say, and to the characteristic dimension d of its cross-sections, by

$$N \sim l/d. \tag{7}$$

This requires the stress acting per unit area on a normal plane to have an appreciable component about the axis of the filament. The couple about the axis accordingly has an average value, of order $H^2d/8\pi$ per unit area, and a value of order $H^2d^3/8\pi$ for the whole cross-section (this estimate will be confirmed by a more precise calculation in the next section).

The same torque acts across all normal planes of the force-free portion of the filament. The torque need not be exactly the same, however, at the non-force-free roots of the filament, because angular momentum about the axis may be in the process of being communicated to the material in the lower parts of the filament. But since the twisting is impressed on the filament at its roots, the torque at the roots can differ only in the sense of being greater than the torque in the force-free part of the filament, i.e. greater than $H^2d^3/8\pi$. Thus in using the latter formula we shall not be underestimating the torque at the roots.

The work done in N relative rotations at the roots is therefore not less in order of magnitude than

$$(2\pi N).(H^2d^3/8\pi).$$

Now we have seen in Section 1 that $H^2/8\pi \simeq 10^3\,\mathrm{ergs\,cm^{-3}}$, so that with $d \sim 2\times 10^8$ cm for the cross-sectional dimension of the filament, the work done is at least of order $5\times 10^{28} N$ ergs. The relation (7) requires N to lie between 10 and 10^2 for a typical ratio of filament length to diameter, so that the total energy storage is not less than 10^{30} ergs, which is of the correct order for a normal flare.

These considerations therefore demonstrate the effectiveness of twisting as a mode of energy storage. A precise assessment of numerical values should be deferred, however, until the end of the next section, when the calculation will be repeated with greater accuracy.

5. *The structure of a force-free twisted filament.*—We shall suppose the curvature of the axis of the filament to be sufficiently small for a limited segment of it to be considered straight. We also make the following assumptions:

(i) all points of a line of force are at the same distance from the axis,
(ii) all lines of force have the same number of turns about the axis,
(iii) the field does not depend on the azimuthal coordinate about the axis, nor on the distance coordinate parallel to the axis, at any rate in first approximation.

Assumption (ii) implies that the angular velocity of twisting is uniform over the roots of the filament. The most questionable of these simplifications is that of axial symmetry in (iii), since in an actual filament the horizontal width is likely to exceed the vertical depth.

We use symbols with the following meanings:

r, θ, z are cylindrical coordinates with respect to the axis,
ϕ is the angle that a line of force makes with the planes normal to the axis,
ν is the number of turns per unit length of a line of force.

Because of (i), the radial component of the field vanishes. Because of (iii), ϕ, ν and the components H_θ, H_z of the field are all independent of θ and z. Because of (ii), and the definitions of ν and ϕ, it follows that

$$\cot\phi = 2\pi\nu r. \tag{8}$$

The force-free condition

$$\mathbf{H}\times\mathrm{curl}\,\mathbf{H} = 0 \tag{9}$$

yields θ and z components that are identically satisfied, while the r-component of (9) gives

$$\frac{d\log H}{dr} = -\frac{\cos^2\phi}{r}, \quad H = |\mathbf{H}|. \tag{10}$$

It is most important in relation to the later discussion of the stability of the flare phenomenon to consider whether a filament satisfying the above conditions can be regarded as an isolated structure. To this end, we first consider the possibility of satisfying (9) for $r \leqslant a$, together with

$$\mathbf{H} = 0, \quad r > a. \tag{11}$$

It will be useful to write F for the flux through a normal section of the filament,

$$F = 2\pi \int_0^a H_z r \, dr. \tag{12}$$

The importance of F lies in the fact that it is independent of the twisting of the filament. It also turns out that F can take the place of the constant that appears in the integration of (10), as will soon be seen.

