On the performance of multi-hop wireless relay networks

3 authors:

Wael Jaafar
Polytechnique Montréal
16 PUBLICATIONS   **51** CITATIONS

SEE PROFILE

Wessam Ajib
Université du Québec à Montréal
129 PUBLICATIONS   **500** CITATIONS

SEE PROFILE

David Haccoun
Polytechnique Montréal
170 PUBLICATIONS   **1,858** CITATIONS

SEE PROFILE

RESEARCH ARTICLE

On the performance of multi-hop wireless relay networks

Wael Jaafar[1*], Wessam Ajib[2] and David Haccoun[1]

[1] Department of Electrical Engineering, École Polytechnique de Montréal, Montreal, Canada
[2] Department of Computer Sciences, Université du Québec à Montréal, Montreal, Canada

ABSTRACT

User cooperation has evolved as a popular coding technique in wireless relay networks (WRNs). Using the neighboring nodes as relays to establish a communication between a source and a destination achieves an increase of the diversity order. The relay nodes can be seen as a distributed multi-antenna system, which can be exploited for transmit diversity by using distributed space–time block coding (STBC). In this paper, we investigate the bit error rate (BER) of multi-hop WRNs employing distributed STBC at the relay nodes. We develop the general model of WRNs using distributed STBC, and we derive the pairwise error probability and an approximation of the BER. We examine the impact of several parameters, such as distributed STBC at the relays, the number of relays, the distances between the nodes, and the channel state information available at the receivers, on the BER performance of the multi-hop WRN. The obtained results provide guidelines about the expected error performance and the design of channel estimation for these networks. Copyright © 2011 John Wiley & Sons, Ltd.

KEYWORDS

relay networks; space–time block coding; channel state information; multi-hop

*Correspondence

Wael Jaafar, Department of Electrical Engineering, École Polytechnique de Montréal, Montreal, Canada.
E-mail: wael.jaafar@polymtl.ca

1. INTRODUCTION

Multiple-input–multiple-output (MIMO) technology is widely recognized as an attractive advance in wireless communications [1,2]. MIMO systems allow improvement of the transmission rate using spatial multiplexing or increase in the diversity gain using space–time coding. However, these gains are obtained at the cost of multiple radio frequency front ends at both the transmitter and the receiver, but the sizes of mobile handsets often do not allow the deployment of multiple antennas. With the purpose of exploiting the potential of MIMO technology, cooperative communication has become an attractive technique to benefit from MIMO gains by employing relays between the source and the destination in order to set up virtual distributed MIMO systems.

In wireless relay networks (WRNs), several relays work jointly to communicate reliably a source node's information to a destination node. The main advantage is the increased diversity provided by several paths between the source and the destination provided by the relay nodes.

User cooperation in wireless networks was first investigated by Sendonaris *et al.* [3,4] for cellular networks and by Laneman *et al.* [5,6] for *ad hoc* networks, whereas the information theoretical capacity of relay channels was investigated earlier [7,8]. A new two-step cooperative strategy, called *distributed space–time coding*, has been proposed in [9]. Then, the authors of [10] proposed new designs of distributed space–time block coding (STBC) from orthogonal STBC and quasi-orthogonal STBC (QOSTBC) for WRNs. They showed that such designs implemented on amplify-and-forward (AF) relays achieve a higher diversity than those implemented on decode-and-forward (DF) relays with multiple relay nodes.

Schemes for multiple antennas in WRNs have also been examined. In [11,12], the authors investigated the spatial diversity of a WRN with multiple antennas at both the source and the destination. Assuming that single-antenna relays exploit distributed STBC, they showed that the system's diversity is proportional to the smallest number of antennas at the source or the destination multiplied by the number of relay nodes.

Table I. Observations and consequences.

Parameter	Impact; consequence
Distributed STBC at one or many relay stages	Use distributed STBC at the stages closer to the source node for AF relaying.
	Use distributed STBC at all stages or all except the last one with a MIMO destination for DF relaying.
Different STBC at different stages	The highest diversity for AF relaying is achieved when stage 1 uses STBC with the largest dimension.
	For DF relaying, the same diversity is achieved for any distributed STBC, but a coding gain is obtained when the STBC with the largest dimension is employed at the first stage.
Number of relays at a stage	For AF relaying, the number of relays at a stage should be close to the dimension of the STBC used at the previous stage.
	For DF relaying, when the number of available relay nodes is limited, it is more advantageous to have more relays at the stages closer to the source node and fewer relays at the last one.
Number of stages	More reliable transmissions as the number of stages increases.
Different distances between nodes	For AF relaying, relay nodes should be closer to the source node.
	For DF relaying, the stages should be ordered in an increasing distance from source to destination.
Imperfect CSI	BER degrades severely when channel estimation errors happen at the hops close to the source node.

STBC, space–time block coding; AF, amplify and forward; MIMO, multiple input–multiple output; DF, decode and forward; CSI, channel state information; BER, bit error rate.

Recently, research has focused on multi-hop (multi-stage) transmission in cooperative communications. One stage is defined by a set of relay nodes, which for simplicity are assumed to be all geographically located at approximately the same distance from the source node. The authors of [13,14] proposed multi-hop protocols for WRNs with AF relaying nodes, and in [15], the authors provided a technique to build distributed STBC for multi-hop WRNs with unitary code matrices.

Even though user cooperation was first developed for single-antenna relay nodes, it is important to investigate the performance of multi-antenna user cooperation. In fact, in the context of mesh networking, different types of inter-connected wireless nodes may have different numbers of antennas, and in the future, wireless nodes will be equipped with multiple antennas, as for example in Long-Term Evolution systems [16]. Moreover, not much work has been devoted to investigating distributed STBC when used in multi-hop WRNs [9–15]. Hence, work in this area is quite important for getting guidelines about the minimal and required resources such as the number of relays, the number of stages, etc., that are needed for the multi-hop WRN to operate efficiently as well to estimate the expected error performance in terms of bit error rate (BER) in real-world environment.[†]

Finally, channel estimation and channel state information (CSI) are key issues for the implementation of the receivers. Previously, perfect CSI was typically assumed to be known at the receiver, which of course may not always correspond to real-world environment. Obtaining accurate CSI requires devoting part of the resources to do it, such as adding pilot symbols to the transmitted information, which may reduce the efficiency of the communication. In [18], the authors showed that for MIMO systems, the mutual information is limited by the channel estimation errors (CEE) at a high signal-to-noise ratio (SNR). In [19,20], we studied the impact of imperfect CSI on the two-hop and L-hop ($L > 2$) WRNs, respectively, showing that CEE occurring on the last hop has always less impact on the error performance than CEE at the hops closer to the source node.

The main contributions of this paper are described as follows. First, we propose a general model of a multi-hop WRN using distributed STBC at the multi-antenna nodes. We then derive the pairwise error probability (PEP) and a bit error probability approximation for such networks where for simplicity we assumed the binary phase-shift keying (BPSK) modulation. Finally, we investigate and evaluate the impact of different parameters on the error performance in terms of the BER. The considered parameters include the used distributed STBC, the number of relays, the number of stages, and distances between nodes and CEE at the receivers. All these results are important for providing observations and guidelines (summarized in Table I) about the configuration of the WRN in order to improve the expected performance of the multi-hop multi-antenna WRN.

The paper is organized as follows. In the next section, we detail the system model including the network and the channel models. In Section 3, we present distributed STBC to be used in multi-hop WRNs with multi-antenna relay nodes. Section 4 derives the BER approximation of the multi-hop multi-antenna WRN using AF or DF relaying. In Section 5, we present and discuss the numerical and simulation results. Finally, Section 6 concludes the paper.

