Progress of Theoretical Physics, Vol. 66, No. 5, November 1981

On the Partial Conservation of the $U(1)$ Current*)

Ken KAWARABAYASHI and Nobuyoshi OHTA

Institute of Physics, University of Tokyo, Komaba, Tokyo 153

(Received June 2, 1981)

Recently proposed partial conservation of the $U(1)$ current ($\mathrm{PCU_1C}$) is applied to estimate the decay rates of various OZI forbidden processes. The results obtained are in good agreement with experiments and thus indicate the important role played by the $U(1)$ axial-vector anomaly in these decay processes. Octet $J^P=\frac{1}{2}^+$ baryons are next introduced into this scheme and low energy theorems related to the θ dependence of the matrix elements are investigated. Physical consequences of non-zero θ (strong CP-violation) are also discussed with the help of the $\mathrm{PCU_1C}$. The results are used to give the bound on θ.

§1. Introduction

In a recent paper,[1] we have taken a particular Lagrangian[2a] as an effective Quantum Chromodynamics (QCD) Lagrangian and have shown that the long-standing $U(1)$ problem[3] (including not only η mass problem[4),5)] but also $\eta\to\pi^+\pi^-\pi^0$ decay[2b)]) can be consistently resolved within the picture based on $1/N$ expanded QCD (N is the number of colors). The proposed model is described by the well-known nonlinear chiral Lagrangian supplemented with the terms corresponding to the kinetic term for a massless axial-vector ghost K_μ and its coupling to the flavor singlet Nambu-Goldstone (NG) boson.

Although no actual derivation has yet been given in QCD for such terms, their presence in the effective Lagrangian is supported by several reasonings; possible important role of states with the nonvanishing topological number $\int d^4x\partial^\mu K_\mu(x)$ for the confinement,[6] presence of the axial-vector anomaly in the singlet channel,[7] and the spontaneous breakdown of chiral $U(L)\otimes U(L)$ symmetry in the large N limit[8] (L is the number of massless quark flavors). Also, it has recently been *proved* that, at the level of the effective Lagrangian approach, the existence of such an axial-*vector* ghost is in fact required as asymptotic field in order for the $U(1)$ problem to be consistently resolved in QCD.[9]

In Ref. 1), we have pointed out that the model can be used to calculate the decay rates for $\psi\to\eta(\eta')\gamma$, and that the predicted rates are in reasonable agreement with experimental results. It is the purpose of the present paper to give further applications of this model and to find phenomenological evidences in support of our picture.

First let us briefly recapitulate the relevant results in Ref. 1). Assuming

*) Paper based on the preprint UT-Komaba 80-15 (1980).

$L=3$, the Lagrangian is described by the 3×3 unitary matrix field M, which transforms like $(3, 3^*)$ under the chiral $U(3)\otimes U(3)$ and can be expressed by the nonet NG bosons appearing as a nonlinear realization of the chiral symmetry. In addition, there are terms representing the contribution from the anomaly in the flavor singlet channel:

$$\mathcal{L}_{\mathrm{PS}}=\frac{F_\pi^2}{16}\mathrm{Tr}\{\partial_\mu M\partial^\mu M^\dagger\}+\frac{1}{4}\sqrt{\frac{3}{2}}F_\pi^2 m^2 u^{(0)}+\frac{1}{4}F_\pi^2\delta m^2 u^{(3)}$$
$$-\frac{1}{2\sqrt{3}}F_\pi^2(m_K^2-m_\pi^2)u^{(8)}+\frac{1}{2F_s^2(m_s^2-m^2)}(\partial^\mu K_\mu)^2$$
$$+\frac{i}{12}(\partial^\mu K_\mu)\{\ln\det M-\ln\det M^\dagger\} \tag{1a}$$

with

$$M=\exp\left\{i\frac{2}{F_\pi}\pi\right\},\qquad \pi\equiv\lambda^0 S+\sum_{i=1}^{8}\lambda^i\pi_i\,, \tag{1b}$$

$$u^{(a)}\equiv\frac{1}{4}\mathrm{Tr}\{\lambda^a(M+M^\dagger)\}, \tag{1c}$$

where $\lambda^a\ (a=0,1,\cdots,8)$ are the ordinary $U(3)$ matrices. The quadratic part of this Lagrangian in terms of the pseudoscalar singlet meson S, pions $\boldsymbol{\pi}=(\pi_1, \pi_2, \pi_3)$ and "eighth" meson π_8 is of the form

$$\mathcal{L}^0_{\mathrm{PS}}=\frac{1}{2}\{(\partial_\mu S)^2-m^2S^2\}+\frac{1}{2}\{(\partial_\mu\boldsymbol{\pi})^2-m_\pi^2\boldsymbol{\pi}^2\}+\frac{1}{2}\{(\partial_\mu\pi_8)^2-m_8^2\pi_8^2\}$$
$$+\frac{2\sqrt{2}}{3}(m_K^2-m_\pi^2)S\cdot\pi_8-\frac{\delta m^2}{\sqrt{3}}\pi_3\cdot(\pi_8+\sqrt{2}S)$$
$$+\frac{1}{2F_s^2(m_s^2-m^2)}(\partial^\mu K_\mu)^2-\frac{1}{F_s}(\partial^\mu K_\mu)S\,, \tag{1$'$}$$

where $m^2=\frac{1}{3}(2m_K^2+m_\pi^2)$, $m_8^2=\frac{1}{3}(4m_K^2-m_\pi^2)$, $\delta m^2=m_{K^+}^2-m_{K^0}^2-m_{\pi^+}^2+m_{\pi^0}^2$, $F_s=\sqrt{3/2}F_\pi$ with $F_\pi\simeq 190$ MeV for the pion decay constant. The divergence of the axial-vector field $\partial^\mu K_\mu$ which appears in the last two terms of $\mathcal{L}^0_{\mathrm{PS}}$ represents the above-mentioned contributions from the anomaly and thus, in QCD, should be identified with

$$\partial^\mu K_\mu=2L\cdot\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\,, \tag{2}$$

where $F^a_{\mu\nu}$ denotes gluonic field strength and $\tilde{F}^{a\mu\nu}=\frac{1}{2}\varepsilon^{\mu\nu\lambda\sigma}F^a_{\lambda\sigma}$.

