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Abstract
In this paper we analyze the outage probability of a cooperative multihop parallel relay network in Nakagami-*m* fading channels. The general closed form expression of the outage probability is derived both for integer and arbitrary Nakagami parameter *m*. We present numerical results on the performance of the network. It shows a careful configuration of the network size and power sharing between nodes can ensure optimal outage performance in the network.

On the Outage of Multihop Parallel Relay Networks

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***Abstract*—In this paper we analyze the outage probability of a cooperative multihop parallel relay network in Nakagami-m fading channels. The general closed form expression of the outage probability is derived both for integer and arbitrary Nakagami parameter m. We present numerical results on the performance of the network. It shows a careful configuration of the network size and power sharing between nodes can ensure optimal outage performance in the network.**

I. Introduction

Wireless networks reliability, severely affected by temporal multipath fading, can be improved through cooperative communication using relays. Past many research, such as [1]–[3] and references therein, have worked on dual hop cooperative systems. Relaying over multiple hops in addition with cooperation has come up with the solution of temporary extension of a network without substantial establishment of infrastructure or bringing the dead-spots into the network coverage of a cellular system [4]–[14]. Multihop relay networks can be divided into two categories, one with relaying layers/stages having a single relay [6]–[9], and the other with multiple relays in each relaying layer [11]–[14]. In this paper the network is divided into arbitrary K parallel relaying paths, each path with arbitrary number of relays. We analyze the outage probability of this system using selective decode and forward (DF) relays [2] in Nakagami-m fading. In contrast to amplify and forward (AF) relaying, DF protocol blocks the propagation of noise at the expense of complex circuitry.

An important statistical analysis of the outage probability with nonregenerative relays has been performed in [6] for a multihop series relaying network. It has presented a limited closed form expression of the MGF (moment generating function) of the end to end SNR with relays that use the inverted channel gain of the following channel for scaling. Recently [4] has investigated the BER (bit error rate) of KPP (K parallel path) network [10] with diversity branch using MGF based approach with AF relaying in generalized fading channels. However the closed form expression of the CDF (cumulative distribution function) or the PDF (probability density function) of the end-end SNR has not been presented. In particular, the exact expression of the PDF and the CDF of the end to end SNR involves inverse Laplace transform in complex mathematical form, which usually confines authors to evaluate the performance numerically rather expressing in closed form [4], [6]. However in many papers authors such as [7]–[9] have come up with the performance bounds in closed form. Boyer in [7] has introduced multihop network with diversity and has derived the outage and error probability bounds for multihop network in both DF and AF relaying cases. Using an inequality of harmonic-geometric mean of end-to-end SNR in Nakagami-m fading, [8], [9] have presented a lower bound over outage and average error probability. On the other hand, high SNR performance analysis has been performed in [11] and [12] for networks considering multiple relays per layer assuming flat Rayleigh fading channels. Using the benefits offered by STBC (space-time block code), [13] has studied a multihop network for both flat and frequency-selective fading channels. Furthermore a list of multihop routing protocols has been proposed in [14] subject to the minimization of outage probability. It has affirmed that the centralized channel state information (CSI) is vital for optimal routing in multihops.

In this paper we extend the result of the model [4] without a diversity link (since the source-destination link will be too weak to contribute any significant diversity signal in practical multihop networks) considering DF relaying. We have derived the closed form expression of the outage probability for a multihop K parallel path network without centralized CSI. The numerical results are presented to analyze the performance behavior with different network parameters such as, power sharing factor among nodes and fading parameters. We observe that the total power constraint plays the dominant role in case of power sharing between the relays and the source or in selecting the size of the network in terms of parallel paths. The interference signals from other adjacent nodes are neglected here by assuming that the distance between two adjacent relaying paths are far away from each other.

