On the perfomance of Hirschman Optimal Transform for Reduction of PAPR in UFMC

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On the performance of Hirschman Optimal Transform Pre-coded UFMC for reduction of PAPR

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Abstract—**Universal filtered multi carrier (UFMC) system is considered to be the best choice for 5G waveform when compared with orthogonal frequency division multiplexing (OFDM), filtered OFDM (f-OFDM), filter bank multi carrier (FBMC) and generalized FDM (GFDM) in terms of spectral leakage, complexity and compatibility with 5G. The all multi carrier systems are suffers from high peak to average power ratio (PAPR) causes non linear functioning in receiver. In this paper, we proposed a Hirschman optimal transform (HOT) based pre-coding for reducing the PAPR in UFMC systems. Results reveals that HOT pre-coded UFMC promising less PAPR compared with discrete cosine transform (DCT), discrete Hartley transform (DHT) and discrete sine transform (DST) and discrete Fourier transform (DFT).**

Keywords – Peak to average power ratio (PAPR); OFDM; Universal filter multi carrier (UFMC); Pre-coding;

I. Introduction

In today's fierce world, the up gradation and exploration on 5G wireless technologies is the current trending topic which helps in advancing the car to car communication, IOT, business and industry. Fifth-generation (5G) mobile networks [1]-[2] provide the enhanced version of the fourth-generation (4G) networks with the remarkable improvements in latency, data capability and energy efficiency. The multi carrier technique which is used in 4G technology is OFDM (Orthogonal Frequency Division Multiplexing) because of its low complexity [3]. In OFDM system, the data is transmitted in the parallel streams by orthogonal subcarriers to avoid inter carrier interference which can be achieved with synchronization.

Although, due to the extensive demand in 5G networks, some of the OFDM characteristics may retard the performance with the use of this waveform. The research is done on the improvements in the OFDM and new waveform contenders like filter bank multi carrier (FBMC), universal filtered multi carrier (UFMC) which can be used in the 5G technology [4]. One of the advances of OFDM is the cyclic prefix (CP) OFDM waveform. The multi path effect in the propagation channel which produces inter symbol interference (ISI) effect in the OFDM is fortified by the CP-OFDM. In CP-OFDM, the cyclic prefix is for the symbol separation which reduces the ISI effect. But with increase in bulk data, the spectral efficiency decreases in CP-OFDM. The waveform contender like FBMC is designed with the staggered modulated multi tone (STM) and a filter which provides a significant feature on interference resistance and by limiting the role of orthogonality compared to the CP-OFDM. The spectral efficiency of the FBMC is observed as 1, for the long stream of data but decreases with the short transmission. The UFMC is the trade-off between the OFDM and FBMC i.e., it is a waveform which acquires both the advantages of OFDM (low complexity) and FBMC (interference resistance). The UFMC performs filter operation on the successive sub carriers which reduces both latency transmission and complexity. Among these multicarrier waveform contenders, the system which shows the superiority in the BER results is selected for the 5G networks i.e., the UFMC technique performs better when compared with other techniques.

PAPR is one of the major problem existed in all the multi carrier .Transmitted signal can have high peak values in the time domain since many sub carriers are added via IFFT operation. Hence Multi carrier communication systems are known to have high PAPR compared with SC communication systems as it reduces the signal to quantization noise ratio (SQNR) of ADC and DAC while degrading the efficiency of the power amplifier in the transmitter. Various techniques based on signal distortion, multiple signaling & probabilistic and coding based techniques are introduced to reduce PAPR. It is proposed to investigate the limitations of existing techniques for channel estimation, offset estimation & PAPR reducing techniques and improve the performance of UFMC System using HOT pre-coding. The paper is structured as follows: the brief view of principles of waveform contenders-OFDM, CP-OFDM, FBMC and UFMC. Then we describe the pre-coded UFMC with various transforms for PAPR reduction and compared with HOT-UFMC.

A. *Orthogonal Frequency Division Multiplexing (OFDM)*

OFDM is ideal platform for both wired and wireless communication i.e., IEEE, Wi-Fi, as it provides its significance in low complexity and high data rate. It is a form of frequency division multiplexing modulation technique which uses the concept of orthogonality. In OFDM signal the data is transmitted in a parallel by splitting the data into multiple carriers. The overall data rate is divided among the multiple carriers i.e., each sub carrier with low data rate and guard between them will avoid the symbol interference [5]. And if the carriers are overlapped in the signal at the receiver, no inter carrier interference is observed since each sub carrier is placed orthogonal to each other. Therefore with the perfect synchronization, the orthogonality between the sub-carriers is achieved [6]. The mapping scheme used is QAM (quadrature amplitude modulation), and at the transmitter side the modulation is carried out through IFFT (Inverse Fast Fourier Transform) whereas at the receiver the demodulation is carried out through FFT (Fast Fourier Transform). As it is easy to maintain zero forcing equalization in the frequency domain, the OFDM system is considered as the effortless choice among the other proposed contenders. Also it is considered as the spectrally effective technique as it can transmit high number of carriers simultaneously with proximity. While the system is simple and effective in nature, it can only be achieved with the strict synchronization which requires high battery consumption (high Peak to Average Power

Ratio). And the filter which converts the data symbols in rectangular shaped (time domain) to sinc shape (frequency domain) will result in undesired levels(high Out Of Band Emission).