When ϕ is eliminated between (8) and (10), the resulting equation yields

$$H = A/\sqrt{1 + 4\pi^2\nu^2 r^2}, \tag{13}$$

where A is an integration constant. Substituting $H_z = H \sin\phi$ in (12), with $\sin\phi$ determined from (8), also gives

$$A = \frac{qF}{\pi a^2 \log(1+q)}, \quad q = 4\pi^2\nu^2 a^2. \tag{14}$$

The results (13), (14), together with $H_\theta = H\cos\phi$, $H_z = H\sin\phi$, lead immediately to

$$\begin{aligned} H &= \frac{Fq}{\pi a^2 \log(1+q)} \frac{1}{(1+qr^2/a^2)^{1/2}}, \\ H_\theta &= \frac{Fq}{\pi a^2 \log(1+q)} \frac{2\pi\nu r}{1+qr^2/a^2}, \\ H_z &= \frac{Fq}{\pi a^2 \log(1+q)} \frac{1}{1+qr^2/a^2}. \end{aligned} \tag{15}$$

We also note for later reference that the total magnetic energy per unit length is given by

$$\tfrac{1}{4}\int_0^a H^2 r \, dr = \tfrac{1}{2}\frac{F^2\nu^2}{\log(1+q)}. \tag{16}$$

The energy per unit length of the axial component is given by

$$\tfrac{1}{4}\int_0^a H_\theta^2 r \, dr = \tfrac{1}{2}\frac{F^2\nu^2}{\log(1+q)}\left[1 - \frac{q}{(1+q)}\cdot\frac{1}{\log(1+q)}\right] \tag{17}$$

while the energy per unit length of the longitudinal component follows immediately on subtracting (17) from (16),

$$\tfrac{1}{4}\int_0^a H_z^2 r \, dr = \tfrac{1}{2}\frac{F^2\nu^2 q}{(1+q)[\log(1+q)]^2}. \tag{18}$$

Manifestly the field does not fall to zero at $r = a$, as had been hoped, in order that (11) might be satisfied. For $r \to \infty$, H declines to zero as $1/r$, H_z as $1/r^2$, H_θ as $1/r$. On the axis $H_\theta = 0$. Hence we have an essentially longitudinal field near the axis giving place to an axial field at large r.

The tendency of the axial field to pinch is resisted by the longitudinal field, and this situation appears stable against a purely radial perturbation. Suppose a, r to be increased by some scale factor τ. Then while H_z decreases as τ^{-2}, H_θ only decreases as τ^{-1}. The pinch thereby becomes the stronger, so that the filament is compressed towards the axis. An opposite situation obviously holds for a decrease of a, r.

The above considerations show that in order to confine the force-free field within a cylinder of radius a, we must surround the filament by a shielding current. Such a shielding current can be thought of as flowing in a boundary sheath outside the cylinder of radius a. Plainly this boundary region cannot be force-free in the electromagnetic sense. Indeed, while the magnetic stress on the outer surface is zero, there is a magnetic pressure

$$\frac{1}{8\pi}(H^2)_{r=a}=\frac{1}{8\pi^3}\frac{F^2q^2}{a^4(1+q)}\cdot\frac{1}{[\log(1+q)]^2} \tag{19}$$

on its inner surface. It appears therefore that the filament cannot be an isolated structure unless the electromagnetic pressure (19) is compensated by an excess material pressure on the outer surface of the boundary zone. The next step is to examine the implications of this requirement.

By suitable choices of ν, F we can arrange that

$$\frac{1}{8\pi}(H^2)_{r=0}\simeq\frac{10^2}{8\pi}(H^2)_{r=a}\simeq 10^3\,\text{ergs cm}^{-3}. \tag{20}$$

We then have a situation giving a storage of magnetic energy near the axis of the filament that is adequate for a flare but which falls at $r=a$ to a value comparable with the thermal energy density of surrounding chromospheric material (cf. Section 1 for numerical values). In these circumstances the filament can be confined by the shielding current of the boundary zone provided that the material pressure is appreciably less within the filament than it is outside.

From (15),

$$(H^2)_{r=0}=(1+q)(H^2)_{r=a}, \tag{21}$$

so that (20) implies $q\simeq 10^2$.

From the definition of q in (15) we then have $\nu a\sim 1$. If N is the total number of turns over the whole length l of the filament $\nu\sim N/l$, from which

$$N\simeq l/a. \tag{22}$$

Thus a is the characteristic cross-sectional dimension introduced in the previous section and used in (7).