II. System Model

Consider a single source-destination pair communicates via total *R* single antenna relays in a multihop parallel network (Fig. 1). There are *K* possible parallel relay paths with arbitrary number of relays in each path. We let path i have a total $N_i - 1$ number of relays in series (i.e. total N_i hops in path i). A selective decode and forward (DAF) protocol has been considered over independent Nakagami-m fading channels. We assume that the relays of different parallel paths are far away

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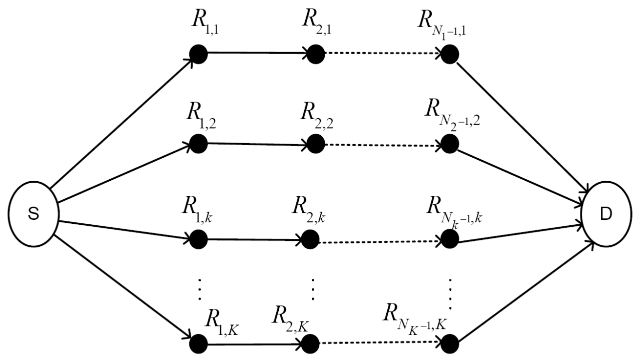

Fig. 1. Multihop parallel relaying network.

from each other to contribute significant interference in the network.

The total power for the whole system is constrained to P_{tot}. This total power is split into the whole network by a power sharing coefficient ξ so that the source and the total relay power is given by, $P_S = (1-\xi)P_{\text{tot}}$ and $P_R = \xi P_{\text{tot}}$ respectively, where, $\xi \in (0, 1]$. We consider the network has no centralized controller, so the relaying paths share a equal proportion of the total power assigned in P_R. For example, the relay path i has total power constraint $P_{R,i}$ such that,

$$P_{R,i} = \frac{P_R}{K} \tag{1}$$

and the individual relays of path i share this total power $P_{R,i}$ equally, i.e. the power of the relay node $R_{j,i}$ (node j in path i) is $P_{R_{j,i}}$, given by,

$$P_{R_{j,i}} = \frac{P_{R,i}}{N_i - 1} \tag{2}$$

We define the instantaneous SNR at the jth relay in ith path as $\gamma_{j,i}$, where $j \in \{1, 2, .., N_i - 1\}$ and i is an element of parallel path set $\mathcal{K}$ such that $\mathcal{K} = \{1, 2, .., K\}$. And thus the average SNR of n_ith hop and ith path is defined as $\overline{\gamma}_{n_i,i} = \Omega_{n_i,i}\frac{P}{N_0}$. Where, $\Omega_{n_i,i} = E\{|h_{n_i,i}|^2\}$, P is the corresponding transmitted power and N_0 is the noise variance modeled as circularly symmetric complex Gaussian random variable. $h_{n_i,i}$ is the Nakagami-m distributed channel gain parameter.

III. Outage Analysis

In a typical KPP model (Fig. 1), we define a relaying path i to be in outage if any link SNR of path i falls below the predefined threshold SNR value γ_{th}. Thus an event where all paths are in outage, will cause the outage of the whole network. We consider all possible relays will try to participate in cooperation if their links are not in outage, and at the last hop the signals from decodable paths are coherently combined by MRC at the destination. To avoid the synchronization problem due to transmission delays between the relay paths at the destination, we assume $N_i \approx N_j$, for $i \neq j$ [15]. Here we define decodable path set D_p as,

$$D_p \triangleq \{i \in \mathcal{K} : \gamma_{n_i,i} \geq \gamma_{\text{th}}\}\,; \qquad n_i \in \{1, 2, ..., N_i - 1\} \tag{3}$$

where, $\mathcal{K}$ is the parallel path set such that, $\mathcal{K} = \{1, 2, ..., K\}$. Thus we can determine the probability of decodable path D_p as,

$$\begin{aligned} \Pr\{D_p\} = &\prod_{i \in D_p} \Pr\left\{\min_{n_i \in \{1,2,..,N_i-1\}} \{\gamma_{n_i,i}\} > \gamma_{\text{th}}\right\} \\ &\times \prod_{j \notin D_p} \Pr\left\{\min_{n_j \in \{1,2,..,N_j-1\}} \{\gamma_{n_j,j}\} < \gamma_{\text{th}}\right\} \end{aligned} \tag{4}$$

where, $\gamma_{n_i,i}$ is the Gamma distributed SNR expressed as [16, eq. (2.21)] of the link of relay hop n_i of path i. Here to calculate the outage, we simplify the model of a multihop series path to a single relay link, denoting the link is active if the path is decodable. Now the total probability of the outage at the destination will be the sum of the signals from all decodable relay paths. Thus the total outage will be conditional on the set of the decodable relay paths.