A communication system with multicarrier modulation transmits N complex valued source symbols S_n where $n = 0, 1, 2,..., N-1$ in parallel on N sub carriers. The source symbols may be obtained after source and channel coding, interleaving and symbol mapping based on the application. The source symbol duration T_d of the serial data becomes $T_s=N\ T_d$ as bandwidth is reduced as $\frac{B}{N}$ in serial to parallel conversion, Hence the symbol duration will be increased in the same scale [7].

The N parallel modulated data streams with N sub carriers are added to form an OFDM signal is

$$x(t)=\sum_{n=0}^{N-1} S_k(n)e^{j2\pi f_n t} \qquad 0 \le t \le T_s \tag{1}$$

All these N orthogonal sub carriers are located at $f_n = \frac{N}{T_s}$; $n=0, 1, 2,.... N-1$ with a carrier spacing of $f_{space} = \frac{1}{T_s}$. The key advantage of using OFDM is that multi-carrier modulation can be implemented by replacing the modulator and demodulator banks with Discrete Fourier Transform (DFT) and Inverse Discrete Fourier Transform (IDFT) or a more computationally efficient Inverse Fast Fourier Transform (IFFT) [8]. When sampling the complex envelope $x(t)$ of an OFDM symbol with rate $\frac{1}{T_d}$ the samples is given as

$$x(v)=\frac{1}{N}\sum_{n=0}^{N-1} S_k(n)e^{\frac{j2\pi n v}{N}} \qquad v=0,1,2,....N-1 \tag{2}$$

Though OFDM is considered as an immense leap in the technological development in 4G, there are some requirements (less PAPR and OOB emission due to rectangular window and cyclic prefix) which hinder the performance of it to be deployed for the 5G technology. So, the research is done in advancing the OFDM in order to operate in 5G standard which is continued in next section.

B. Universal Filtered Multi Carrier

An F-OFDM performs filtering operation on whole frequency band which results in the overlapping of multicarrier symbols. The ISI among them is eliminated by adding CP to each symbol. The OOB emission in F-OFDM is less compared to other contenders. The FBMC applies filtering on per-subcarrier wise with poor localization in frequency domain presents strong OOB emission. The new waveform contender UFMC is originated from the filtered OFDM and FBMC. The UFMC as shown in Fig. 1 performs block wise filtering which gives additional flexibility to the system and eliminates the FBMC drawbacks [9]. At the transmitter side, s_k is a set of complex valued data symbols which are divided into B blocks and are transformed with the N-point IFFT. The filtering is applied on each sub-band independently. The filter length is decided based on the tradeoff between latency and OOB emission. For the longer filters, the OOB emission and frequency misalignment is less whereas short filters leads to shorter multicarrier symbols i.e., low latency at the receiver side, the signals are windowed to convert from serial to parallel. The zero padding is added while transforming with $2N$ point FFT. The data symbols are estimated with the frequency domain processing and subcarrier equalizer. For a specific multicarrier symbol of K the time domain vector is calculated with filtered components on sub band basis, FFT length N and filtering length L. Where indexed i and complex QAM symbol n_i for each of B sub bands. The symbols are time domain with the IDFT matrix. V_i is the inverse Fourier matrix with respective sub-band positions and within the available frequency range. With the filter length and FFT size the symbol duration with $N+L-1$ is initiated. The user can use different subcarrier spacing and filter length. E.g. let a user a uses N_1 as FFT size and L_1 as filter length whereas user b uses N_2 and L_2 respectively. The UFMC is designed with the same symbol