The canonical quantization of the system (1) can be carried out by introducing a "gauge" fixing term

$$\frac{1}{4F_s^2(m_s^2-m^2)\alpha}(\partial_\mu K_\nu-\partial_\nu K_\mu)^2\ , \tag{3}$$

which violates the underlying QCD color gauge invariance, with α being a parameter. It then follows that the singlet field S acquires a nonvanishing mass m_S even in the chiral limit $m_K{}^2$ and $m_\pi{}^2\to 0$. One of the important consequences of (1) is the partial conservation of the $U(1)$ current ($\mathrm{PCU_1C}$)[10]

$$\langle\partial^\mu K_\mu\rangle_{\rm phys}=F_s(m_s^2-m^2)\langle S\rangle_{\rm phys}\ . \tag{4}$$

Physically the above relationship implies that the flavor singlet field contains a certain fraction of gluonic component.

It might be argued that in the soft regions the divergence of the current $\partial^\mu K_\mu$ is naturally dominated by soft operators, such as S, and thus the notion of the $\mathrm{PCU_1C}$ itself may not reflect specific properties of the underlying QCD. This may indeed be the case. However, an important point of (4) is that in the present picture its coefficient is uniquely fixed and is given in terms of known parameters and thus, independently of such questions, one can test (4) by confronting its predictions with experiments.

This paper is organized as follows. In § 2, we use the $\mathrm{PCU_1C}$ to evaluate the decay rates of various OZI forbiden processes.[11] The results obtained are consistent with experiments. Predictions to Υ decays are also included here. In § 3, we introduce baryons in this scheme and derive, with the help of the $\mathrm{PCU_1C}$, the low energy theorems related to the θ dependence of the physical matrix elements. Experimental consequences of the angle θ are discussed in some detail and the bounds on θ are estimated from *two independent* physical quantities, giving roughly $\theta\lesssim 10^{-(7\sim 8)}$. Section 4 is reserved for discussions and conclusions. In particular, it is pointed out that the $\mathrm{PCU_1C}$ relation should be modified in the presence of θ and that our model is also consistent with the experiments on $\eta(\eta',\pi^0)\to 2\gamma$ decays.

§ 2. OZI forbidden processes*)

As the first application of the $\mathrm{PCU_1C}$, we extend our previous estimation[1] for the decay rates of $\psi\to\eta(\eta')\gamma$, by including various OZI forbidden processes;[11] $\psi\to\pi^0\gamma$, $\psi'\to\psi\pi^0$, $\psi'\to\psi\eta$ and Υ decays.

*) The contents of this section overlap with those of the paper by J. M. Gérard, J. Pestieau and J. Weyers (Phys. Letters **94B** (1980), 227), which came into our attention after the completion of our work. It will, therefore, be appropriate to make a few comments on the differences between two approaches. Besides the difference of the numerical imputs for various parameters, we note that their A_η does not vanish in the flavor $SU(3)$ limit ($\theta_i=0$), which we think rather unnatural. This is because they took the $SU(3)$ *asymmetric* limit [quark mass ratios $M_u/M_d\neq 1$ and $M_u(M_d)/M_s\ll 1$] in their equation (6) whereas the $SU(3)$ *symmetric* limit ($M_u=M_d=M_s$) was actually taken in their Eqs. (10) and (11).

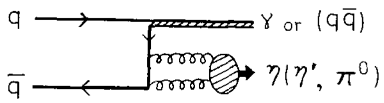


Fig. 1. OZI forbidden processes; meson $q\bar{q}$ decays into (a photon or $q\bar{q}$ meson state) $+(\eta, \eta'$ or $\pi^0)$. Here blob stands for the transition of two gluons into $\eta(\eta', \pi^0)$ by the matrix element $A_{\eta(\eta',\pi^0)}$.

In a standard picture, these processes proceed as follows;[12)~14)] first the quark-antiquark state annihilates into a state consisting of gluons (and photons), and second gluons decay into hadronic states, which we restrict here to η, η' or π^0 states (Fig. 1). To a good approximation, these latter processes are, then, described by the matrix elements

$$A_{\eta(\eta',\pi^0)} \equiv \frac{g^2}{16\pi^2}\langle 0|F^a_{\mu\nu}\tilde{F}^{a\mu\nu}|\eta(\eta', \pi^0)\rangle , \tag{5}$$

which, according to the PCU$_1$C relation (4), are transformed into

$$A_{\eta(\eta',\pi^0)} = \frac{1}{3}\sqrt{\frac{3}{2}}F_\pi(m_S{}^2 - m^2)\langle 0|S|\eta(\eta', \pi^0)\rangle . \tag{6}$$