Taking the cardinality of D_p as p, we can write the outage probability of the system using the law of total probability as,

$$P_{\text{out}} = \sum_{p=0}^{K} \sum_{D_p} P\{\text{outage}|D_p\}\Pr\{D_p\} \tag{5}$$

where

$$P\{\text{outage}\,|D_p\} = \Pr\left\{\sum \gamma_{N_i,i} < \gamma_{\text{th}}\,|i \in D_p\right\} \tag{6}$$

using (4) and (6) in (5),

$$\begin{aligned} P_{\text{out}} &= \sum_{p=0}^{K} \sum_{D_p} \Pr\left\{\sum_{i \in D_p} \gamma_{N_i,i} < \gamma_{\text{th}}\right\} \Pr\{D_p\} \\ &= \sum_{p=0}^{K} \sum_{D_p} \Pr\left\{\sum_{i \in D_p} \gamma_{N_i,i} < \gamma_{\text{th}}\right\} \\ &\quad \times \prod_{i \in D_p} \Pr\left\{\min_{n_i \in \{1,2,...N_i-1\}} \{\gamma_{n_i,i}\} > \gamma_{\text{th}}\right\} \\ &\quad \times \prod_{j \notin D_p} \Pr\left\{\min_{n_j \in \{1,2,...N_j-1\}} \{\gamma_{n_j,j}\} < \gamma_{\text{th}}\right\} \end{aligned} \tag{7}$$

Now invoking the result from order statistics, the equation (7) can be written as,

$$\begin{aligned} P_{\text{out}} = &\sum_{p=0}^{K} \sum_{D_p} \left[Pr\left\{\sum_{i \in D_p} \gamma_{N_i,i} < \gamma_{\text{th}}\right\} \right. \\ &\times \prod_{i \in D_p} \prod_{n=1}^{N_i-1} \left[\frac{\Gamma\left(m_{n,i}, m_{n,i}\gamma_{\text{th}}/\bar{\gamma}_{m_n,i}\right)}{\Gamma\left(m_{n,i}\right)}\right] \\ &\times \left. \prod_{j \notin D_p} \left(1 - \prod_{n=1}^{N_j-1} \frac{\Gamma\left(m_{n,j}, m_{n,j}\gamma_{\text{th}}/\bar{\gamma}_{m_n,j}\right)}{\Gamma\left(m_{n,j}\right)}\right)\right] \end{aligned} \tag{8}$$

The CDF sum of eq.(8) can be calculated by using the random sum of random variables. If the gamma variates with parameter

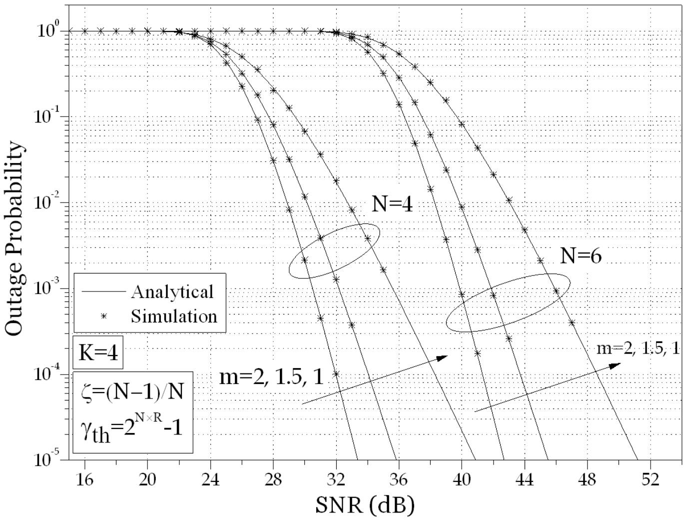

Fig. 2. Outage probability of 4 path multihop relay network as a function of total power in Nakagami-m fading channels.