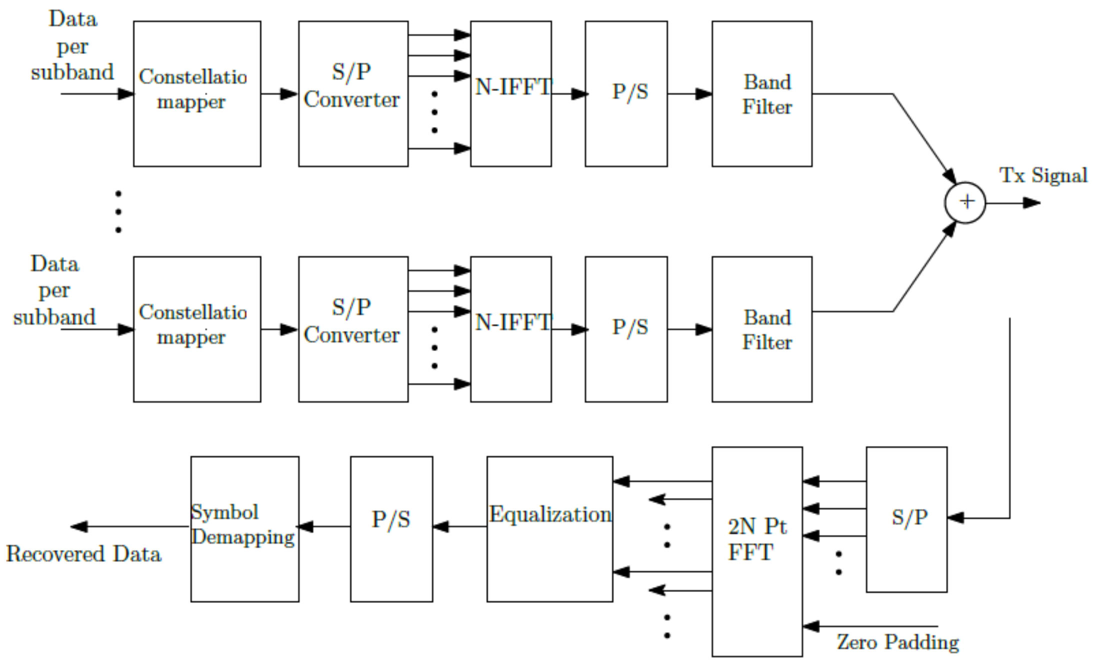


Figure 1. Basic UFMC structure

duration it means $N_1+L_1-1 = N_2+L_2-1$.This flexibility makes the UFMC more attractive to the adaptive modeling schemes. UFMC captures the strengths of F-OFDM and FBMC. Sub carrier grouping and Sub band filtering and UFMC is perfect solution for 5G[10],[11],[12]. UFMC band data is demonstrated in detailed way for the following parameters: FFT length is 64, Band size is 10 (10 data carriers per band), number of bands are 4, band pass filter length as 16 same as length of cyclic prefix in case of OFDM and Sub band offset is 12. Statistics of individual bands, filtered bands and UFMC transmitting signal is represented Figure 2 and in Figure 3. Sub band data spectrum, filter magnitude response and UFMC filtered output spectrum for a single band are shown in the Figure 4