The matrix elements (6) can be evaluated after diagonalizing the mass matrix in (1) by the orthogonal transformation

$$\begin{bmatrix} S \\ \pi_8 \\ \pi_3 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & \theta_2 \\ \sin\theta_3 & -\cos\theta_3 & -\theta_1 \\ \theta_1\sin\theta_3 - \theta_2\cos\theta_3 & -\theta_1\cos\theta_3 - \theta_2\sin\theta_3 & 1 \end{bmatrix} \begin{bmatrix} \eta' \\ \eta \\ \pi^0 \end{bmatrix} , \tag{7}$$

where θ_1, θ_2, θ_3 are the mixing angles between π_3 and π_8, π_3 and S, and S and π_8, respectively.*) Here we have taken account of the fact that θ_1, $\theta_2 \ll 1$ and have neglected terms higher than quadratic in θ_1 and θ_2. After straightforward calculation, we find

$$\theta_1 \simeq -1.7\times 10^{-2} , \qquad \theta_2 \simeq 0.9\times 10^{-2} , \qquad \theta_3 \simeq -0.31(\simeq -18^\circ), \tag{8}$$

and the mass m_S given by

$$m_S{}^2 = m^2_{\eta'} + m_\eta{}^2 - m_8{}^2 . \tag{9}$$

From Eqs. (6)~(9), we obtain

$$A_\eta = \frac{\sin\theta_3}{3}\sqrt{\frac{3}{2}}F_\pi(m^2_{\eta'} + m_\eta{}^2 - 2m_K{}^2) \simeq -0.31F_\pi m_\eta{}^2 ,$$

$$A_{\eta'}/A_\eta = \cot\theta_3 \simeq -3.0 ,$$

$$A_{\pi^0}/A_\eta = \theta_2/\sin\theta_3 \simeq -0.029 . \tag{10}$$

*) The angle θ_3 is identical with the mixing angle ϕ defined in Ref. 1).

Let us now consider $\psi \to \eta(\eta')\gamma$ decays. The diagram depicted in Fig. 1 immediately leads us to conclude

$$r \equiv \Gamma(\psi \to \eta'\gamma)/\Gamma(\psi \to \eta\gamma) = \left(\frac{p_{\eta'}}{p_\eta}\right)^3 \left(\frac{A_{\eta'}}{A_\eta}\right)^2 \simeq 7.3 . \tag{11}$$

The absolute decay rates, on the other hand, cannot be determined without further dynamical imputs. We only note that recent computation of $\Gamma(\psi \to \eta\gamma)$ by Novikov et al.[12] based on the QCD sum rules, when it is translated into our result, implies

$$\Gamma(\psi \to \eta\gamma) \simeq 90 \text{ eV} , \tag{12}$$

consistent with the observation (74 ± 15)eV.[15]

In exactly the same way, one can estimate other decay ratios, the results of which are summarized in Table I. We see from Table I that the agreement of our results with experiments[15] is generally satisfactory. Here we have also included predictions for Υ decays.[*] It should be recalled that our results for mixing angles θ_i as well as various A's are all determined by the Lagrangian (1) and, besides F_π and masses for pseudoscalar mesons, no other imput was introduced to fix these parameters.

Table I. Comparison of our prediction for decay ratios with experiments.[15] For the decays into $\pi^0\gamma$, see the footnote.

	our prediction	experiments
$\Gamma(\psi \to \eta'\gamma)/\Gamma(\psi \to \eta\gamma)$	7.3	5.9 ± 1.5
$\Gamma(\psi \to \pi^0\gamma)/\Gamma(\psi \to \eta\gamma)$	8.6×10^{-4}	$\lesssim 90 \times 10^{-3}$
$\Gamma(\psi' \to \psi\pi^0)/\Gamma(\psi' \to \psi\eta)$	16×10^{-3}	$(22 \pm 5) \times 10^{-3}$
$\Gamma(\Upsilon \to \eta'\gamma)/\Gamma(\Upsilon \to \eta\gamma)$	8.8	—
$\Gamma(\Upsilon \to \pi^0\gamma)/\Gamma(\Upsilon \to \eta\gamma)$	8.2×10^{-4}	—
$\Gamma(\Upsilon' \to \Upsilon\pi^0)/\Gamma(\Upsilon' \to \Upsilon\eta)$	4.6×10^{-3}	—

There have been several estimates in the past concerning these decays.[12)~14),17),18)] In these evaluations the relative magnitude of A_η and $A_{\eta'}$ is determined either from experimental ratio r,[17] or from the value at the origin of nonrelativistic wave function,[14] or from other phenomenological analyses.[12),13),18)] According to our model, the $A_{\eta(\eta',\pi^0)}$ are determined by the $\mathrm{PCU_1C}$ relation with the results which simply depend on the mixing angles and known masses for the pseudoscalar mesons. The fact that our analysis yields a consistent picture for

[*] The masses of Υ and Υ' used in these estimations are taken from Ref. 16). Here we have estimated the decay rates of ψ and Υ into $\pi^0\gamma$ through mixings of π_3, π_8 and S. The result turns out to be very small but is consistent with the fact that in this decay mode there is another channel through $\pi^0\rho$ state, which almost dominates $\psi \to \pi^0\gamma$.

the decay ratios (as well as absolute decay rates) for $\psi\to\eta(\eta', \pi^0)\gamma$ will then suggest that the $U(1)$ anomaly is in fact responsible for these decays.

§ 3. Incorporating baryons

We now introduce $J^P=\frac{1}{2}^+$ octet baryons into our theory. Incorporation of baryons in the chiral Lagrangians has already been formulated in the literatures[19] and we state here only the results. Following Ref. 19), the Lagrangian is given by

$$\mathcal{L}_{\rm tot}=\mathcal{L}_{\rm PS}+\mathcal{L}_{\rm BP}+\mathcal{L}_{\rm B}\,, \tag{13a}$$

$$\begin{aligned}\mathcal{L}_{\rm BP}=&\frac{1}{4}\,{\rm Tr}\{\bar{L}(i\partial-m_B)L+\bar{R}(i\partial-m_B)R\}\\&+f\,{\rm Tr}\{\bar{L}\gamma_\mu\gamma_5(M\partial^\mu M^\dagger-M^\dagger\partial^\mu M)L+\bar{R}\gamma_\mu\gamma_5(M\partial^\mu M^\dagger-M^\dagger\partial^\mu M)R\},\end{aligned} \tag{13b}$$