Throughout the paper, we will use the following notation. Bold letters denote matrices (uppercase) and vectors

---

[†]This work has been partially published in [17].

(lowercase). $\mathbf{A}^t$, $\mathbf{A}^*$, $\bar{\mathbf{A}}$, and $\|\mathbf{A}\|_F$ denote the transpose, the conjugate transpose, the conjugate, and the Frobenius norm of a complex matrix $\mathbf{A}$, respectively. $E\{.\}$ denotes the expectation, and $\mathbf{I}_{T_0}$, $\mathbf{0}_{T_0}$, and $\mathbf{1}_{T_0}$ are the $T_0 \times T_0$ identity matrix, matrix of 0's, and matrix of 1's, respectively. We also define the "matrix transpose" as

$$\begin{bmatrix} \mathbf{A}_1 & \dots & \mathbf{A}_{T_0} \end{bmatrix}^{mt} = \begin{bmatrix} \mathbf{A}_1^t & \dots & \mathbf{A}_{T_0}^t \end{bmatrix}^t = \begin{bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_{T_0} \end{bmatrix},$$

where $\mathbf{A}_i$ can be a vector or a matrix ($i = 1, \dots, T_0$).

2. SYSTEM MODEL

2.1. Network model

As depicted in Figure 1, we assume an L-hop WRN composed of one source node with M_0 antennas, K relay nodes distributed over $L-1$ relay stages (for simplicity, we call it a stage), and one destination node with M_L antennas. We assume that the source node is denoted "node 1 at stage 0" and the destination is "node 1 at stage L." The number of relays at the lth stage is K_l, with $\sum_{l=1}^{L-1} K_l = K$, and the kth relay at the lth stage is equipped with $M_k^{(l)}$ antennas. The total number of antennas at stage l is then $M^{(l)} = \sum_{k=1}^{K_l} M_k^{(l)}$. Let $d_{s,d}$ and $d_{k,k'}^{(l)}$ be the physical distances between the source and the destination and between node k at the $(l-1)$th stage and node k' at the lth stage ($l = 1, \dots, L$), respectively. We assume that $M_L \geq M^{(l)}$, $\forall l = 1, \dots, L-1$. The relays do not provide additional traffic to the destination, and the nodes are assumed to be "half duplex."

In order to realize a multi-hop communication, we assume that a path ("chain of stages") has been set up by a "routing protocol" on layer 3 and that a certain resource (time or frequency) is available for the end-to-end transmission [21]. At the medium access control (MAC) layer, different protocols could be considered for the multi-hop WRN using distributed STBC. In [22,23],

the authors explored a cross-layer protocol where centralized distributed STBC is used at the physical layer in a multi-hop fashion. The proposed MAC protocol adopts a hop-by-hop approach. In contrast to this proposition, the authors in [24] implemented a path-centric MAC protocol, which reserves a multi-hop path between the source and the destination, facilitates the relay selection, and coordinates the cooperative transmissions in the multi-hop WRN. The proposed MAC protocol is a modified path access control protocol [24,25]. Implementing distributed STBC in a realistic cooperative system is difficult and generates many challenges. In [24], these challenges, occurring at the physical, MAC, and routing layers, have been discussed, and some practical solutions have been proposed.

The communication is performed over L hops, where each hop duration is T symbol periods. At hop 1, the source node broadcasts T information symbols to the relays of stage 1. The relays at stage l ($l = 1, \dots, L-1$) amplify (or detect and decode) the received signals and send them over hop $(l + 1)$. It is important to note that DF relaying in this case differs from selection DF, that is, the relay nodes do not have to satisfy a given SNR threshold in order to forward the regenerated source's data. At this point, the relay nodes of a given stage transmit simultaneously the received signals using a distributed STBC scheme during T symbol periods, where T is also the dimension of the STBC used at any stage.

We assume perfect synchronization between the relays of each stage. In our model, the relay stages are chosen in such a way that all relay nodes of adjacent stages are connected and there is no communication between relays of non-adjacent stages. This assumption is used when successive relay stages are located in increasing distances from the source to the destination. Relay nodes are chosen to belong to the same stage when they all have good SNRs. In practical systems, the reception of a node at stage l is interfered by the simultaneous transmissions of the nodes at stages $l-1-2l'$ and $l+1+2l''$ ($0 \leq l' \leq [(l-1)/2]$ and $0 \leq l'' \leq [(L-l-2)/2]$, where [.] is the floor function). In our model, because of the relatively long physical distances between non-adjacent relay stages, this interference is assumed to be negligible.
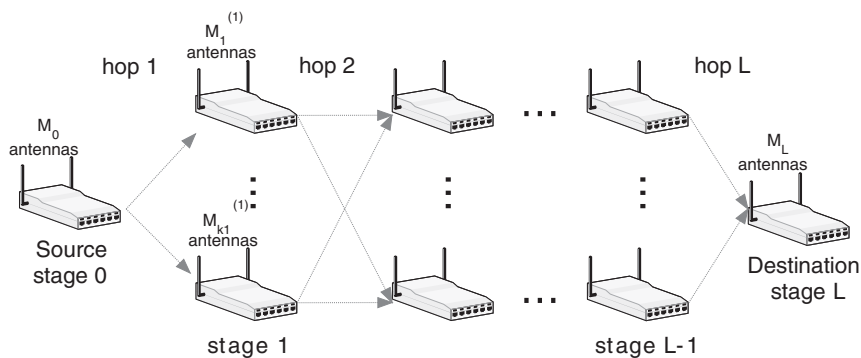

Figure 1. L-hop multi-antenna wireless relay network.

2.2. Assumptions and channel model

2.2.1. Channel model.

We assume stationary terminals and quasi-static channel conditions for each period T. We also assume flat fading Rayleigh channels with independent identically distributed zero mean complex Gaussian distributed circularly symmetric random variables. We assume that CSI is not available at the transmitters whereas it is available at the relays and/or the destination. Let $\mathbf{H}_{k,k'}^{(l)}$ be the gain of the channel between node k at the $(l-1)$th stage and node k' at the lth stage where each independent coefficient has variance $\sigma_{k,k'}^{(l)\,2} = \left(d_{\mathrm{s,d}}/d_{k,k'}^{(l)} \right)^{\alpha}$ and α is the path loss exponent [26]. In AF relaying, we assume that the destination has knowledge of the entire equivalent channel calculated as a combination of the channels $\mathbf{H}_{k,k'}^{(l)}$. In DF relaying, we assume that each node has only knowledge of its channels with the nodes of the previous stage.

2.2.2. Imperfect channel state information.

Imperfect CSI can be assumed at the relay nodes and at the destination. In order to determine the impact of CEE on the BER, we assume the following channel model:

$$\tilde{\mathbf{H}}_{k,k'}^{(l)} = \mathbf{H}_{k,k'}^{(l)} + \frac{\Delta \mathbf{H}_{k,k'}^{(l)}}{\nu^{(l)}}, \forall l = 1, \ldots, L \qquad (1)$$

where $\Delta \mathbf{H}_{k,k'}^{(l)}$ is the matrix of errors between node k at stage $(l-1)$ and node k' at stage l. We assume that $\Delta \mathbf{H}_{k,k'}^{(l)}$ is composed of independent identically distributed zero mean complex Gaussian distributed circularly symmetric coefficients with variances inversely related to the channel estimation accuracy $e_{k,k'}^{(l)\,2} = \sigma_{k,k'}^{(l)\,2}/SNR_{\mathrm{ch}}^{(l)}(k,k')$ [27]. $\nu^{(l)}$ is a scalar coefficient whose value determines the dependency of the CEE on the SNR.