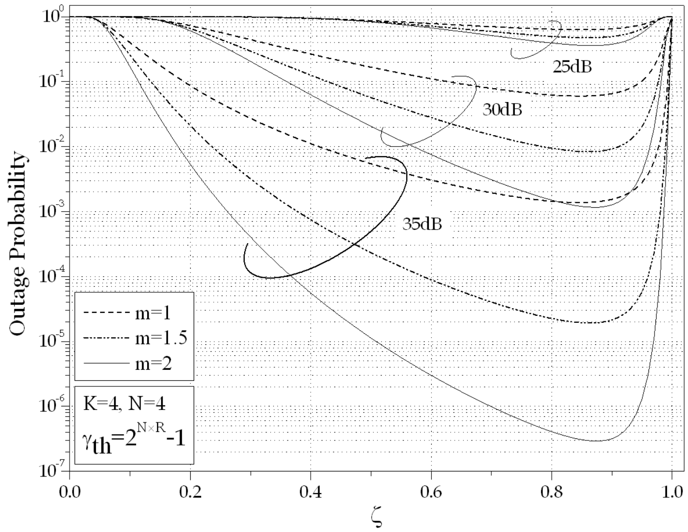

Fig. 3. Outage probability of 4 path and 4 hop relay network as a function of power sharing coefficient ξ with different SNR groups in Nakagami-m fading channels.

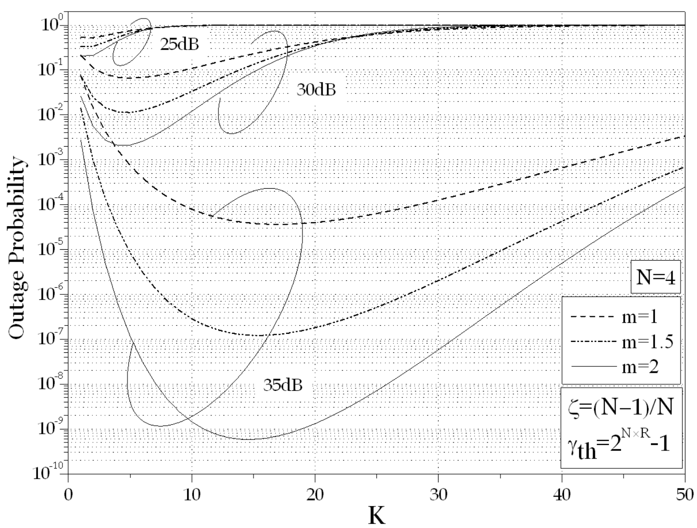

Fig. 4. Outage probability of 4 hop relay network as a function of number of relaying paths K with different SNR groups in Nakagami-m fading channels.

m and λ are restricted to only integer m's, then for distinct λ's we have the CDF of the random sum Y,

$$F_Y(y) = \left(\prod_{i=1}^{p} \frac{1}{\lambda_i^{m_i}}\right) \sum_{i=1}^{p} \sum_{l=1}^{m_i} \frac{d^{l-1}}{ds^{l-1}} \left\{ \prod_{\substack{q=1 \\ q\neq i}}^{p} \left(s + \frac{1}{\lambda_q}\right)^{-m_q} \right\} \Bigg|_{s=-\frac{1}{\lambda_i}}$$
$$\times \frac{\lambda_i^{a_i+1}}{(a_i)!\,(l-1)!} \left\{ \Gamma(a_i+1) - \Gamma\left(a_i+1,\ \frac{y}{\lambda_i}\right) \right\} \quad (9)$$

where, $a_i = m_i - l$, Gamma distribution parameter $\lambda_i = \frac{\omega_i}{m_i}$ and $\Gamma(a, z)$ is upper incomplete Gamma function defined as, $\Gamma(a, z) \triangleq \int_z^{\infty} t^{a-1} e^{-t} dt$