$$\begin{aligned}\mathcal{L}_{\rm B}=&-\frac{1}{2}\,{\rm Tr}[\{(D_0+F_0)\lambda_0+(D_8+F_8)\lambda_8\}\{\bar{L}LM+\bar{R}RM^\dagger\}]\\&-\frac{1}{2}\,{\rm Tr}[\{(D_0-F_0)\lambda_0+(D_8-F_8)\lambda_8\}\{L\bar{L}M+R\bar{R}M^\dagger\}]\\&+\frac{1}{4}\,{\rm Tr}[\{(D_0+F_0)\lambda_0+(D_8+F_8)\lambda_8\}(M+M^\dagger)]{\rm Tr}(\bar{L}L+\bar{R}R),\end{aligned} \tag{13c}$$

where $\mathcal{L}_{\rm PS}$ is the Lagrangian for nonet pseudoscalar mesons given by (1). $\mathcal{L}_{\rm BP}$ describes baryons interacting with the nonet mesons in the chiral symmetric way, while $\mathcal{L}_{\rm B}$, which corresponds to the quark mass terms in QCD, explicitly breaks chiral invariance and transforms like $(3, 3^*)\oplus(3^*, 3)$ under $SU_L(3)\otimes SU_R(3)$.*) Here L and R, defined by $\sqrt{M}\,B\sqrt{M^\dagger}$ and $\sqrt{M^\dagger}\,B\sqrt{M}$, transform as $(8, 1)$ and $(1, 8)$, respectively, with $B(\equiv\sum_{i=1}^{8}B_i\lambda_i)$ being the physical octet baryon matrix *invariant* under the chiral transformation. In (13b), the chiral and flavor $SU(3)$ invariant piece of the baryon mass is denoted by m_B. By writing baryons in terms of three quarks, we have fixed the coefficient of the last term in (13c) as above so as to assure the OZI rule.[11]

In terms of unknown coefficients D_0, D_8, F_0, F_8 and m_B, the octet baryon masses are given by

$$m_N=m_B-2\sqrt{\frac{2}{3}}(D_0+3F_0)-\frac{2}{\sqrt{3}}D_8-2\sqrt{3}F_8\,,$$

$$m_\Lambda=m_B-2\sqrt{\frac{2}{3}}(D_0+3F_0)-\frac{4}{\sqrt{3}}D_8\,,$$

*) Breaking of the flavor $SU(2)$ is neglected in (13c).

$$m_\Sigma = m_B - 2\sqrt{\frac{2}{3}}(D_0 + 3F_0) + \frac{4}{\sqrt{3}} D_8\ ,$$

$$m_\Xi = m_B - 2\sqrt{\frac{2}{3}}(D_0 + 3F_0) - \frac{2}{\sqrt{3}} D_8 + 2\sqrt{3} F_8\ ,$$

which satisfy the Gell-Mann-Okubo mass formula

$$m_\Sigma + 3m_\Lambda = 2(m_N + m_\Xi).$$

Among the various F's and D's, D_8 and F_8 can be determined from the baryon mass splittings, with the result

$$D_8 = \frac{\sqrt{3}}{8}(m_\Sigma - m_\Lambda) \simeq 16.7\ \text{MeV}\ ,$$

$$F_8 = \frac{\sqrt{3}}{12}(m_\Xi - m_N) \simeq 54.7\ \text{MeV}\ . \tag{14}$$

The remaining parameters D_0 and F_0 (and m_B) cannot be determined from the baryon mass spectrum alone. In order to fix these coefficients, we expand $\mathcal{L}_B$ in powers of $1/F_\pi$ and extract the interaction parts between mesons and nucleons, which break both chiral and flavor $SU(3)$ invariance. In this manner, we find

$$\begin{aligned}\mathcal{L}_B = &-\frac{4}{\sqrt{3}F_\pi^2}\{(\sqrt{2}D_0 + D_8) + 3(\sqrt{2}F_0 + F_8)\}\bar{N}N\boldsymbol{\pi}^2 \\ &+ \frac{8}{3F_\pi^2}\{(\sqrt{2}D_0 + D_8) - (\sqrt{2}F_0 + F_8)\}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N(\sqrt{2}S + \pi_8) \\ &+ \frac{8}{\sqrt{3}F_\pi^2}\{-2(\sqrt{2}F_0 + F_8) + 3F_8\}\bar{p}pK^+K^- \\ &+ \frac{4}{\sqrt{3}F_\pi^2}\{-2(\sqrt{2}D_0 + D_8) + 3D_8 - 2(\sqrt{2}F_0 + F_8) + 3F_8\}\bar{n}nK^+K^- \\ &+ \cdots\ ,\end{aligned} \tag{15}$$

where chiral breaking baryon mass terms as well as other interactions are not written explicitly. It is clear that interactions in (15) give deviations of the S-wave scattering lengths of π-N and K-N scattering from the current algebra formulae:[20)]

$$\Delta a_{\pi N} \equiv \frac{1}{2}(a_{\pi^+ p} + a_{\pi^- p})$$

$$= \frac{-2}{\sqrt{3}\pi F_\pi^2}\{(\sqrt{2}D_0 + D_8) + 3(\sqrt{2}F_0 + F_8)\}, \tag{16a}$$

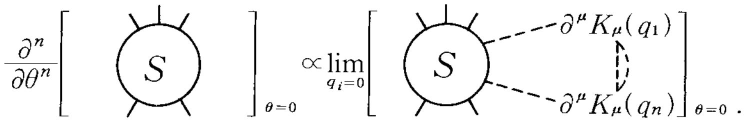
$$\Delta a_{KN} \equiv a_{K^+ p} - 2a_{K^+ n} = \frac{2}{\sqrt{3}\pi F_\pi^2}\{2(\sqrt{2}D_0 + D_8) - 3D_8\}, \tag{16b}$$

where $a_{\pi^+p}(a_{\pi^-p})$ denotes the S-wave scattering length of $\pi^+(\pi^-)$ off the proton and similar definitions for a_{K^+p} and a_{K^+n}.