With the help of (15) it is easy to work out the total torque across a normal plane more accurately than was done in the previous section. Ignoring a small contribution from the boundary zone, the torque is given by

$$\tfrac{1}{2}\int_0^a H_z H_\theta r^2\,dr=\frac{a^2(H^2)_{r=0}}{8\pi\nu}\left[\frac{\log(1+q)}{q}-\frac{1}{1+q}\right]. \tag{23}$$

With $\nu a\simeq 1$, $q\simeq 10^2$, the right hand side of (23) can be written to sufficient accuracy as

$$\frac{1}{8\pi q}a^3(H^2)_{r=0}\log q. \tag{24}$$

This result is similar in form to the expression $H^2d^3/8\pi$, $(d=a)$, used in the previous section. It differs in detail, however, in that the torque is reduced below the previous rough formula by the factor $q^{-1}\log q$.

To end the present section, a more accurate assessment of the energy storage in a twisted filament will now be given. From (14) and (15) it follows that

$$(H)_{r=a}\simeq 2\nu F/a\log q, \tag{25}$$

the approximation $q\simeq 1+q$ being used. Eliminating νF and $(H)_{r=a}$ between (16), (20), and (25) we thus have $10\pi a^2\log q$ for the energy storage per unit length of the filament. For a filament of length l the total energy storage is therefore $10\pi a^2 l\log q$.

This expression can be put in a convenient form by noting that $a^2 l \sim Ah$, where A is the area of the filament projected against the solar disk and h represents height above the photosphere. Writing $A = \pi R^2 f$, where R is the solar radius, our result becomes $\sim 10\pi^2 R^2 hf \log q$ and this is not sensitive either to q or to h, the height being always of order 10^8 cm. The energy storage is accordingly dependent essentially on f, the fraction of the solar disk covered by the filament. Inserting $h \cong 10^8$ cm, $q \cong 10^2$, $R \cong 6 \cdot 9 \times 10^{10}$ cm, gives $\sim 2 \cdot 10^{32} f$ ergs, so that for $f \sim 10^{-2}$ the energy storage $\sim 10^{30}$ ergs—as obtained in our approximate considerations.

6. *The boundary zone.*—In place of (11) we now write

$$\mathbf{H} = 0, \quad r > b > a. \tag{11'}$$

We still regard the field for $r \leqslant a$ as being given by (15).

The zone between the cylinders of radii a, b carries the non-force-free shielding current. The nature of this current is easily understood. At

$$r = a, \quad H_\theta = 2\pi\nu a H_z,$$

so that for $\nu a \cong 1$, the field is mainly axial. Thus the shielding current must be mainly parallel to the axis of the filament, and if $F > 0$ must be directed in the negative z-direction. The total current required to flow in the boundary can readily be shown to be $\sim \frac{1}{2} ac(H)_{r=a}$.

If the current density $\mathbf{j}$ is assumed for simplicity to be uniform over a normal cross-section then

$$j = |\mathbf{j}| = \frac{ac(H)_{r=a}}{2\pi(b^2 - a^2)}, \tag{26}$$

and the total heat produced per unit length of the filament is

$$\pi(b^2 - a^2) \cdot \frac{j^2}{\sigma} \cong \frac{2a^2c^2}{\sigma(b^2 - a^2)} \cdot \frac{1}{8\pi} (H^2)_{r=a}, \tag{27}$$

where σ is the effective conductivity. In the present case the current is nearly perpendicular to $\mathbf{H}$, so that (6) of Section 3 must be included in estimating the conductivity to be used in (27). In fact

$$\sigma/c^2 \cong 10^{-32} \frac{n^2 x}{(H^2)_{r=a}}. \tag{28}$$

The next step is to estimate b. This can be done from the usual skin-depth formula for an alternating field of angular frequency ω. This gives a depth $c/(\omega\sigma)^{1/2}$, i.e.

$$b - a \cong \frac{c}{(\omega\sigma)^{1/2}}.$$

For ω we set T^{-1}, where T is the time scale for setting up the twisted filament. Thus

$$b - a \cong \left(\frac{Tc^2}{\sigma}\right)^{1/2} \tag{29}$$

We now consider numerical values. With

$$(H^2)_{r=a}/8\pi = 10 \text{ erg cm}^{-3}, \quad n = 10^{13} \text{ atoms cm}^{-3}, \quad x = 10^{-2},$$