2.2.3. Power.

We assume that the transmit power is the same over all hops and is distributed in proportion to the number of antennas of each relay among the nodes of the same stage [19]. This power allocation may not provide an optimal solution but was chosen for simplicity. However, the authors in [10] established that this distribution is optimal in some cases and maximizes the expected receive SNR in two-hop WRNs with single-antenna relay nodes. We note by $P^{(l)}$ and P the transmit power at stage l and the total transmit power of the network, respectively, that is, $P^{(l)} = P/L$, $l = 0, \ldots, L-1$. The transmit power of node k at stage l is given by $P_k^{(l)} = P^{(l)} M_k^{(l)}/M^{(l)}$, where $M_k^{(l)}$ and $M^{(l)}$ are defined in the network model.

2.2.4. Analysis of the transmission on hop 1.

We denote by $\mathbf{R}_k^{(l)}$ and $\mathbf{X}_k^{(l)}$ the matrices of signals received and transmitted by node k at stage l ($l = 1, \ldots, L-1$), respectively. Hence, $\mathbf{R}_k^{(1)}$, whose size is $M_k^{(1)} \times T$, is given by

$$\mathbf{R}_k^{(1)} = c_0 \mathbf{H}_{1,k}^{(1)} \mathbf{S} + \mathbf{N}_k^{(1)}, \; k = 1, \ldots, K_1 \qquad (2)$$

where $c_0^2 = PT/LM_0$ is the transmit energy of any antenna at the source and $\mathbf{S}$ is an $M_0 \times T$ matrix of the transmitted symbols. $\mathbf{N}_k^{(l)}$ denotes the $M_k^{(l)} \times T$ matrix of an additive white Gaussian noise received by node k at the lth stage. $\mathbf{N}_k^{(l)}$ has zero mean and diagonal covariance matrix $E\left\{ \left(\mathbf{N}_k^{(l)} \right)^* \mathbf{N}_k^{(l)} \right\} = N_0 \mathbf{I}_T$, $l = 1, \ldots, L-1$. We have

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & \cdots & \mathbf{s}_{M_0} \end{bmatrix}^{mt}, \; \mathbf{H}_{1,k}^{(1)} = \begin{bmatrix} h_{1,k}^{(1)}(1,1) & \cdots & h_{1,k}^{(1)}(1,M_0) \\ \vdots & & \vdots \\ h_{1,k}^{(1)}(M_k^{(1)},1) & \cdots & h_{1,k}^{(1)}(M_k^{(1)},M_0) \end{bmatrix}$$

$$\mathbf{N}_k^{(1)} = \begin{bmatrix} \mathbf{n}_{k,1}^{(1)} & \cdots & \mathbf{n}_{k,M_k^{(1)}}^{(1)} \end{bmatrix}^{mt} \text{ and } \mathbf{R}_k^{(1)} = \begin{bmatrix} \mathbf{r}_{k,1}^{(1)} & \cdots & \mathbf{r}_{k,M_k^{(1)}}^{(1)} \end{bmatrix}^{mt}$$

where $\mathbf{s}_i = \mathbf{s}\mathbf{A}_{1,i}^{(0)} + \bar{\mathbf{s}}\mathbf{B}_{1,i}^{(0)} = \mathbf{s}^{\{1\}}\mathbf{C}_{1,i}^{(0)}$, $\mathbf{s}$ is the vector $1 \times T$ of the source information symbols, and $\mathbf{A}_{1,i}^{(0)}$ and $\mathbf{B}_{1,i}^{(0)}$ are the STBC code matrices associated with the ith antenna of node 1 at stage 0 (source node) of dimension $T \times T$. Because we consider only orthogonal or quasi-orthogonal codes, then $\mathbf{A}_{1,i}^{(0)} \neq \mathbf{0}_T$ and $\mathbf{B}_{1,i}^{(0)} = \mathbf{0}_T$ or $\mathbf{B}_{1,i}^{(0)} \neq \mathbf{0}_T$ and $\mathbf{A}_{1,i}^{(0)} = \mathbf{0}_T$; hence, $\mathbf{C}_{1,i}^{(0)}$ is the applied code matrix, and $\mathbf{s}^{\{1\}}$ is the vector of transmitted symbols or its conjugate. $h_{1,k}^{(1)}(a,b)$ denotes the coefficient of the channel between the ath antenna of node k at stage $l-1$ and the bth antenna of node k' at stage l, and $\mathbf{r}_{k,i}^{(1)}$ is the vector of signals received at the ith antenna of node k and is expressed by

$$\mathbf{r}_{k,i}^{(1)} = c_0 \mathbf{h}_{1,k}^{(1)}(i)\mathbf{S} + \mathbf{n}_{k,i}^{(1)} \qquad (3)$$

where $\mathbf{h}_{1,k}^{(1)}(i) = \left[\begin{array}{ccc} h_{1,k}^{(1)}(i,1) & \dots & h_{1,k}^{(1)}(i,M_0) \end{array} \right]$. In the next section, we study distributed STBC in multi-hop WRNs where the relay nodes perform AF or DF relaying and are equipped with multiple antennas.

3. DISTRIBUTED SPACE–TIME BLOCK CODING IN MULTI-HOP MULTI-ANTENNA WIRELESS RELAY NETWORK MODEL

In order to simplify the understanding of the model, we summarize in Table II all the notations and symbols used in the next section.

3.1. Amplify-and-forward relaying

At the second hop, the $M_k^{(1)} \times T$ matrix of signals sent by relay k at stage 1 is given by

$$\mathbf{X}_k^{(1)} = \left[\begin{array}{ccc} \mathbf{x}_{k,1}^{(1)} & \dots & \mathbf{x}_{k,M_k^{(1)}}^{(1)} \end{array} \right]^{mt} \tag{4}$$
$$\text{with} \quad \mathbf{x}_{k,i}^{(1)} = c_1 \left(\mathbf{r}_{k,i}^{(1)} \mathbf{A}_{k,i}^{(1)} + \bar{\mathbf{r}}_{k,i}^{(1)} \mathbf{B}_{k,i}^{(1)} \right)$$

where c_1 is the normalizing amplification factor applied at the antennas of any relay node, calculated in such a way that the transmission energy at each hop is the same (Appendix A). $\mathbf{A}_{k,i}^{(1)}$ and $\mathbf{B}_{k,i}^{(1)}$, of dimension $T \times T$, are the code matrices associated with the ith antenna of relay k, $\forall i = 1, \dots, M_k^{(1)}$ and $k = 1, \dots, K_1$. Combining (3) and (4), we obtain

$$\mathbf{x}_{k,i}^{(1)} = c_0 c_1 \left(\mathbf{h}_{1,k}^{(1)}(i) \mathbf{S} \mathbf{A}_{k,i}^{(1)} + \bar{\mathbf{h}}_{1,k}^{(1)}(i) \bar{\mathbf{S}} \mathbf{B}_{k,i}^{(1)} \right)$$
$$+ c_1 \left(\mathbf{n}_{k,i}^{(1)} \mathbf{A}_{k,i}^{(1)} + \bar{\mathbf{n}}_{k,i}^{(1)} \mathbf{B}_{k,i}^{(1)} \right) \tag{5}$$