A particular combination $\sqrt{2}D_0+D_8(\sqrt{2}F_0+F_8)$ which appears in (16a) implies that $\Delta a_{\pi N}$ is proportional to the non-strange quark mass M_u $(=M_d)$ and thus vanishes in the limit of chiral $SU(2)\otimes SU(2)$ symmetry $(M_u=M_d=0)$. On the other hand, Δa_{KN} remains finite, because in this limit it is proportional to the strange quark mass M_s which is non-vanishing. Experimentally, it has been well known that $\Delta a_{\pi N}$ is numerically very small, whereas sizable deviations are observed for Δa_{KN}, indicating $M_u(M_d)\ll M_s$ in QCD. If we substitute experimental data[21] $\Delta a_{\pi N}\simeq -0.01m_\pi^{-1}$ and $\Delta a_{Kp}\simeq -0.04m_\pi^{-1}$ in Eqs. (16a) and (16b), we find

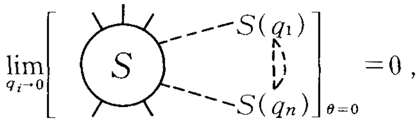
$$D_0=-4.3\,\mathrm{MeV}\,, \qquad F_0=-39.0\,\mathrm{MeV}\,. \tag{17}$$

Let us next consider the θ dependence of the present theory. For this purpose, we simply add the term $(\theta/6)\partial^\mu K_\mu$ to the Lagrangian. It is then clear that the θ derivative of any S-matrix element is related to the corresponding S-matrix involving $\partial^\mu K_\mu$ with zero momentum:



(18)

According to the PCU$_1$C (4), however, the r.h.s. of Eq. (18) is proportional to the S-matrix with $\partial^\mu K_\mu(q)$ replaced by the flavor singlet field $S(q)$. Since θ dependence of any physical quantity must disappear in the chiral limit, we obtain low energy theorems,[4]



(19)

in the chiral limit. On the other hand, if we keep the chiral breaking terms, the l.h.s. of Eq. (18) no longer vanishes and thus θ may have physical consequences.

In order to investigate these effects, we note that, in the presence of θ, the Lagrangian (1) contains a term which is linear in S. This can be easily seen by eliminating $\partial_\mu K^\mu$ from Eq. (1). As is well known this is an indication that the vacuum is not stable. To find the correct vacuum for $\theta\neq 0$, we shift S and π_8 (due to its mixing with S) with a condition that no linear term in NG bosons should remain in $(1)'$ for the shifted fields S' and $\pi_8{}'$:

$$S=S'+\langle S\rangle_\theta\,, \tag{20a}$$

$$\pi_8=\pi_8{}'+\langle\pi_8\rangle_\theta\,. \tag{20b}$$

Obviously, this is equivalent to rotating the matrix field M so as to minimize the vacuum energy. It is then easy to show that, for small θ, the non-vanishing values for $\langle S\rangle_\theta$ and $\langle\pi_8\rangle_\theta$ are given by

$$\langle S\rangle_\theta=\frac{F_S}{6}\frac{(m_S{}^2-m^2)(4m_K{}^2-m_\pi{}^2)}{m_S{}^2(4m_K{}^2-m_\pi{}^2)-\frac{8}{3}(m_K{}^2-m_\pi{}^2)^2}\theta\,, \tag{20a)$'$}$$

$$\langle\pi_8\rangle_\theta=\frac{F_S}{6}\frac{2\sqrt{2}(m_S{}^2-m^2)(m_K{}^2-m_\pi{}^2)}{m_S{}^2(4m_K{}^2-m_\pi{}^2)-\frac{8}{3}(m_K{}^2-m_\pi{}^2)^2}\theta\,, \tag{20b)$'$}$$

where we have neglected δm^2 for simplicity.

The physical consequence of all this is clear; there is strong CP violation, whose strength is fixed by the vacuum expectation values (VEV's) given in Eqs. (20a, b)$'$. Indeed, as can be seen from Eqs. (20a, b), the CP violating effect can be obtained if we simply replace the fields S and π_8 by their VEV's. In this manner, we find that the second term of Eq. (15) gives the following CP violating π-N interaction

$$\mathcal{L}_{CP}=\theta g'\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N\,, \tag{21a}$$

where

$$\begin{aligned} g'&=\frac{4}{\sqrt{3}F_\pi}\frac{(m_S{}^2-m^2)(2m_K{}^2-m_\pi{}^2)}{m_S{}^2(4m_K{}^2-m_\pi{}^2)-\frac{8}{3}(m_K{}^2-m_\pi{}^2)^2}\\ &\quad\times\{(\sqrt{2}D_0+D_8)-(\sqrt{2}F_0+F_8)\}\\ &=0.063\,, \end{aligned} \tag{21b}$$

where for the numerical evaluation use has been made of Eqs. (9), (14) and (17).

Notice that, as we already mentioned below Eqs. (16a, b), the last curly bracket factor in Eq. (21b) is proportional to non-strange quark mass $M_u(=M_d)$, while $(2m_K{}^2-m_\pi{}^2)$ is proportional to the strange quark mass M_s. Hence g' vanishes if any of the quark masses is zero. This is consistent with the fact that all physical effects of θ should disappear in the limit of chiral $U(1)$ symmetry.