(28) gives $\sigma/c^2 \cong 3 \cdot 10^{-11}$. With 10^5 sec as a reasonable estimate for T, we have $b - a \cong 5 \times 10^7$ cm, from which it follows that the shielding current can be confined within a zone of reasonable thickness. The zone becomes thinner at higher n,

but somewhat thicker at smaller n. Thus, lowering the particle density to 10^{12} atoms cm^{-3}, $x = 10^{-1}$, but keeping T, $(H)_{r=a}$ the same, gives

$$b - a \cong 1{\cdot}5 \times 10^8 \text{ cm}.$$

Since we have considered $a \cong 10^8$ cm it is clear that the boundary zone is such that $b - a \cong a$ over the relevant chromospheric portions of the filament.

If as an example we put $b - a = a$ in (27) and (29), σ can be eliminated to give

$$\frac{2}{3}\frac{a^2}{T} \cdot \frac{(H^2)_{r=a}}{8\pi} \tag{30}$$

for the rate of heat production per unit length of the filament. Multiplying (30) by T gives $a^2(H^2)_{r=a}/12\pi$ for the total heat production per unit length taken over the whole time interval of the winding process. Using (25) for $(H)_{r=a}$ then gives

$$\frac{F^2\nu^2}{3\pi(\log q)^2}. \tag{31}$$

A comparison with (16), which gives the total magnetic energy storage per unit length, shows that for $q \cong 10^2$ the dissipation amounts to only about 3 per cent of the stored energy. The fact that energy dissipation is so small is of course already clear from the form of (30). This shows that only the energy content of a magnetic field of intensity $(H)_{r=a}$ is dissipated by ohmic losses, whereas the main energy content of the field within the filament comes from regions near the axis, where the magnetic intensity is much greater than it is at $r = a$.

It will be recalled that it was assumed in Section 4 that ohmic decay does not prevent the filament field from becoming twisted. The present result gives strong support to this assumption.

7. *The instability of twisted force-free filaments.*—Although it is tempting to argue that filaments of the sort considered above become dynamically unstable (and that the instability constitutes the flare phenomenon), this point of view seems to us untenable. We have stressed at the outset that two quite distinct problems are involved in the flare phenomenon, the problem of energy storage, and the problem of almost catastrophic dissipation. The process of storage is comparatively slow and continuous. If such a process were interrupted by a catastrophic instability then one must understand clearly why the instability of necessity must arise at a sharply defined moment (it must not happen until there has been adequate energy storage!). We have not been able to see any plausible grounds for the existence of such a sharply defined moment during the winding of a single twisted filament.

The solution to the problem would seem to lie in considering two filaments. (cf. Fig. 3) *This we can properly do, since the filaments are distinct structures, not parts of a connected magnetic complex.* Suppose two parallel filaments touch each other along a line. The currents flowing in their boundary regions are either substantially parallel or anti-parallel. If the axial components of the fields within the filaments have the *same* sense about their respective axes the boundary currents are parallel, and the filaments attract each other at their line of contact (the boundary zones are not force-free). Moreover the electromagnetic forces are of necessity as large as the thermal pressure gradients in surrounding material, and can therefore be adequate to keep the filaments pressed together for a considerable time interval.

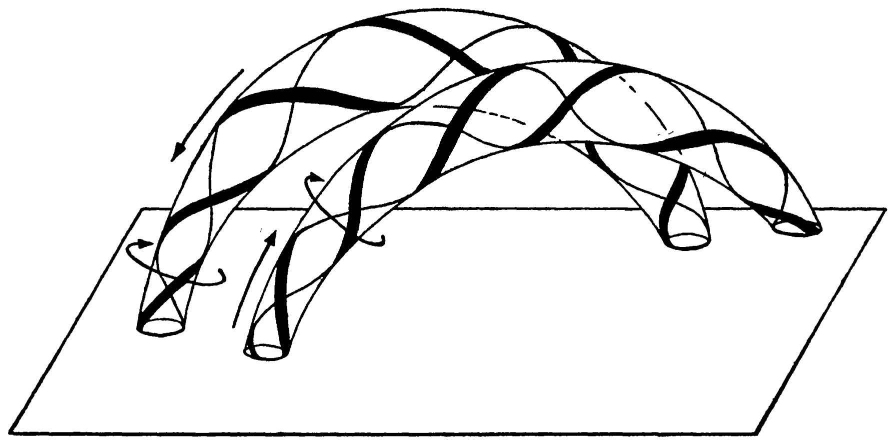

FIG. 3.—*Two bundles of lines of force, both twisted up. The initial longitudinal field is in opposite directions, and the twisting has occurred in the opposite sense. The arrows indicate the two components of the field. The electric current responsible for the circumferential component is in the same direction in each one.*