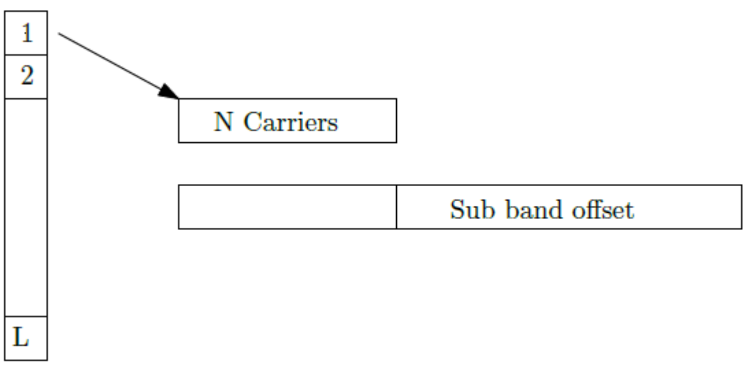


Figure 2. UFMC band

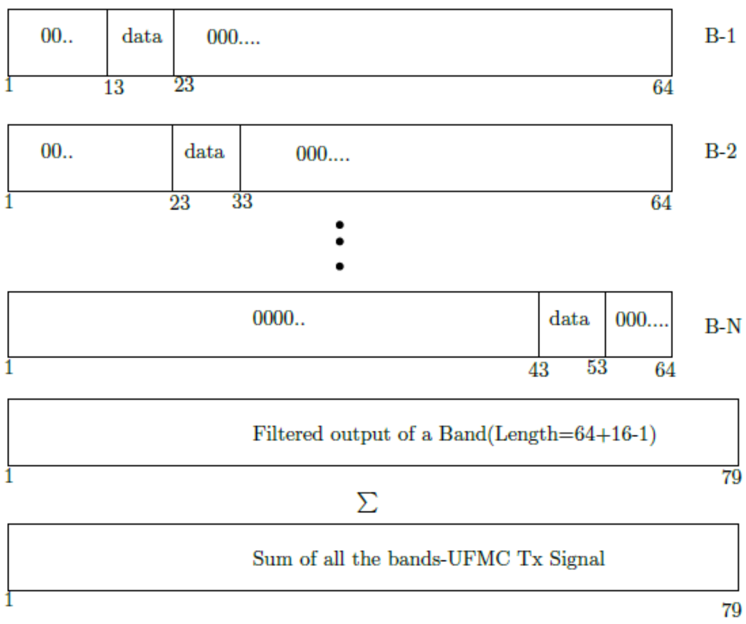
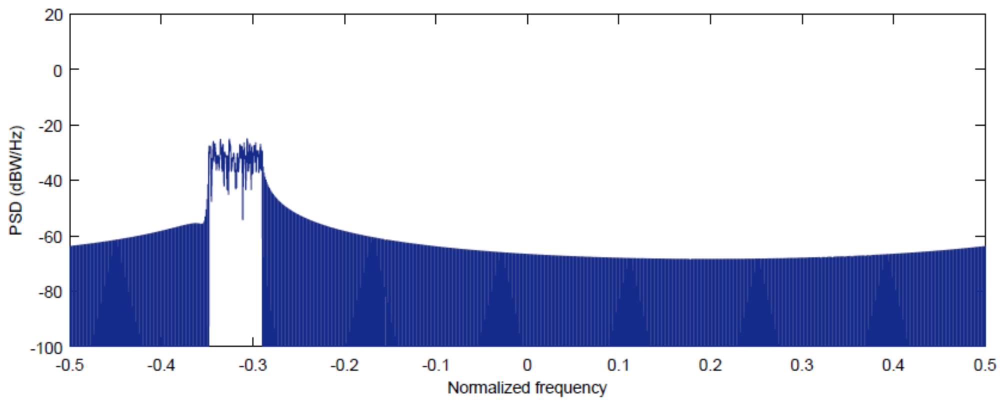
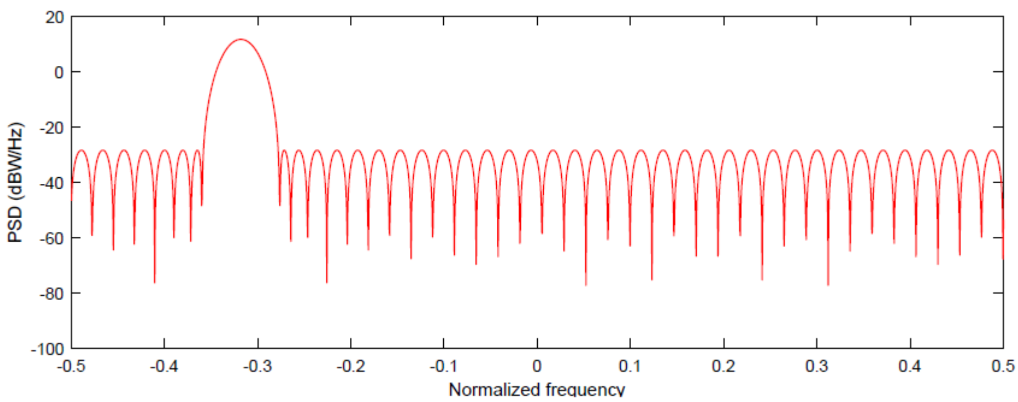


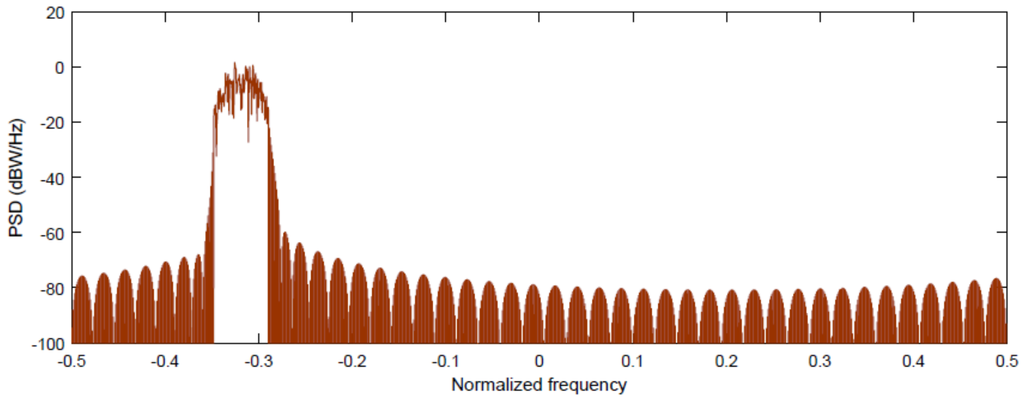
Figure 3. UFMC band Structure



(a) UFMC Sub-band-1 spectrum



(b) UFMC Sub-band-1 Filter response



(c) UFMC filtered output spectrum

Figure 4. Band-1 Simulations

C. *PAPR*

PAPR is one of the major challenges in multicarrier communication systems. High PAPR can be observed due to ifft operation or combining various data carrier sub carriers for transmission. large amount of peak values in the transmitting signal causes high PAPR and makes non linear function in system blocks. It can also increase as the number of sub carriers in a symbol while a system with large number of sub carrier increases the efficiency of the system. PAPR is calculated the following expression is

$$PAPR = \frac{max\left(\left|z(n)\right|^2\right)}{E\left[\left|z(n)\right|^2\right]} \tag{3}$$

CCDF is used to analyze the efficiency of a PAPR reduction technique. The high PAPR level in the transmitter leads to the performance degradation of power amplifiers and increases the OOB radiation. Therefore, to increase the efficiency of the UFMC system there is a need to reduce the PAPR.