Note that because we are considering only orthogonal or quasi-orthogonal codes, we have two possible cases. In the first one, $\mathbf{A}_{k,i}^{(1)} \neq \mathbf{0}_T$ and $\mathbf{B}_{k,i}^{(1)} = \mathbf{0}_T$; hence, $\mathbf{x}_{k,i}^{(1)} = c_1 r_{k,i}^{(1)} \mathbf{A}_{k,i}^{(1)}$. In the second case, $\mathbf{B}_{k,i}^{(1)} \neq \mathbf{0}_T$ and $\mathbf{A}_{k,i}^{(1)} = \mathbf{0}_T$. Then, $\mathbf{x}_{k,i}^{(1)} = c_1 \bar{r}_{k,i}^{(1)} \mathbf{B}_{k,i}^{(1)}$. For simplicity of notation, we note $\mathbf{A}$ as any code matrix associated with the signal to

transmit and $\mathbf{B}$ as any code matrix associated with the conjugate of the signal to transmit. Let $\mathbf{C}$ be a matrix of complex elements; we use the following notation: $\forall u \in \mathbb{N}$, $u \geq 1$,

$$\mathbf{C}^{\{u\}} = \left\{ \begin{array}{l} \mathbf{C}^{\{u-1\}} \text{ if the applied code matrix is } \mathbf{A} \\ \bar{\mathbf{C}}^{\{u-1\}} \text{ if the applied code matrix is } \mathbf{B} \end{array} \right\} \tag{6}$$

and $\mathbf{C}^{\{0\}} = \mathbf{C}$. Let $\mathbf{C}_{k,i}^{(1)}$ be the code matrix associated with the vector of symbols transmitted by the ith antenna of relay k at the first stage. Then,

$$\mathbf{C}_{k,i}^{(1)} = \left\{ \begin{array}{l} \mathbf{A}_{k,i}^{(1)} \text{ if the applied code matrix is } \mathbf{A}_{k,i}^{(1)} \\ \mathbf{B}_{k,i}^{(1)} \text{ if the applied code matrix is } \mathbf{B}_{k,i}^{(1)} \end{array} \right\} \tag{7}$$

We denote by $\mathbf{S}_{k,i}^{\{1\}}$ the equivalent matrix of symbols to be transmitted by the ith antenna of relay k at the first stage. Because we are using orthogonal STBC or QOSTBC, then $\mathbf{S}_{k,i}^{\{1\}} = \mathbf{S}^{\{1\}}$. Substituting the expressions of $\mathbf{S}_{k,i}^{\{1\}}$ and Equation (7) in Equation (5), we get

$$\mathbf{x}_{k,i}^{(1)} = c_0 c_1 \mathbf{h}_{1,k}^{(1)\{1\}}(i) \mathbf{S}_{k,i}^{\{1\}} \mathbf{C}_{k,i}^{(1)} + c_1 \mathbf{n}_{k,i}^{(1)\{1\}} \mathbf{C}_{k,i}^{(1)} \tag{8}$$

With the iteration over l stages, the received signals at node k_l of stage l ($k_l = 1, \dots, K_l$, $l = 1, \dots, L$) is given by

$$\mathbf{R}_{k_l}^{(l)} = \left[\begin{array}{ccc} \mathbf{r}_{k_l,1}^{(l)} & \dots & \mathbf{r}_{k_l,M_{k_l}^{(l)}} \end{array} \right]^{mt} \tag{9}$$

where

$$\mathbf{r}_{k_l,i_l}^{(l)} = \left(\prod_{s=0}^{l-1} c_s \right) \sum_{k_1=1}^{K_1} \dots \sum_{k_{l-1}=1}^{K_{l-1}} \sum_{i_1=1}^{M_{k_1}^{(1)}} \dots \sum_{i_{l-1}=1}^{M_{k_{l-1}}^{(l-1)}} \left(\prod_{p=2}^{l} h_{k_{p-1},k_p}^{(p)\{l-p\}}(i_p, i_{p-1}) \mathbf{h}_{1,k_1}^{(1)\{l-1\}}(i_1) \mathbf{S}^{\{l-1\}} \prod_{p'=1}^{l-1} \mathbf{C}_{k_{p'},i_{p'}}^{(p')} \right)$$
$$+ \sum_{s'=1}^{l-1} \left(\sum_{k_{s'}=1}^{K_{s'}} \dots \sum_{k_{l-1}=1}^{K_{l-1}} \sum_{i_{s'}=1}^{M_{k_{s'}}^{(s')}} \dots \sum_{i_{l-1}=1}^{M_{k_{l-1}}^{(l-1)}} \left(\mathbf{n}_{k_{s'},i_{s'}}^{(s')\{l-s'\}} \prod_{j=s'}^{l-1} \left(c_j \mathbf{C}_{k_j,i_j}^{(j)} \right) \prod_{j'=s'+1}^{l} h_{k_{j'-1},k_{j'}}^{(j')\{l-j'\}}(i_{j'}, i_{j'-1}) \right) \right)$$
$$+ \mathbf{n}_{k_l,i_l}^{(l)} \tag{10}$$

and c_s ($s = 1, \dots, L-1$) is the normalization factor of the antennas of any relay node at stage l (Appendix A). We note by $\mathbf{d}_{k_l,i_l}^{(l)}$ the first term in Equation (10) and by $\hat{\mathbf{n}}_{k_l,i_l}^{(l)}$ the sum of the second and last terms in Equation (10). To obtain the received signals at the destination, we can simply substitute $l = L$, $k_L = 1$, $M_{k_L}^{(L)} = M_L$ and $i_L = 1, \dots, M_L$ in Equations (9) and (10). If we assume that the source and relay nodes are all equipped with a single antenna, a simpler expression of $\mathbf{R}_1^{(L)}$ is found [17]. We see that the equivalent overall channel estimated at the destination is a combination of all channels in

Table II. Notations

Symbols	Definition	Dimension
L	Number of hops in the cooperative communication	
K_l	Number of nodes at stage l ($l = 1, \ldots, L-1$)	
$K = \sum_{l=1}^{L-1} K_l$	Total number of relay nodes in the system	
$M^{(0)} = M_1^{(0)} = M_0$	Number of antennas at the source node	
$M^{(L)} = M_1^{(L)} = M_L$	Number of antennas at the destination node	
$M_k^{(l)}$	Number of antennas at node k of stage l	
$M^{(l)} = \sum_{k=1}^{K_l} M_k^{(l)}$	Total number of antennas at stage l	
$d_{\text{s,d}}$	Distance between the source and destination nodes	
$d_{k,k'}^{(l)}$	Distance between node k of stage $l-1$ and node k' of stage l	
T	Time duration of a hop (a transmission between two stages)	
P	System transmit power	
$P^{(l)} = P/L$	Transmit power of stage l ($l = 0, \ldots, L-1$)	
$P_k^{(l)} = P^{(l)}(M_k^{(l)}/M^{(l)})$	Transmit power of node k at stage l	
$\mathbf{s}, \mathbf{y}, \mathbf{z}$	Information symbols	$1 \times T$
$\mathbf{S}, \mathbf{Y}, \mathbf{Z}$	Transmitted symbols by the source node	$M_0 \times T$
$\boldsymbol{H}_{k,k'}^{(l)}$	Channel gain between node k at stage $l-1$ and node k' at stage l	$M_{k'}^{(l)} \times M_k^{(l-1)}$
$h_{k,k'}^{(l)}(i,j)$	(i,j)th element of channel matrix $\boldsymbol{H}_{k,k'}^{(l)}$ $\left(i = 1, \ldots, M_{k'}^{(l)} \text{ and } j = 1, \ldots, M_k^{(l-1)}\right)$	
$\boldsymbol{h}_{k,k'}^{(l)}(i) = \begin{bmatrix} h_{k,k'}^{(l)}(i,1) \\ \vdots \\ h_{k,k'}^{(l)}(i,M_k^{(l-1)}) \end{bmatrix}^t$	Channel gain between node k at stage $l-1$ and ith antenna of node k' at stage l	$1 \times M_k^{(l-1)}$
α	Path loss exponent	
$\sigma_{k,k'}^{(l)\,2} = \left(d_{\text{s,d}}/d_{k,k'}^{(l)}\right)^{\alpha}$	Variance of any channel coefficient $h_{k,k'}^{(l)}(i,j)$	
$c_0^2 = PT/LM_0$	Transmit energy of any antenna at the source	
$c_l^2, c_l^{DF\,2}$	Transmit energy of any antenna at any node of stage l (AF, DF)	
$\boldsymbol{A}_{k,i}^{(l)}, \boldsymbol{B}_{k,i}^{(l)}, \boldsymbol{C}_{k,i}^{(l)}$	Code matrix associated with the signal, the conjugate of the signal, and the signal or its conjugate, transmitted by antenna i of node k on stage l	$T \times T$
$\boldsymbol{r}_{k,i}^{(l)}$	Received signals at antenna i of node k at stage l	$1 \times T$
$\boldsymbol{R}_k^{(l)} = \begin{bmatrix} \boldsymbol{r}_{k,1}^{(l)} & \cdots & \boldsymbol{r}_{k,M_k^{(l)}}^{(l)} \end{bmatrix}^{mt}$	Received signals at node k of stage l	$M_k^{(l)} \times T$
$\boldsymbol{x}_{k,i}^{(l)}$	Transmitted signals by antenna i of node k at stage l	$1 \times T$
$\boldsymbol{X}_k^{(l)} = \begin{bmatrix} \boldsymbol{x}_{k,1}^{(l)} & \cdots & \boldsymbol{x}_{k,M_k^{(l)}}^{(l)} \end{bmatrix}^{mt}$	Transmitted signals by node k of stage l	$M_k^{(l)} \times T$
$\boldsymbol{n}_{k,i}^{(l)}$	Received additive white Gaussian noise at antenna i of node k at stage l	$1 \times T$
$\boldsymbol{N}_k^{(l)} = \begin{bmatrix} \boldsymbol{n}_{k,1}^{(l)} & \cdots & \boldsymbol{n}_{k,M_k^{(l)}}^{(l)} \end{bmatrix}^{mt}$	Received additive white Gaussian noise at node k of stage l	$M_k^{(l)} \times T$