We also note that g' (and D_0 and F_0) is rather sensitive to Δa_{KN} and $\Delta a_{\pi N}$ due to their large coefficients [see Eqs. (16a, b)]. For example, if we take $\Delta a_{KN}=-0.05m_\pi^{-1}$, g' is equal to 0.038. Thus we find that, within experimental uncertainties, our results for g' is not inconsistent with that due to Crewther et al.[22] (See also Note added.)

There is another related quantity of physical interest; that is the electric dipole moment of the neutron D_n. In Ref. 22), this was evaluated by extracting out a term proportional to $m_\pi{}^2\ln m_\pi{}^2$. Here we wish, by making use of the PCU$_1$C, to express D_n in terms of other physical amplitudes.

For this purpose, let us define

$$\mathcal{M}\equiv\frac{i\theta}{6}\int d^4xe^{iq\cdot x}T\langle P'|j_\mu^{\mathrm{em}}(0)\partial^\nu K_\nu(x)|P\rangle\,, \tag{22}$$

where $|P\rangle$ is the one neutron state with momentum P. In the soft q limit, we then find

$$\lim_{q\to 0}\mathcal{M}=-\left(\frac{m_N{}^2}{E'E}\right)^{1/2}D_n\bar{u}(P')\sigma_{\mu\nu}k^\nu\gamma_5u(P)+\cdots\,, \tag{23}$$

where k is the momentum of photon. If, on the other hand, the $\mathrm{PCU_1C}$ relation (4) is substituted in Eq. (22), one obtains*)

$$\lim_{q\to 0}\mathcal{M}=\frac{i\theta F_s}{6}\left(\frac{m_s{}^2-m^2}{m_s{}^2}\right)\lim_{q\to 0}\int d^4xe^{iq\cdot x}T\langle P'|j_\mu^{\mathrm{em}}(0)j_s(x)|P\rangle\,, \tag{24}$$

where $j_s(x)\equiv(\Box+m_s{}^2)S(x)$.

It is now clear that the r.h.s. of Eq. (24) is related to the photo-soft S production amplitude, which, in the standard notation of CGLN,[23] can be expanded as

$$\varepsilon^\mu\int d^4xe^{iq\cdot x}T\langle P'|j_\mu^{\mathrm{em}}(0)j_s(x)|P\rangle=i\left(\frac{m_N{}^2}{E'E}\right)^{1/2}\bar{u}(P')\,T(\nu,\,\nu_B\,;\,q^2)u(P) \tag{25a}$$

with

$$T(\nu,\,\nu_B\,;\,q^2)=M_AA+M_BB+M_CC+M_DD\,. \tag{25b}$$

Here M_i $(i=A,\,B,\cdots)$ are independent gauge invariant kinematical factors and their coefficients A, B, C and D are Lorentz invariant functions of ν, ν_B and q^2. By comparing Eqs. (22) and (23) with Eqs. (24) and (25) and expressing S in terms of η and η', we find

$$D_n=\frac{\theta F_s}{6}\,\frac{m_s{}^2-m^2}{m_s{}^2}\{A^{\gamma\eta}(\nu=\nu_B=0;\,q^2=0)\sin\theta_3$$
$$+A^{\gamma\eta'}(\nu=\nu_B=0;\,q^2=0)\cos\theta_3\}, \tag{26}$$

which is our desired result. This also becomes zero in the chiral $U(1)$ limit owing to the low energy theorems (19).

Unfortunately, because of the limited data for photo-η and η' productions at threshold, we are unable to make reliable estimations even for the relevant (on-the-mass-shell) amplitudes $A^{\gamma\eta}$ and $A^{\gamma\eta'}$. For a rough estimate, however, we assume that the $A^{\gamma\eta}$ amplitude [the coefficient function of M_A] dominates Eq. (25)

*) In the presence of θ, the exact form of the $\mathrm{PCU_1C}$ is modified. However, to obtain the lowest term in θ, we can use Eq. (4) in Eq. (22). See § 4 for detail.

and neglect the contribution from other amplitudes altogether. Also neglected is the contribution from photo-η' production process. Under these very crude approximations, we find

$$|D_n| \sim \left|\frac{\theta F_S}{6}\sin\theta_3\right| \frac{m_S{}^2 - m^2}{m_S{}^2} \frac{4\pi}{m_\eta} \sqrt{\frac{\sigma^{\gamma\eta}_{\text{threshold}}}{4\pi}}$$

$$\sim 1\times 10^{-16}|\theta|\ \text{cm}\,, \tag{27}$$

on the basis of the data, $\sigma^{\gamma\eta} \sim 2.5\times 10^{-30}\ \text{cm}^2$ at $E_\gamma \simeq 1.24$ GeV.[24)]

We note that our numerical result (27) is somewhat small compared with other estimates.[22)] However, because of many approximations involved, we do not think there is any serious discrepancy between our result and other estimates for D_n.

The bound on θ obtained from (27) is found to be

$$|\theta| \lesssim 10^{-8}\,, \tag{28}$$

whereas a less severe bound

$$|\theta| \lesssim (2\sim 3)\times 10^{-7}\,, \tag{29}$$

is given from (21a) and (21b). The latter bound is obtained on the basis of the observation of parity violating effects in nuclear interactions,[25)] giving $|g'\theta| \lesssim (1\sim 2)\times 10^{-8}$.

§ 4. Concluding remarks

We have investigated physical effects of the PCU$_1$C relation. One may argue that the ghost terms in (1) can be eliminated from the Lagrangian at the very beginning.[2)] The mass of S then becomes m_S and all physical results concerning *nonet mesons* can be obtained without these terms. However, as we have demonstrated, the term $\partial^\mu K_\mu$ plays an important role in a variety of other processes through the PCU$_1$C relation.