D. *Proposed Method*

Proposed method for reducing PAPR in UFMC system utilizes pre-coding techniques at the transmitter [13]. In pre-coding, the performance various transforms like discrete Hartley transform (DHT), discrete sine transform (DST), discrete cosine transform (DCT) and Finally Hirschman optimal transform (HOT) are studied. The proposed method for PAPR reduction in UFMC system is shown in Figure 5.

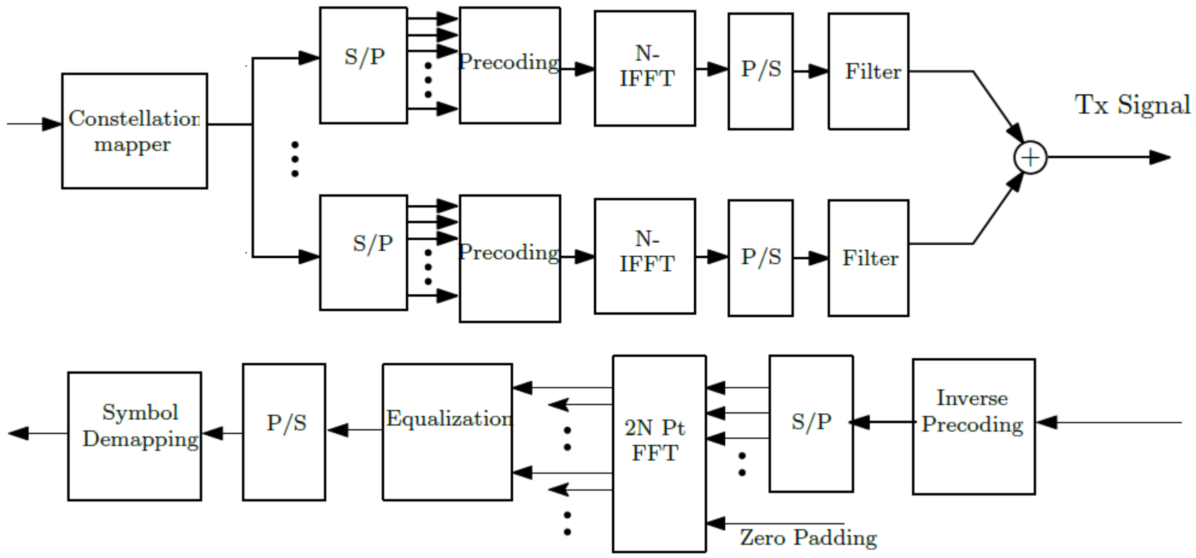


Figure 5. Proposed UFMC with HOT precoding

In this paper, a pre-coding based UFMC system is proposed in combination with conventional companders in order to minimize PAPR. The following transforms are used for comparative analysis. Form the above block diagram, each sub-band is pre-coded with a pre-coding matrix H. the pre-coded transmitting signals is represented as

$$x(v)=\frac{1}{N}\sum_{n=0}^{N-1}H_k(n)e^{\frac{j2\pi nv}{N}} \qquad v=0,1,2,....N-1 \tag{4}$$

where H_k is precoded k^{th} symbol.

Discrete Hartley Transform (DHT): Discrete Hartley transform is a real valued transform which uses additive *sin* and *cos* as basis signal sets. The forward and inverse transformations of DHT ia given as

$$X_k=\sum_{n=0}^{N-1}x_n\, cas\left(\frac{2\pi nk}{N}\right) \tag{5}$$

$$x_n=\frac{1}{N}\sum_{n=0}^{N-1}X_k\, cas\left(\frac{2\pi nk}{N}\right) \tag{6}$$

where $cas(\theta) = cos(\theta) + sin(\theta)$. DHT can also derive from DFT by subtracting real and imaginary parts of the coefficients.

Discrete Sine Transform (DST): Discrete Sine Transform uses only sine waves as basis signals. DST coefficients can be calculated using as

$$X_k = \sum_{n=0}^{N-1} x_n \sin\left(\frac{\pi n m}{N+1}\right) \quad (7)$$

$$x_n = \frac{1}{N} \sum_{n=0}^{N-1} X_k \sin\left(\frac{\pi n m}{N+1}\right) \quad (8)$$

Discrete Fourier Transform (DFT): Discrete Fourier Transform use complex exponential signals as basis signals. It utilizes the advantages of both DST and DCT. It also helps analyzing the phase information.