the system (from the source node to the destination, passing through the relay nodes). The equivalent information signals is a combination of the transmitted signals by the source node and the STBC code matrices applied by the relays in the communication.

At the destination, we assume maximum likelihood detection (MLD) delivering the vector $\tilde{\mathbf{s}}_1^{(L)}$, of length $1 \times T$, from a set S of all possible transmitted vectors of symbols.

3.2. Decode-and-forward relaying

For simplicity, we keep the same notations as in Section 3.1, but we do not add a DF index on all symbols. However, it is important to observe that all the notations and results included in this section are different from those presented for AF relaying.

Once the kth relay at stage 1 has received the signal expressed in Equation (2), it attempts to estimate the information symbols using MLD. Then, the signal that it transmits is given by

$$\mathbf{X}_k^{(1)} = c_1^{DF} \tilde{\mathbf{S}}_k^{(1)} \tag{11}$$

where $c_l^{DF} = \sqrt{PT/LM^{(l)}}$ is the normalization amplification factor of the antennas at any relay of stage l ($l = 1, \ldots, L-1$), $\tilde{\mathbf{S}}_k^{(l)} = \begin{bmatrix} \tilde{\mathbf{s}}_k^{(l)} \mathbf{C}_{k,1}^{(l)} & \cdots & \tilde{\mathbf{s}}_k^{(l)} \mathbf{C}_{k,M_k^{(l)}}^{(l)} \end{bmatrix}^{mt}$ is the estimation of the transmitted signals at node k of stage l (after application of the code matrices), and $\tilde{\mathbf{s}}_k^{(l)}$ is the estimated vector of information symbols at node k of stage l ($l = 1, \ldots, L-1$).

With iteration over l stages, the received signals at node k of the lth stage is given by

$$\mathbf{R}_k^{(l)} = \sum_{k'=1}^{K_{l-1}} c_{l-1}^{DF} \mathbf{H}_{k',k}^{(l)} \tilde{\mathbf{S}}_{k'}^{(l-1)} + \mathbf{N}_k^{(l)}$$

$$= c_{l-1}^{DF} \left[\mathbf{H}_{1,k}^{(l)} \cdots H_{K_{l-1},k}^{(l)} \right] \begin{bmatrix} \tilde{\mathbf{S}}_1^{(l-1)} \\ \vdots \\ \tilde{\mathbf{S}}_{K_{l-1}}^{(l-1)} \end{bmatrix} + \mathbf{N}_k^{(l)}$$

$$= c_{l-1}^{DF} \mathbf{H}_{k,eq}^{(l)} \tilde{\mathbf{S}}_{k,eq}^{(l)} + \mathbf{N}_k^{(l)} \qquad (12)$$

By substituting $l = L$, $k = 1$, and $M_k^{(L)} = M_L$ in Equation (12), we obtain the expression of the received signals at the destination. The latter proceeds using MLD to estimate the vector of information symbols $\tilde{\mathbf{s}}_1^{(L)}$.

In this paper, we do not need to present examples of the general model because some particular examples of cooperative systems with two or three hops, multi-antenna or single-antenna nodes, have been already detailed in our previous work [17,19,20].

4. BIT ERROR RATE DERIVATION

In this section, we compute the BER of the WRN by starting to investigate the PEP.

The mean of $\mathbf{R}_{k_l}^{(l)}$ is given as

$$E\{\mathbf{R}_{k_l}^{(l)}\} = \left[\mathbf{d}_{k_l,1}^{(l)} \cdots \mathbf{d}_{k_l,M_{k_l}^{(l)}}^{(l)} \right]^{mt} \qquad (14)$$

and its variance is expressed by

$$Var\left(\mathbf{R}_{k_l}^{(l)}\right) = \sum_{i_l=1}^{M_{k_l}^{(l)}} \gamma_{k_l,i_l}^{(l)} \mathbf{I}_T = M_{k_l}^{(l)} \gamma_{k_l,i_l}^{(l)} \mathbf{I}_T = \gamma_{k_l}^{(l)} \mathbf{I}_T \qquad (15)$$

By substituting $l = L$, $k_L = 1$, and $M_{k_L}^{(L)} = M_L$ in Equations (14) and (15), we obtain the mean and variance of $\mathbf{R}_1^{(L)}$, the received signal at the destination. Let $\mathbf{D}_1^{(l)}(\mathbf{y})$ be the matrix of the signals received at the destination when $\mathbf{y}$ is sent by the source node, without accounting for the noise term. In other words, $\mathbf{D}_1^{(L)}(\mathbf{y})$ is written as

$$\mathbf{D}_1^{(L)}(\mathbf{y}) = \left[\mathbf{d}_{1,1}^{(L)} \cdots \mathbf{d}_{1,M_L}^{(L)} \right]^{mt} \qquad (16)$$

with $\mathbf{d}_{1,k}^{(L)}$ given in Equation (10) and where the information signal $\mathbf{s}$ is substituted by $\mathbf{y}$. Using Equations (15) and (16), the PEP of the WRN is given by [28]

$$P^{(L)}(\mathbf{y} \longrightarrow \mathbf{z}) \approx E_{\mathbf{H}_{k,k'}^{(l)}, \forall k, k', l} \left\{ Q\left(\sqrt{\frac{1}{2\gamma_1^{(L)}} \left\| \mathbf{D}_1^{(L)}(\mathbf{y}) - \mathbf{D}_1^{(L)}(\mathbf{z}) \right\|_F^2} \right) \right\} \qquad (17)$$