We note that similar gluonic excitations may exist in QCD for other channels of different spin and parity. Thus it may be equally important to know whether or not the mixing of ω and ϕ, for instance, can be consistently explained in QCD in terms of such gluonic modes.[26)]

In § 3, we have first eliminated $\partial^\mu K_\mu$ in terms of S and obtained, in the presence of small non-zero θ, the VEV's of the fields S and π_8. Actually the field $\partial^\mu K_\mu$ also may acquire a non-vanishing VEV and thus should be shifted:

$$\partial^\mu K_\mu = \partial^\mu K_\mu' + \langle \partial^\mu K_\mu \rangle_\theta\,. \tag{30}$$

By minimizing the vacuum energy in the Lagrangian with $\theta \neq 0$

[$\mathcal{L}_\theta \equiv (1)+(\theta/6)\partial_\mu K_\mu$], we find, for small θ, Eqs. (20a, b)$'$ and

$$\langle \partial^\mu K_\mu \rangle_\theta = -\frac{F_S^2}{2} \frac{(m_S^2-m^2)(2m_K^2-m_\pi^2)m_\pi^2}{m_S^2(4m_K^2-m_\pi^2)-\frac{8}{3}(m_k^2-m_\pi^2)^2}\theta$$

$$\simeq -0.35 F_\pi^2 m_\pi^2 \theta\ , \tag{31}$$

which is consistent with the estimation by Shifman et al.[27] Note that Eq. (31) vanishes in the limit of chiral $U(1)$ symmetry [$M_u(=M_d)=0$ *and/or* $M_s=0$]. On the contrary, the VEV's (20a, b)$'$ remain finite in a similar limit [for example, $M_u(=M_d)=0$ *and* $M_s \neq 0$]. It simply reflects the fact that, in this limit, a finite $U(1)$ rotation of the matrix field M is necessary to eliminate θ from the theory altogether.*)

It should be noticed that the Lagrangian with $\theta \neq 0$ takes the same form as (1) when it is expressed in terms of the shifted fields S', π_8' and $\partial^\mu K_\mu'$. The PCU$_1$C relation (4) is therefore valid for the shifted fields. Then, using Eqs. (20a), (20a)$'$, (30) and (31), we get the modified form of the PCU$_1$C for the fields S and $\partial^\mu K_\mu$:

$$\langle \partial^\mu K_\mu \rangle_{\text{physical}} = F_S(m_S^2-m^2)\left(\langle S \rangle_{\text{physical}} - \frac{F_S}{6}\theta\right), \tag{32}$$

where the suffix "physical" stands for the correct physical states in the presence of θ.

Let us finally estimate the $\eta\,(\eta', \pi^0) \to 2\gamma$ decay amplitudes. Following Adler,[7),13)] we obtain the ratio of the amplitudes

$$A_{\eta'\to 2\gamma} : A_{\eta\to 2\gamma} : A_{\pi^0\to 2\gamma} = 1.9 : 1.0 : 1.0\ , \tag{33}$$

which should be compared with experimental results[15)] 1.9 : 1.0 : 1.3.

In conclusion, we point out that it is quite interesting that we have a consistent picture on the $U(1)$ problem and on the related physical processes based on the PCU$_1$C relation.

Acknowledgements

We would like to thank S. Kitakado for valuable discussions and a careful reading of the manuscript. One of us (N.O.) is grateful to Professor T. Muta for helpful advices.

---

*) From Eqs. (1b), (20a, b) and (20a, b)$'$, we see that for small θ the rotation which, in the chiral $U(1)$ limit, eliminates the θ dependence is in fact given by $M'=M\exp(-iA)$ with

$$A \equiv \theta\cdot \frac{m_S^2-m^2}{m_S^2(4m_K^2-m_\pi^2)-\frac{8}{3}(m_K^2-m_\pi^2)^2} \times \mathrm{diag}(2m_K^2-m_\pi^2,\ 2m_K^2-m_\pi^2,\ m_\pi^2).$$