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{j2\pi nk}{N}} \quad (9)$$

$$x_n = \frac{1}{N} \sum_{n=0}^{N-1} X_k e^{\frac{j2\pi nk}{N}} \quad (10)$$

Discrete Cosine Transform (DCT): A Discrete Cosine Transforms (DCT) performs with limited coefficients and only on real values, as DFT is complex (real and imaginary) and has poor energy compaction (the ability to pack the energy of spatial sequence into the few coefficients) compared with DCT as it uses only cosine signals as basis.DCT coefficients are calculated as

$$X(k) = \alpha(k) \sum_{n=0}^{N-1} x(n) \cos\left[\frac{\pi(2n+1)k}{2N}\right] \qquad k = 0,1,\ldots.. N-1 \quad (11)$$

Where $\alpha(0) = \sqrt{\frac{1}{N}}, \quad \alpha(k) = \sqrt{\frac{2}{N}}; \quad k = 0,1,2,\ldots..N-1$

Hirschman Optimal Transform (HOT): We use the *K*-dimensional DFT as the originator signals for our $N=K^2$ dimensional HOT basis. Each of these basis functions must then be shifted and interpolated to produce the sufficient number of orthogonal basis functions that define the HOT [14]. We note that the DFT basis can be extended in a similar manner to produce an $N=KL$ dimensional transform. The number of sampling points and the up-sampling rate are not required to be equal. In other words, $N=K\times L$, where N is the length of the input, K is the number of sampling points and L is the up-sampling rate. When the up-sampling rate is one, the DHT equals the DFT. When the up-sampling rate equals the length of the input, the result obtained after the transform is the same as the input. Also if $K = L$, we have the HOT.

Because the HOT is based on periodic shifts of the DFT, the $N = K^2$-point HOT can be accomplished using K separate K-point DFT computations. Because the HOT requires lengths N that are squares of integers the HOT transform coefficients are determined using DFT in a faster way.

$$H\left(kr+l\right)=\frac{1}{\sqrt{K}}\sum_{n=0}^{N-1}x\left(kn+L\right)e^{-\frac{2\pi rn}{k}} \qquad 0\le r,\, l\le K-1 \tag{12}$$

and inverse transform can be computed using

$$x\left(kn+L\right)=\frac{1}{\sqrt{K}}\sum_{n=0}^{N-1}H\left(kr+l\right)e^{\frac{2\pi rn}{k}} \qquad 0\le n,\, l\le K-1 \tag{13}$$

The DHT transform matrix for a one dimensional input of length 6 is given in . In this matrix, $K = 3$ and $L = 2$. One can observe that DHT uses sparse matrices (i.e., most of the elements are zeros). The sparsity of this matrix increases as L increases. The non-zero elements are the twiddle factors used in DFT.

$K^2 \times K^2$ HOT Transform matrix is defined by $H_{K^2}=F_K\otimes I_K$, where I_K is $K\times K$ identity matrix.

$$F_{m,n}=\frac{1}{\sqrt{K}}e^{-\frac{j2\pi mn}{K}}; \quad 0\le m,n\le K-1$$

$$H = \begin{bmatrix} I & I & I \\ I & e^{-j2\pi/3} I & e^{-j4\pi/3} I \\ I & e^{-j4\pi/3} I & e^{-j8\pi/3} I \end{bmatrix}$$

Which is equivalent to

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & e^{-j2\pi/3} & 0 & 0 & e^{-j4\pi/3} & 0 & 0 \\ 0 & 1 & 0 & 0 & e^{-j2\pi/3} & 0 & 0 & e^{-j4\pi/3} & 0 \\ 0 & 0 & 1 & 0 & 0 & e^{-j2\pi/3} & 0 & 0 & e^{-j4\pi/3} \\ 1 & 0 & 0 & e^{-j6\pi/3} & 0 & 0 & e^{-j8\pi/3} & 0 & 0 \\ 0 & 1 & 0 & 0 & e^{-j6\pi/3} & 0 & 0 & e^{-j8\pi/3} & 0 \\ 0 & 0 & 1 & 0 & 0 & e^{-j6\pi/3} & 0 & 0 & e^{-j8\pi/3} \end{bmatrix}$$

Due to the sparsity of the DHT matrix, finding the DHT is computationally more efficient than the DFT. The zeros in the DHT matrix are not stored, which reduces the memory cost. The zeros also reduce the computational load since multiplying by zeros is selectively avoided by the algorithm.

II. Results And Conclusion

Simulations were performed in MATLAB on all the 5G waveform contenders for the following specifications provided in Table as per the standards provided in [15], [16], [17]. It can be clearly observed that the out band emissions with various waveforms are depicted in Figure7. Even though FBMC has sharp magnitude spectrum it has some limitations in terms of complexity. High Peak to Average Power Ratio (PAPR) is one of the challenging problems in OFDM based communication systems [18] appeared as result of addition of sub carriers at the transmitter. So many methods are proposed to reduce PAPR in [19], [20], [19], [21]. Simulations were performed with the following parameters: FFT length is 1024, Number of sub-bands are 10, subcarriers per band are 60 and filter length is chosen as 40. With these specifications, PAPR of each pre-coded UFMC compared and analyzed with 4-

QAM, 16-QAM, 64-QAM and 256-QAM. It can be clearly observed from the Table I and simulation results show the superiority of HOT pre-coded UFMC and from the spectral comparisons shown in Figure 6, UFMC is also has promising spectral efficiency compared with OFDM, F-OFDM and other waveform contenders except FBMC.