Now formula (29) for the size of the boundary zone is a skin-depth formula with the meaning that lines of force can diffuse relative to matter by a distance $b-a$ in a time T. Thus if the two filaments remain pressed together for this length of time they must begin to penetrate into each other. If we now add the second requirement, that the longitudinal fields within the filaments be directed in *opposite* senses, the respective current systems must continue to attract each other however far the interpenetration proceeds. Thus the axial field components are generated by longitudinal currents directed in some sense—which are clearly attractive, while the longitudinal field components are generated by axial currents in opposite senses about the respective axes—and at a line of contact such axial currents are parallel (not antiparallel!) and therefore are again attractive. Furthermore, as the axes of the filaments approach each other the neighbouring current elements become stronger, so that forces much greater than those in the boundary zones can be developed.

All this can be stated more clearly in the following way. We are considering two filaments with the same sense in the axial field component and with opposite senses in the longitudinal component; this requires the lines of force to be right-handed spirals in one case and left-handed in the other. Interpenetration tends to augment the axial component and to annihilate the longitudinal component, leaving conditions suitable for the pinch effect to operate (4).

The most important point still remains, however. To produce an almost catastrophic effect, the axial parts of the filaments, where the fields are most intense, must approach one another at an increasing rate. We have seen that penetration through the boundary zones requires a time of the same order as the winding time T, which we took $\sim 10^5$ sec. Unless interpenetration is much speeded up beyond this there is clearly no possibility of a marked instability arising.

It is here that the dependence on the magnetic intensity of the conductivity for a non-force-free current plays a crucial role. Thus the conductivity at magnetic intensity H is given by

$$\frac{\sigma}{c^2} = 10^{-32}\frac{n^2 x}{H^2}, \tag{32}$$

which agrees with (28) at $H=(H)_{r=a}$. Reference to the skin-depth formula (29) shows that the time required for diffusion to take place through a given distance decreases as σ, and hence as H^{-2}, if n, x are fixed. It follows that the axial parts of the filaments interpenetrate each much more rapidly than the boundary zones. Moreover the strong part of the field is confined within a distance $aq^{-1/2}$ of the axis. We therefore require penetration not through a distance $b-a$ which $\sim a$, but through a distance less than this by the factor $q^{1/2}$. Taking both these factors into consideration the strong axial parts of the filaments interpenetrate each other in a time

$$\frac{1}{q} \cdot \frac{(H^2)_{r=a}}{(H^2)_{r=0}} T, \tag{33}$$

assuming n, x to stay fixed. With $(H^2)_{r=a}/(H^2)_{r=0} \cong q^{-1}$, $q \cong 10^2$, $T \cong 10^5$ sec, (33) gives an interpenetration time of ~ 10 seconds.

It must now be emphasized that our present intention is only to discuss events leading to instability. The present considerations are not therefore intended to apply to the final phases of extreme catastrophic instability when dynamical considerations must be introduced.

The assumptions of n, x constant were made in the above investigation. If strong attractive forces develop between the two filaments the particle density n must rise at their interface until the gas pressure becomes sufficient to withstand the compressive magnetic forces. It is therefore possible that in a shallow layer at the interface n may rise substantially above the value $\sim 10^{13}$ atom cm^{-3} used above. Our result is not affected by any such one-dimensional compressive effect, however (the compression occurs in a direction normal to the surface of separation of the filaments). To see this, we first note that the interpenetration time of the filaments is proportional to σD^2, where D is the interpenetration distance. With σ proportional to n^2, the time is thus proportional to $(nD)^2$—and for a fixed quantity of gas within the filaments nD is independent of the degree of compression, this being one-dimensional. Thus the more the magnetic forces cause n to increase the shorter the distance through which the lines of force need interpenetrate each other—the two effects simply cancel each other.