Proof: See Appendix A. ∎

While the value of m is not restricted to integer values the CDF can be written as Moschopoulos et. al. [17]

$$F_Y(y) = \prod_{r=1}^{p} \left(\frac{\lambda_1}{\lambda_r}\right)^{m_r} \sum_{k=0}^{\infty} \theta_k \left[1 - \frac{\Gamma(\beta+k, y/\lambda_1)}{\Gamma(\beta+k)}\right], \quad y \geq 0; \quad (10)$$

where, $\lambda_1 = \min_r \{\lambda_r\}$, $\beta = \sum_{r=1}^{p} m_r$ and,

$$\begin{cases} \theta_0 &= 1 \\ \theta_{k+1} &= \frac{1}{k+1} \sum_{i=1}^{k+1} \left[\sum_{j=1}^{N} m_j \left(1 - \frac{\beta_1}{\beta_j}\right)^i \right] \theta_{k+1-i};\ k = 0, 1, 2, .. \end{cases}$$

Substituting the above result in eq. (9) and putting $\lambda_i = \frac{\overline{\gamma}_i}{m_i}$ we will have the total outage expression as eq. (11), shown at the bottom of the next page.

Again using (10) we can express the outage probability of the system as (12) shown at the bottom of the next page, where θ_k is given by eq. (10)

IV. Numerical Results

Numerical results are plotted by taking SNR threshold $\gamma_{\text{th}} = 2^{NR} - 1$, with a data rate $R = 1$bps and noise variance as unity. We consider all relaying paths have equal number of relays, i.e. $N_i = N_j = N$. Furthermore figures are plotted for three distinct Nakagami parameter m, that is m=1 (Rayleigh fading), 1.5 and 2.

Fig. 2 plots the outage probability in Nakagami-m fading channels as a function of total power measured in SNRdB with 4 relays per layer scenario in 4 and 6 hop networks. In general outage probability is inversely related to the SNR. The network size in terms of the number of hops exposes a similar behavior on the outage. The figure indicates that an increase of 2 hops from 4, requires an SNR penalty of around 9dB to ensure an outage about 10^{-3}. Moreover the high SNR slope suggests that the diversity of the network is not effected by the number of hops rather than by the number of relaying paths. Fig. 3 and Fig. 4 show the outage probability behavior as a function of power sharing factor, ξ and the number of relaying paths, K respectively in a 4 hop network. The outage probability in Fig. 3 is divided into three different total power groups, that is 25dB, 30dB and 35dB. It reveals that the outage performance will be optimum if the total power is equally divided into the source and the relay stages. However surprisingly the increasing number of parallel paths does not always guarantee the lower outage as shown in Fig. 4. With the same total power groups as in Fig. 3, it shows that the outage probability take an upward bell shaped curves while plotted as a function of K. The observation shows that, the total power constraint of the network limits the number of optimum paths to ascertain minimum outage. For example, with a total power constraint of 35dB we can use about 15 relays per relaying layers for optimal outage performance while the source and each relaying layers share equal aggregated power.

V. Conclusion

In this paper the general expression of the outage probability of a cooperative multihop DF relay network is presented. The numerical results show that the outage probability can be reduced by increasing the number of relay nodes in each stage with sufficiently high SNR. However the total power constraint plays the dominant role in case of power sharing between the relays and the source or in selecting the size of the network in terms of parallel paths.

Appendix

A. Proof of CDF

The sum of the received SNR, $\gamma_{n_i,i}$ is a random sum of independent random variables. Let, $Y=\sum_{k\in D_p}=X_k,\ X$ is gamma distributed random variable with parameters (m_k,ω_k) such that,

$$f_X\left(x\right)=\frac{x^{m_k-1}m_k^{m_k}}{\omega_k^{m_k}\Gamma\left(m_k\right)}e^{-\frac{xm_k}{\omega_k}};\qquad x\geq 0.$$