Table 1: PAPR comparisons

Scheme	4-QAM	16-QAM	64-QAM	256-QAM
UFMC	7.5406	8.9114	8.1418	7.4515
HOT-UFMC	7.274	7.2611	7.8521	6.6305
DCT-UFMC	7.599	7.8594	9.0851	8.1169
DFT-UFMC	7.452	9.029	8.6081	7.6530
DST-UFMC	7.763	8.4859	9.2035	8.3376
DHT-UFMC	8.496	7.7821	8.0763	7.7673

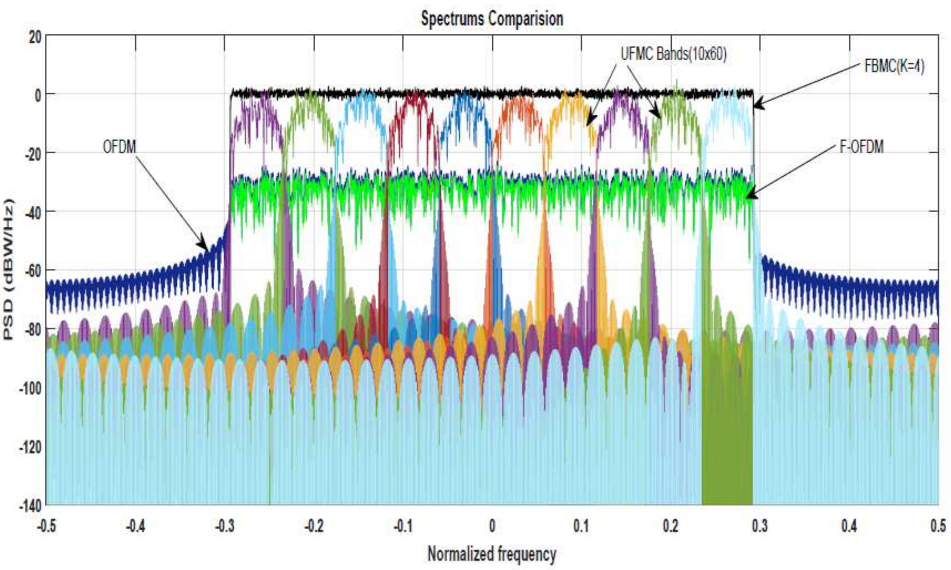
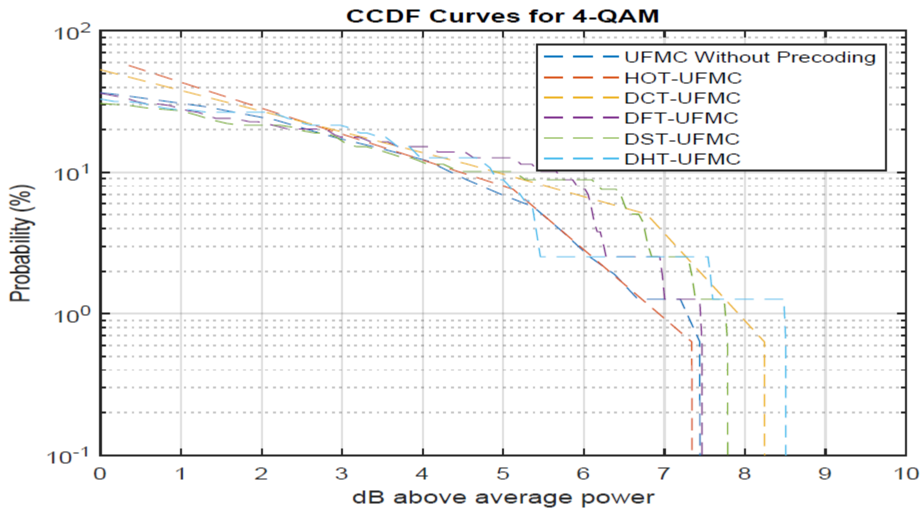
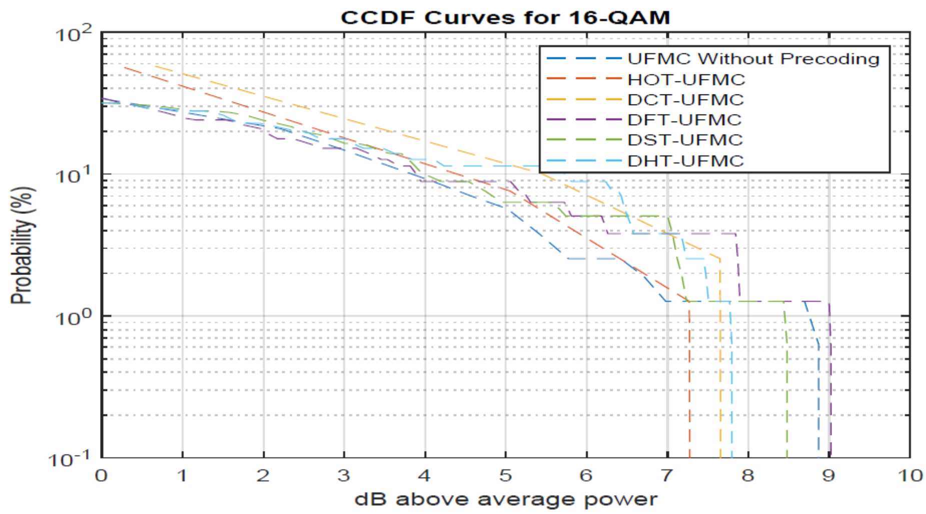