4.1. Amplify-and-forward relaying

In Equation (10), $\mathbf{n}_{k_l,i_l}^{(l)}$ is Gaussian with zero mean; then, $\mathbf{r}_{k_l,i_l}^{(l)}$ and $\mathbf{R}_{k_l}^{(l)}$ are Gaussian. The mean of $\mathbf{r}_{k_l,i_l}^{(l)}$ is $E\{\mathbf{r}_{k_l,i_l}^{(l)}\} = \mathbf{d}_{k_l,i_l}^{(l)}$ (as defined in Equation (10)), and its variance is given by

where $Q(u) = \int_u^\infty \left(e^{-(v^2/2)} / \sqrt{2\pi} \right) dv$. For BPSK symbols and by averaging PEP over the number of erroneous symbols between $\mathbf{y}$ and $\mathbf{z}$, we express the BER by[‡]

$$BER_{AF} = \sum_{\mathbf{y}=[1...1]; \mathbf{z} \in S; \mathbf{z} \neq \mathbf{y}} P^{(L)}(\mathbf{y} \longrightarrow \mathbf{z}) \frac{tr(diag(\mathbf{y} - \mathbf{z}))}{T} \qquad (18)$$

$$Var\left(\mathbf{r}_{k_l,i_l}^{(l)}\right) = E\left\{ \left(\mathbf{r}_{k_l,i_l}^{(l)} - E\left\{\mathbf{r}_{k_l,i_l}^{(l)}\right\}\right)^* \left(\mathbf{r}_{k_l,i_l}^{(l)} - E\left\{\mathbf{r}_{k_l,i_l}^{(l)}\right\}\right)\right\} = E\left\{ \hat{\mathbf{n}}_{k_l,i_l}^{(l)*} \hat{\mathbf{n}}_{k_l,i_l}^{(l)} \right\}$$

$$= N_0 \left(\sum_{s'=1}^{l-1} \left(\sum_{k_{s'}=1}^{K_{s'}} \cdots \sum_{k_{l-1}=1}^{K_{l-1}} \sum_{i_{s'}=1}^{M_{k_{s'}}^{(s')}} \cdots \sum_{i_{l-1}=1}^{M_{k_{l-1}}^{(l-1)}} \left(\prod_{j=s'}^{l-1} c_j^2 \prod_{j'=s'+1}^{l} \sigma_{k_{j'-1},k_{j'}}^{(j')2} \right) \right) + 1 \right) \mathbf{I}_T$$

$$= \gamma_{k_l,i_l}^{(l)} \mathbf{I}_T \qquad (13)$$

This expression is available because we have $\mathbf{C}_{i,j}^{(l)*} \mathbf{C}_{i,j}^{(l)} = \mathbf{I}_T$, and we assume that $\mathbf{n}_{i,j}^{(l)}$, $\mathbf{n}_{i',j'}^{(l')}$, $h_{i,j}^{(l)}(a,b)$, and $h_{i',j'}^{(l')}(a',b')$ are independent random variables $\forall i \neq i'$, $\forall j \neq j'$, $\forall a \neq a'$, $\forall b \neq b'$, and $\forall l, l'$, respectively.

where $tr(diag(\mathbf{y} - \mathbf{z}))/T$ is the probability of obtaining $tr(diag(\mathbf{y} - \mathbf{z}))$ erroneous bits when T bits are transmitted. Without loss of generality, we assume that $\mathbf{y} = \mathbf{1}_{1 \times T}$.

[‡]If we choose any other modulation, the following expression will concern the symbol error rate.

4.2. DF relaying

Let $\mathbf{Y}_{k,eq}^{(l)}$ and $\mathbf{Z}_{k,eq}^{(l)}$ be the equivalent matrices at node k of the lth stage (as in Equation (12)) obtained when $\mathbf{y}$ and $\mathbf{z}$ are sent. The PEP at any node k of any stage l is given by

$$P_{k,DF}^{(l)}(\mathbf{y} \longrightarrow \mathbf{z}) \approx E_{\mathbf{H}_{k',k''}^{(l)},\forall k',k'',l}\left\{ Q\left(\sqrt{\frac{{c_{l-1}^{DF}}^2}{2N_0}\left\|\mathbf{H}_{k,eq}^{(l)}\left(\mathbf{Y}_{k,eq}^{(l)}-\mathbf{Z}_{k,eq}^{(l)}\right)\right\|_F^2}\right)\right\}$$ (19)

The BER of the system is obtained for BPSK symbols using the expression [29]

$$BER_{DF}^{(L)} = \sum_{l=1}^{L} P_{e_l} \prod_{l'=1;l'\neq l}^{L} \left(1 - P_{e_{l'}}\right)$$ (20)

where

$$P_{e_l} = \frac{1}{K_l}\sum_{k_l=1}^{K_l}\sum_{\mathbf{y}=[1...1];\mathbf{z}\in S;\mathbf{y}\neq\mathbf{z}} P_{k_l,DF}^{(l)}(\mathbf{y}\longrightarrow\mathbf{z})\frac{tr(diag(\mathbf{y}-\mathbf{z}))}{T}$$ (21)

5. RESULTS AND DISCUSSION

In all simulations, we assume for simplicity that $M_0 = 1$, $\alpha = 4$, and BPSK modulation. We also assume that the distances between all stages are equal, that is, $d_{s,d} = 1$ and $d_{k,k'}^{(l)} = d_{s,d}/L$, $l = 1,\ldots,L$ (unless otherwise stated). We evaluate by simulation the average BER as a function of the average SNR. Without loss of generality, the orthogonal STBC and QOSTBC used in our simulations are Alamouti and QOSTBC 4×4, respectively, having the following code matrices:

- Alamouti: $\mathbf{A}_1 = \mathbf{I}_2$, $\mathbf{B}_1 = \mathbf{A}_2 = \mathbf{0}_2$, and $\mathbf{B}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
- QOSTBC 4×4: $\mathbf{A}_1 = \mathbf{I}_4$, $\mathbf{A}_2 = \mathbf{A}_3 = \mathbf{B}_1 = \mathbf{B}_4 = \mathbf{0}_4$, $\mathbf{A}_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$,

$\mathbf{B}_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, and $\mathbf{B}_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

We use for our simulations the WRN models developed in Equations (9), (10), and (12), and we validate the BER results by the analytical expressions given by Equations (18) and (20).

5.1. Amplify-and-forward versus decode-and-forward relaying: achieved diversity

In Figure 2, we show the BERs of WRNs exploiting Alamouti at the AF or DF single-antenna relaying nodes

where $K_l = 2$, $l = 1, 2, 3$. At high SNR, AF relaying outperforms significantly DF relaying and achieves the maximal diversity order $d = 2$. When distributed STBC is used at successive hops with AF relaying, the diversity is never altered because it is maximal on each hop. However, $d = 1$ is achieved with DF relaying. Indeed, the diversity of the

system is equal to the minimal diversity achieved at one of the hops (in this case the first one) [12]. Finally, the analytical results agree very well with the simulation results. For clarity of presentation, analytical results will be omitted in the next figures.

In Figure 3, we present the BER results for different configurations as explained in Table III. Without loss of generality, we chose QOSTBC 4×4. For AF relaying, the BERs of all configurations are the same and have $d = 4$. This is expected because the cooperation is exploited in a distributed fashion. For DF relaying, the BER improves when the configuration converges to a MIMO relay system. Even though the diversity order at the second hop is maximal (equals to 4), d is controlled by the diversity order achieved at hop 1. Therefore, d for Config. 1, 2, and 3 is equal to 4, 2, and 1, respectively.