The extreme catastrophic instability will, however, be mitigated by an increase in the ionization factor x, but even a rise to $x \sim 1$ would still give an interpenetration time $\sim 10^3$ sec. It is perhaps worth writing down the relevant results again. The skin depth formula shows that interpenetration takes place through a distance $aq^{-1/2}$ in a time $\sigma a^2/qc^2$, and with σ/c^2 given by (32), this can be written

$$10^{-32} \frac{n^2 a^2 x}{qH^2}. \tag{34}$$

The most favourable case occurs when n is small and H large. With $n=10^{12}$ atoms cm^{-3}, $H=200$ gauss, $q=10^2$, $a=2 \times 10^8$ cm, $x \sim 1$, the formula (34) gives a time $\sim 10^2$ sec. These estimates are exactly of the right order to explain the observed rapidity of onset of a flare. A value as low as 10^{12} atoms cm^{-3} for n is not at all implausible, since the pinch effect that follows the annihilation of the longitudinal field must increase the density, so that the density after the pinch could well be as high as 10^{13} atoms cm^{-3}.

One point remains, however. It will be recalled that the quantity x was defined as the ratio of the ion density n_i to neutral atom density n. Evidently x is not limited to the value unity. Indeed x takes a value $\gg 1$ if almost all the hydrogen becomes ionized. This would greatly increase the time estimate given by (34) and would destroy the catastrophic instability of the process of interpenetration. The consideration of this point allows a calculation to be made of the total flare energy, and this calculation is nearly independent of all the foregoing results.

Plainly x cannot become very large if the longitudinal fields within the filaments are not intense enough for annihilation to provide sufficient energy to ionize entirely the whole of the filament material. It is easy to work out what this limitation implies. If A is the area of the flare projected against the disk, and h is the height range of the flare, then the total material content of the flare $\sim AhMn$. The energy required to ionize this mass of material $\sim 10^{13} AhMn$ ergs. For $x \leqslant 1$, we thus require the energy of the longitudinal field component to be less than $\sim 10^{13} AhMn$. And since the total field energy exceeds that of the longitudinal component by a factor ~ 10, our present condition therefore requires the total magnetic energy to be less than $\sim 10^{14}\, AhMn$. This gives $\sim 10^{31}$ ergs, when $n \cong 10^{13}$ atoms cm^{-3}, $h \cong 10^8$ cm, and $A \cong 10^{20}$ cm^2 (i.e. about 1 per cent of the disk). In view of what was said above n should probably be taken less than 10^{13} atoms cm^{-3}, and the energy limit should be reduced correspondingly.

If the energy limit is exceeded, the longitudinal component of the filaments do not interpenetrate completely at the first instability. Then the material will probably radiate without any dynamical instability developing from the axial component of the field, since the axial field in this case can pinch on to a longitudinal field in both filaments. Eventually, however, the hot hydrogen must cool, and recombination of the ionized atoms decreases the conductivity. A further interpenetration of the longitudinal field components must then take place, until the interpenetration is again reduced by further heating. This will go on until the longitudinal field is annihilated in one or the other of the filaments—it is of course unlikely that the intensities of the two filaments will be so exactly equal that both longitudinal components become effectively annihilated. Since the whole process is slow it seems most unlikely that any strong dynamical instability will arise from the axial field. Rather does it seem likely that the axial components from both filaments will pinch in a stable way on to the residue of the longitudinal field.

It is a possibility that chromospheric flocculi are a manifestation of some such process of steady emission.

To end this section it is worth noting that (16) requires the total energy of the whole length l of the filament to be

$$\frac{1}{2}\frac{F^2\nu^2 l}{\log(1+q)} \cong \frac{1}{2}\frac{F^2N^2}{l\log q},$$

where $N \cong \nu l$ is the total number of turns of the lines of force. If we now think of l as variable with F, N fixed, it follows that the longer the filament the smaller the stored energy (the logarithmic term cannot vary appreciably with l). This means that random perturbations will always tend to lengthen a filament. In our view this is just why filaments are in fact long. An examination of Hα spectroheliograms of areas where flares occur shows the filaments to meander apparently aimlessly over the areas in question, in just the way that would be expected for systems that lengthen under the influence of perturbations, but which are otherwise stable.