Figure 6. Spectrum comparisons

CCDF comparisons were made on various pre-coding techniques i.e, DFT-UFMC, DHT-UFMC, DCT-UFMC and DSTUFMC with the proposed method HOT-UFMC with the mentioned simulation parameters. The CCDF curves for various pre-coded UFMC are depicted in Figure7. From the CCDF curves it can be clearly observed that the probability of amount of dB greater than the average power is less compared to others. For example in Figure 7(a) HOT pre-coded UFMC is better than around 3dB when compared to UFMC and 2dB when compared to other pre-coding techniques.





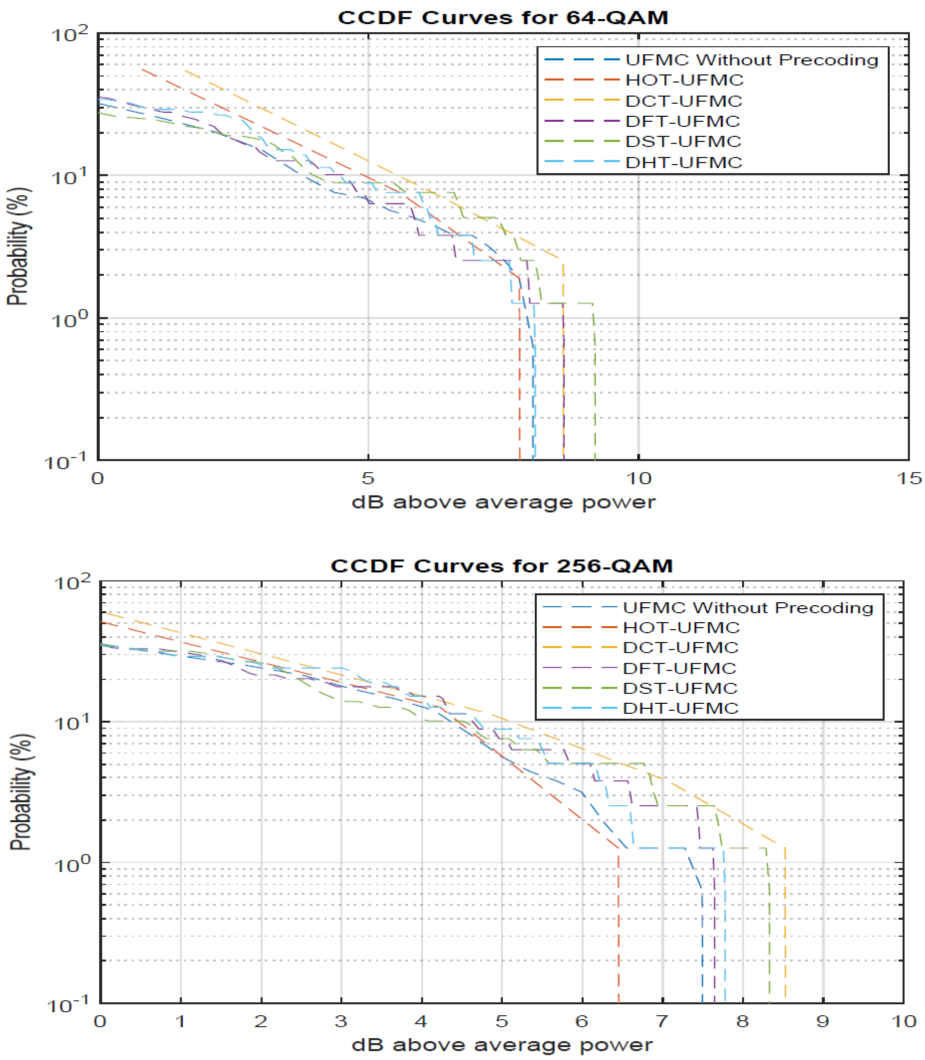


Figure 7. CCDF comparisons

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Declarations

Funding: The authors did not receive support from any organization for the submitted work.

Conflicts of interest: Not applicable

Availability of data and material (data transparency): Yes

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

*Code availability (software application or custom code): Yes

The code generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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