5.2. Impact of using distributed space–time blocking code at different stages

In Figure 4, we evaluate the BER for the scenarios presented in Table IV. We assume that $K_l = 2$ and $M_k^{(l)} = 1$, $\forall k = 1, 2$ and $l = 1, 2, 3$. For AF relaying, the BER results are very close for scenarios 1, 4, and 6. Indeed, using Alamouti at the first stages provides enough protection to the transmitted symbols until reaching the destination. The slight gain of scenario 1 comes from the preservation of the structure of the code at the destination. However, when the code is used many times, its initial structure is altered, causing it to be less powerful. Using Alamouti at the second or last two stages (scenarios 2 and 5) leads to the same
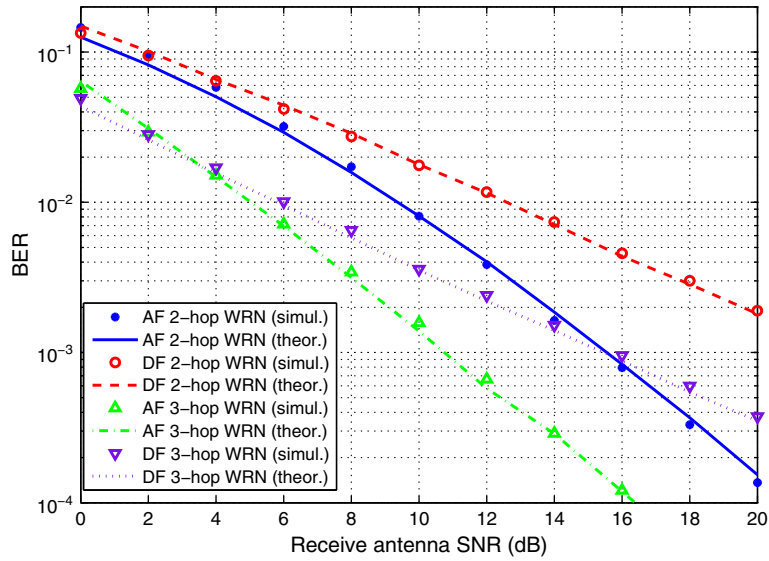
Figure 2. Bit error rate (BER) versus receive antenna signal-to-noise ratio (SNR) for L-hop wireless relay networks (WRNs) with distributed Alamouti. AF, amplify and forward; DF, decode and forward.
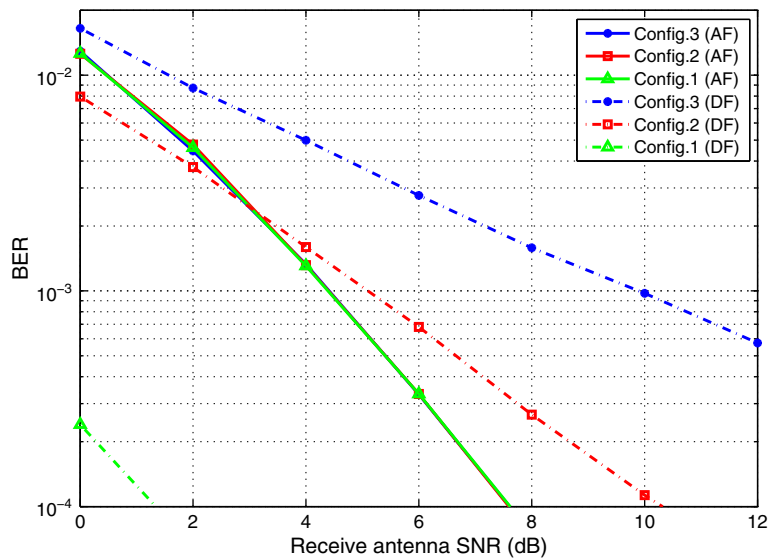

Figure 3. Bit error rate (BER) versus receive antenna signal-to-noise ratio (SNR) for two-hop wireless relay network with distributed 4×4 QOSTBC. AF, amplify and forward; DF, decode and forward.

Table III. Two-hop wireless relay network configurations.

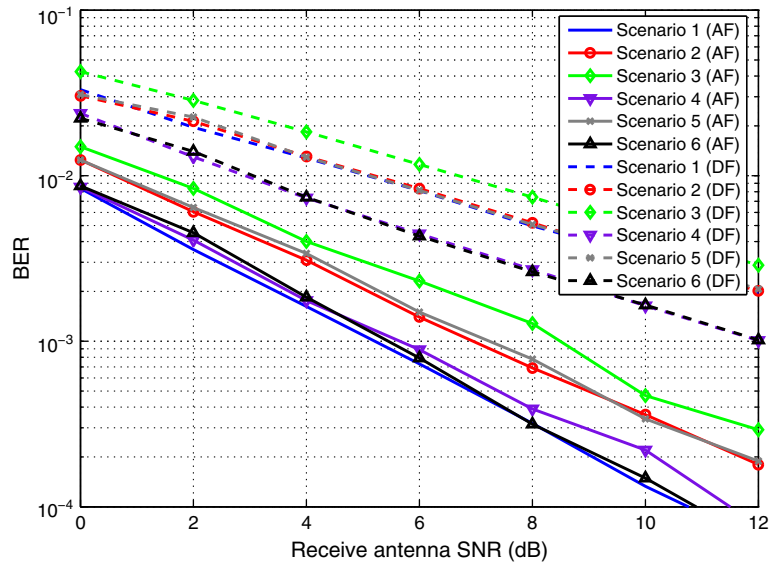|	Number of relays	Number of antennas at each relay
Config. 1	1	4
Config. 2	2	2
Config. 3	4	1

Figure 4. Bit error rate (BER) versus receive antenna signal-to-noise ratio (SNR) for four-hop wireless relay network with different scenarios. AF, amplify and forward; DF, decode and forward.

Table IV. Distributed Alamouti used at which stages.

	Stage 1	Stage 2	Stage 3
Scenario 1	X		
Scenario 2		X	
Scenario 3			X
Scenario 4	X	X	
Scenario 5		X	X
Scenario 6	X	X	X

conclusion. However, the BER is the worst for scenario 3. This is explained by the lack of protection over the first hops. For DF relaying, scenarios 4 and 6 outperform all the other ones. Because the first hops present the highest error risk, using Alamouti in the first two hops provides an important protection. Moreover, the MIMO destination allows recovery of the symbols at the last hop even without using distributed STBC. Scenarios 1, 2, and 5 provide similar BERs that are worse than those provided by scenarios 4 and 6. These results show that the first and second stages have the same error risk level whereas using Alamouti at the last stage is unnecessary.

5.3. Impact of using different distributed space–time blocking codes at the stages

In Figure 5, we present the BER of the WRN when different STBC are used at the stages. For AF relaying, when Alamouti is used at the first stage, $d = 2$. Meanwhile, $d = 4$ when QOSTBC 4×4 is used at the first stage. At low SNR, having a small number of relays at the first stage

reduces the risk of error propagation in the next stages. At high SNR, hop 1 is reliable enough to use a larger number of relays. For DF relaying, the case of QOSTBC 4×4 at the first stage outperforms that of Alamouti for any SNR value because using STBC at the last hop is unnecessary.

5.4. Impact of the number of relays

In Figure 6, we present the BERs when Alamouti is used at stage 1 and for different K_2. For AF relaying and $BER = 10^{-2}$, the SNR gain when K_2 increases from 1 to 2 and 3 is about 2 and 2.5 dB, respectively. For larger K_2, a small additional gain is obtained (less than 1 dB for $K_2 \geq 4$). Therefore, it is attractive to have a number of relays close to the dimension of the distributed code used at the previous stage. Having a larger number of relays does not improve significantly the BER. For DF relaying, we assume that each relay node at the second stage chooses arbitrarily the matrix code to use for its transmission. In our simulations, we choose to dedicate $\lceil K_2/2 \rceil$ for the first code matrix ($\lceil x \rceil$ denotes ceiling of x) and $K_2 - \lceil K_2/2 \rceil$ for the second code matrix. When K_2 increases from 1 to 5, the BERs are almost the same. Thus, it is very beneficial to have the minimal number of relays at the last stage.