Defining $\lambda_k=\frac{\omega_k}{m_k}$, we have the MGF of Y as,

$$\begin{aligned}\Phi_Y(s)&=E\left\{e^{-sY}||D_p|=p\right\}\\&=\prod_{k=1}^{p}\left(1+s\lambda_k\right)^{-m_k}\end{aligned}\qquad(13)$$

For distinct values of m_n, the PDF of Y is the inverse Laplace transform of eq. (13), using [18, eq. (2.1.4.8)],

$$f_Y\left(y\right)=\left(\prod_{k=1}^{p}\frac{1}{\lambda_k^{m_k}}\right)\sum_{k=1}^{p}\sum_{l=1}^{m_k}\frac{\mathcal{A}_{k_l}\left(-\frac{1}{\lambda_k}\right)y^{m_k-l}}{\left(m_k-l\right)!\left(l-1\right)!}e^{-\frac{y}{\lambda_k}},\qquad y\geq 0.\qquad(14)$$

where $\mathcal{A}_{k_l}\left(s\right)=\frac{d^{l-1}}{ds^{l-1}}\left\{\prod_{\substack{q=1\\q\neq k}}^{p}\left(s+\frac{1}{\lambda_q}\right)^{-m_q}\right\}$

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$$\begin{aligned}P_{\text{out}}=\sum_{p=0}^{K}\sum_{D_p}\Bigg[&\prod_{i\in D_p}\prod_{n=1}^{N_i-1}\left(\frac{\Gamma\left(m_{n,i},m_{n,i}\gamma_{\text{th}}/\overline{\gamma}_{m_{n,i}}\right)}{\Gamma\left(m_{n,i}\right)}\right)\prod_{j\notin D_p}\left(1-\prod_{n=1}^{N_j-1}\frac{\Gamma\left(m_{n,j},m_{n,j}\gamma_{\text{th}}/\overline{\gamma}_{m_{n,j}}\right)}{\Gamma\left(m_{n,j}\right)}\right)\\&\times\left\{\left(\prod_{k\in D_p}\frac{m_k^{m_k}}{\overline{\gamma}_k^{m_k}}\right)\sum_{k\in D_p}\sum_{l=1}^{m_k}\frac{d^{l-1}}{ds^{l-1}}\left\{\prod_{\substack{q=1\\q\neq k}}^{p}\left(s+\frac{m_q}{\overline{\gamma}_q}\right)^{-m_q}\right\}\Bigg|_{s=-\frac{m_k}{\overline{\gamma}_k}}\frac{\overline{\gamma}_k^{m_k-l+1}}{m_k^{m_k-l+1}\left(m_k-l\right)!\left(l-1\right)!}\right.\\&\left.\times\left\{\Gamma\left(m_k-l+1\right)-\Gamma(m_k-l+1,m_k\gamma_{\text{th}}/\overline{\gamma}_k)\right\}\right\}\Bigg]\end{aligned}\qquad(11)$$

$$\begin{aligned}P_{\text{out}}=\sum_{p=0}^{K}\sum_{D_p}\Bigg[\Bigg[&\prod_{r=1}^{|D_p|}\left(\frac{m_r\overline{\gamma}_1}{m_1\overline{\gamma}_r}\right)^{m_r}\sum_{k=0}^{\infty}\theta_k\left[1-\frac{\Gamma\left(\sum_{r\in D_p}m_r+k,\gamma_{\text{th}}m_1/\overline{\gamma}_1\right)}{\left(\sum_{r\in D_p}m_r+k\right)}\right]\Bigg]\\&\times\prod_{i\in D_p}\prod_{n=1}^{N_i-1}\left(\frac{\Gamma\left(m_{n,i},m_{n,i}\gamma_{\text{th}}/\overline{\gamma}_{m_{n,i}}\right)}{\Gamma\left(m_{n,i}\right)}\right)\prod_{j\notin D_p}\left(1-\prod_{n=1}^{N_j-1}\frac{\Gamma\left(m_{n,j},m_{n,j}\gamma_{\text{th}}/\overline{\gamma}_{m_{n,j}}\right)}{\Gamma\left(m_{n,j}\right)}\right)\Bigg]\end{aligned}\qquad(12)$$

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