8. *General remarks on the dynamical instability.*—Although we have not been able to consider the dynamics of the instability in a serious quantitative way, there are several features that seem worth mentioning.

Instability seems likely to start at a particular point of the coupled filaments, rather than simultaneously along their whole length. This arises from the increasing speed of the interpenetration, which demands some unevenness of interpenetration. Plainly one point must run somewhat ahead of the others and instability will set in first at this special point. So much is clear. It is then plausible to suppose that instability spreads both ways along the filaments from this special point. This means that instead of waiting for instability to be reached separately at each individual place along the filaments we expect that the first point of instability will simply trigger off the whole length over which the filaments are pressed together.

Recent work in the laboratory shows that a powerful pinch is followed by a second and even by a third pinch. If we assume this to happen in a similar fashion in the solar case we can understand at any rate empirically the emission of high energy particles by flares, since such particles are in fact produced in the laboratory case.

We have spoken of the interpenetration of two parallel filaments, but it is clear that since the filaments have different roots they can only be parallel over a limited segment of their lengths. Thus three portions can be distinguished for each filament, a portion that is common with the other filament, and two portions, one for each root, that are not common with the other filament.

The destruction of the longitudinal components of the fields of two filaments over their common portion allows a considerable change to take place in the linkages of the lines of force at the junctions of the common and non-common portions of the filaments. Indeed a line of force emerging from the photosphere can change the root to which it returns. Thus for instance a line of force that is twisted into a right-handed screw on its way out from the photosphere to a junction point can return to the photosphere along a left-handed screw. Many of the turns of such a twisted line of force can then simply run up to the junction point and untwist themselves. These considerations also suggest that filaments can bifurcate, that not all the lines of force emerging at a particular root need re-enter the photosphere at one unique other root.

9. *A remark on the structure of the solar magnetic field.*—Throughout the above discussion we have thought of a twisted filament as being built up in two stages: a first phase in which the filament emerges from the photosphere, and a second phase in which the filament becomes twisted by mechanical motions at, and below, its roots. Nothing in the main argument would be seriously affected, however, if the filament were to emerge from the photosphere in an already twisted condition.

This prompts the thought that the phenomenon of the flare may be an essential part of a process whereby internal solar fields become untwisted. It is well known that the Sun possesses a deep and highly active convection zone. (Not only are there strong theoretical reasons for expecting such a zone, but many stars not very dissimilar from the Sun—dwarfs somewhat further down the main-sequence—possess structures that can only be understood on the basis of the presence of a convection zone.) The magnetic field within this zone must tend to become seriously twisted, so much so that it has always been something of a mystery to understand how the twisting comes to be limited in its complexity. Since it seems clear that the twisting does not inhibit convection, one possibility is that both the convection and the magnetic field assume highly correlated patterns that are specifically arranged to prevent any overall increase in the complexity of the field. A second possibility arises from the present discussion, namely that a balance is reached through an untwisting of the lines of force during discharges in the solar atmosphere. On this basis it would scarcely seem to be by chance that the somewhat peculiar physical conditions in the chromosphere happen to be favourable to the occurrence of flares. Rather should the chromosphere then be thought of as an important factor in the structure and equilibrium of the whole Sun.

The authors understand that somewhat similar considerations to those given above are being published by P. A. Sweet. The reader is referred to the following papers already published by Sweet (*I.A.U. Symposium on Electromagnetic Phenomena in Cosmical Physics*, No. 6, 123 (1958); and *Nuovo Cim.*, Supp. **8**, Ser. 10, 188 (1958).)

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References

(1) Giovanelli, R. G., *M.N.*, **107**, 338, 1947.
(2) Hoyle, F., *Some Recent Researches in Solar Physics*, (Cambridge, 1949, p. 93).
(3) Dungey, J. W., *Cosmic Electrodynamics* (Cambridge, 1958).
(4) Severny, A. B., *Pub. Crimean Astr. Obs.*, **20**, 22, 1958.
(5) Spitzer, L., *Physics of Fully Ionized Gases* (*Interscience*, p. 81, 1956).
(6) Piddington, J. H., *M.N.*, **94**, 638 and 651, 1954.
(7) Cowling, T. G., *Magnetohydrodynamics* (Interscience, p. 99, 1957).