References

1) K. Kawarabayashi and N. Ohta, Nucl. Phys. **B175** (1980), 477.
2a) C. Rosenzweig, J. Schechter and G. Trahern, Phys. Rev. **D21** (1980), 3388.
P. Nath and R. Arnowitt, Phys. Rev. **D23** (1981), 473.
P. Di Vecchia and G. Veneziano, Nucl. Phys. **B171** (1980), 253.
E. Witten, Ann. of Phys. **128** (1980), 363.
N. Ohta, Prog. Theor. Phys. **66** (1981), 1408.
b) P. Di Vecchia, F. Nicodemi, R. Pettorino and G. Veneziano, Nucl. Phys. **B181** (1981), 318.
3) D. G. Sutherland, Phys. Letters **23** (1966), 384.
S. Weinberg, Phys. Rev. **D11** (1975), 3583.
J. Kogut and L. Susskind, Phys. Rev. **D11** (1975), 3594.
G. 't Hooft, Phys. Rev. Letters **37** (1976), 8; Phys. Rev. **D14** (1976), 3432.
R. J. Crewther, Phys. Letters **70B** (1977), 349.
For a review, R. J. Crewther, Rivista Nuovo Cim. **2** (1979), No. 8, 63.
R. J. Crewther, Phys. Letters **93B** (1980), 75; **95B** (1980), 461E.
4) E. Witten, Nucl. Phys. **B149** (1979), 285; **B156** (1979), 269.
G. Veneziano, Nucl. Phys. **B159** (1979), 213.
5) P. Di Vechhia, Phys. Letters **85B** (1979), 349.
A. Aurillia, Y. Takahashi and P. K. Townsend, Phys. Letters **95B** (1980), 265.
6) C. G. Callan, R. Dashen and D. J. Gross, Phys. Rev. **D17** (1978), 2717.
7) S. L. Adler, Phys. Rev. **177** (1969), 2426.
J. S. Bell and R. Jackiw, Nuovo Cim. **60A** (1969), 47.
A. A. Migdal, Phys. Letters **80B** (1979), 275; **81B** (1979), 37.
8) S. Coleman and E. Witten, Phys. Rev. Letters **45** (1980), 100.
G. Veneziano, Phys. Letters **95B** (1980), 90.
9) H. Hata, T. Kugo and N. Ohta, Nucl. Phys. **B178** (1981), 527.
10) G. Veneziano, Nucl. Phys. **B159** (1979), 213.
K. A. Milton, W. F. Palmer and S. S. Pinsky, Phys. Rev. **D22** (1980), 1124, 1647.
11) S. Okubo, Phys. Letters **5** (1963), 165.
J. Iizuka, K. Okada and O. Shito, Prog. Theor. Phys. **35** (1966), 1061.
12) V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B165** (1980), 55.
13) H. Goldberg, Phys. Rev. Letters **44** (1980), 363.
See also N. Aizawa, Z. Maki, I. Umemura and K. Yamamoto, Prog. Theor. Phys. **62** (1979), 1085; **63** (1980), 962.
14) P. Langacker, Phys. Letters **90B** (1980), 447 and references therein.
15) Particle Data Group, Rev. Mod. Phys. **52** (1980), S1.
R. Partridge et al., Phys. Rev. Letters **44** (1980), 712.
M. Oreglia et al., Phys. Rev. Letters **45** (1980), 959.
16) D. Andrews et al., Phys. Rev. Letters **44** (1980), 1108.
T. Böhringer et al., Phys. Rev. Letters **44** (1980), 1111.
17) G. Segrê and J. Weyers, Phys. Letters **62B** (1976), 91.
H. Genz, Lett. Nuovo Cim. **21** (1978), 270.
18) T. Inami, K. Kawarabayashi and S. Kitakado, Phys. Letters **61B** (1976), 60.
C. Rosenzweig, Phys. Rev. **D13** (1976), 3080.
H. Fritzsch and J. D. Jackson, Phys. Letters **66B** (1977), 365.
T. N. Pham, Phys. Letters **87B** (1979), 267.
19) Y. Yamaguchi, in *1967 Tokyo Summer Lectures in Theoretical Physics,* edited by G. Takeda and Y. Hara (Syōkabō and W. A. Benjamin inc., 1968), p. 24.
K. Kawarabayashi and S. Kitakado, Prog. Theor. Phys. **40** (1968), 1398.
20) S. Weinberg, Phys. Rev. Letters **17** (1966), 616.
Y. Tomozawa, Nuovo Cim. **46A** (1966), 707.
S. Weinberg, in *A Festschrift for I. I. Rabi,* edited by Lloyd Motz (New York Academy of Sciences, 1977).
21) G. Ebel, H. Pilkuhn and E. Steiner, *Proceedings of the Lund International Conference on Elementary Particles* (1969), 411.

22) R. J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys. Letters **88B** (1979), 123; **91B** (1980), 487E.
See also, V. Baluni, Phys. Rev. **D19** (1979), 2227.
23) G. F. Chew, M. L. Goldberger, F. E. Low and Y. Nambu, Phys. Rev. **106** (1957), 1345.
24) H. R. Hicks et al., Phys. Rev. **D7** (1973), 2614.
25) M. Konuma, K. Ohya, T. Oka and Y. Yamamoto, Prog. Theor. Phys. Suppl. No. 60 (1976), 203.
26) P. G. O. Freund and Y. Nambu, Phys. Rev. Letters **34** (1975), 1645.
J. Arafune, M. Fukugita and Y. Oyanagi, Phys. Letters **70B** (1977), 221.
M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979), 519.
See also Refs. 18) and G. Chew and C. Rosenzweig, Phys. Reports **41C** (1978), 5.
27) M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B166** (1980), 493.

Note added:

An evaluation of g' was also made by Di Vecchia using an effective Lagrangian involving baryons (Schladming Lectures, 1980). Here we comment on main differences between his and our analyses. First, baryons in his scheme are assigned differently from ours; that is, baryons in his scheme transform like $(3, 3^*)$ and $(3^*, 3)$ under the chiral $U(3)\otimes U(3)$, while in our scheme they (L and R) transform like $(8, 1)$ and $(1, 8)$. Thus there is *no flavor singlet baryons in our scheme.* Second, although we have taken D_0 and D_8 (F_0 and F_8) as free parameters, it may be argued that they should be fixed by the quark mass matrix $M_q = \mathrm{diag}(M_u, M_d, M_s) \propto \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$. It is then shown that D_0 and F_0 are determined from the baryon mass spectrum with the results: $D_0 = -13.3\,\mathrm{MeV}$, $F_0 = -43.6\,\mathrm{MeV}$. These give $|g'| = 0.028$, which coincides with his result. In this case, we *predict* $\Delta a_{\pi N} = 0.03 m_\pi^{-1}$ and $\Delta a_{KN} = -0.07 m_\pi^{-1}$. Third, the Lagrangian adopted by Di Vecchia is not of the most general type consistent with the OZI rule. In order to assure the OZI rule for interactions involving baryons made of three quarks, particular combinations of interaction terms such as $\mathrm{Tr}(\bar{L}M_qRU) - \mathrm{Tr}(\bar{L}RM_q)\cdot\mathrm{Tr}(U)$ must appear in the Lagrangian. [See Refs. 19) and also H. Sugawara and F. von Hippel, Phys. Rev. **145** (1966), 1331.] Although these modifications are irrelevant for evaluating g', they affect other physical quantities (for example, Δa_{KN} and $\Delta a_{\pi N}$). In the $1/N$ expansion scheme for mesons it was legitimate to drop interactions which are expressed as products of traces, since those terms are in fact higher orders of $1/N$expansion. However, there does not seem to exist any convincing argument for neglecting such terms involving baryons.