5.5. Impact of the number of stages

In Figure 2, as the number of hops increases (for fixed $d_{s,d} = 1$), the BER decreases rapidly. Indeed, because the distances between the stages are smaller, the quality of the radio channels is better, and thus the transmissions are more reliable.
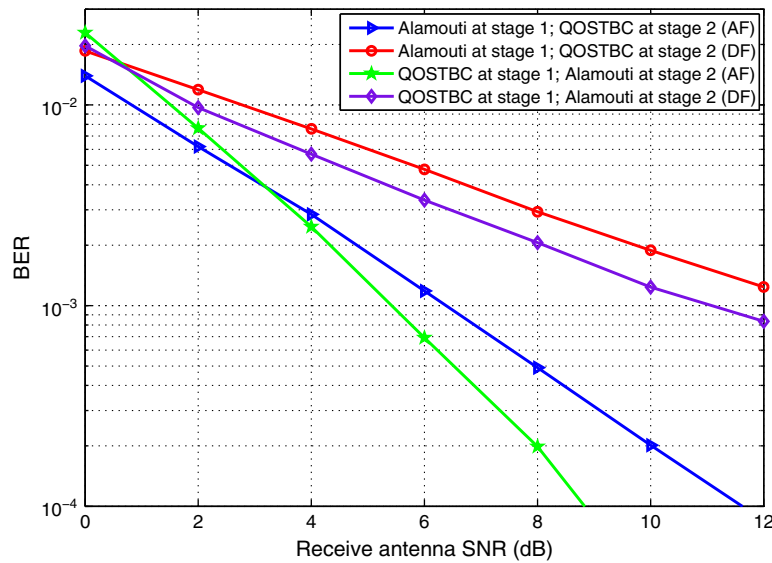
Figure 5. Bit error rate (BER) versus receive antenna signal-to-noise ratio (SNR) for three-hop wireless relay network with different space–time blocking codes at each stage. AF, amplify and forward; DF, decode and forward; QOSTBC, quasi-orthogonal space–time blocking code.
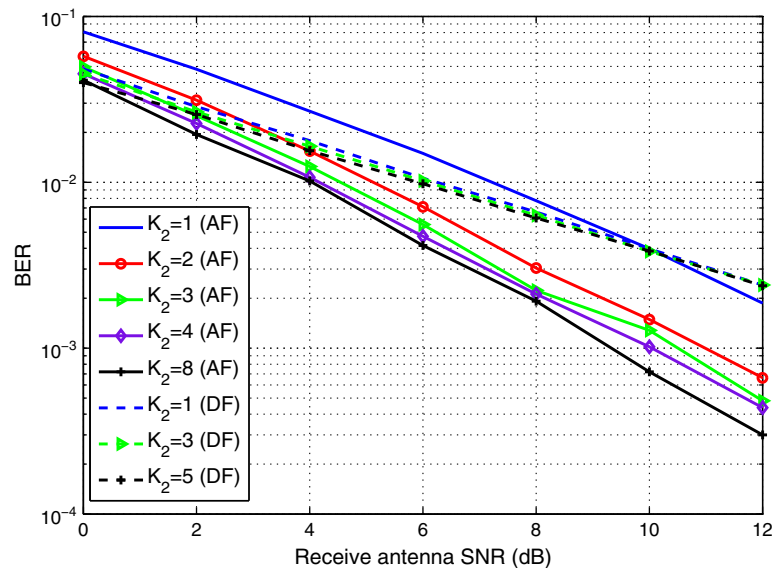

Figure 6. Bit error rate (BER) versus receive antenna signal-to-noise ratio (SNR) for three-hop wireless relay network with $K_1 = 2$. AF, amplify and forward; DF, decode and forward.

5.6. Impact of the distances between nodes

In Figure 7, we assume that the distances between the source, the stages, and the destination are not equal in a three-hop WRN. We define the distance configurations as triplets $\left(d_{1,k}^{(1)}, d_{k,k'}^{(2)}, d_{k',1}^{(3)} \right)$. For AF relaying (Figure 7a), when the stages are closer to the source node, we obtain the best BER performance. Closer stages to the source

node means better channel conditions and thus more reliable transmissions. Even though the distance $d_{k',1}^{(3)}$ is large ($k' = 1, 2$), distributed Alamouti at the last hop and the MIMO destination provide enough protection to the transmitted signals. For DF relaying (Figure 7b), the best BER performance is obtained for the distance configuration $(0.2, 0.3, 0.5)$. As the distance "stage 1–stage 2" or "stage 2–destination" increases (for a short distance "source–stage 1"), the BER degrades significantly. The
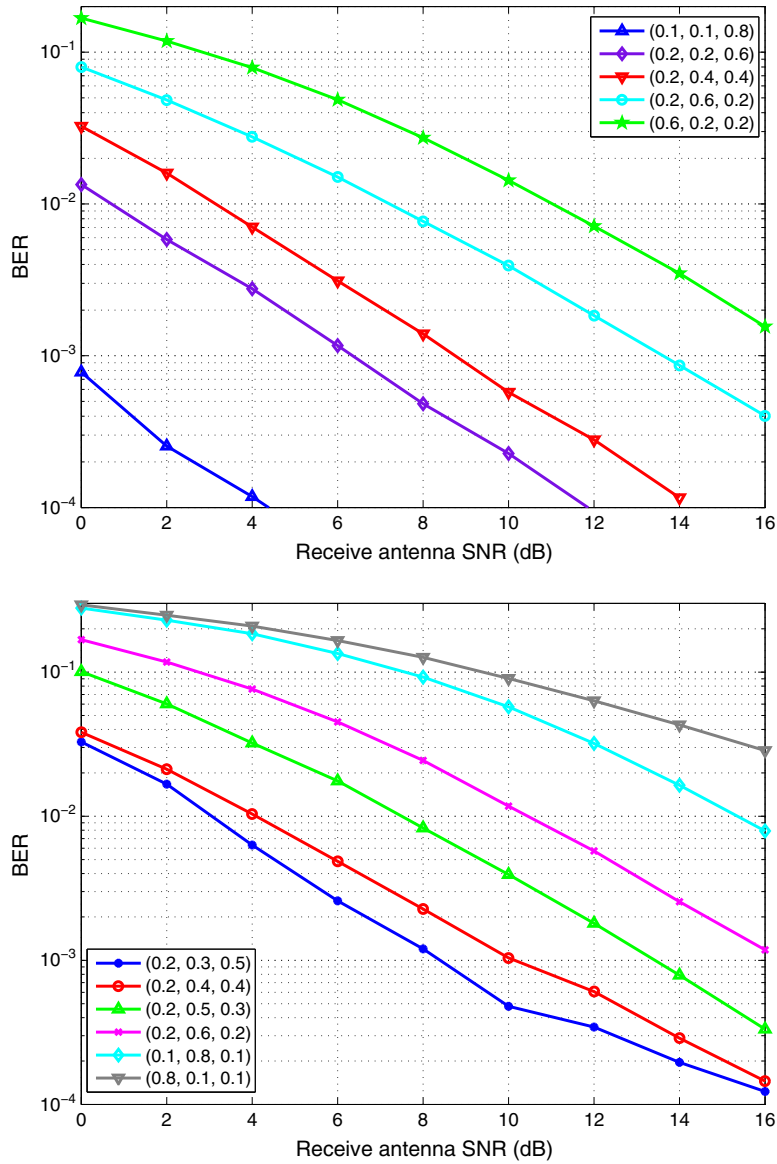
Figure 7. Bit error rate (BER) versus receive antenna signal-to-noise ratio (SNR) for three-hop wireless relay network with different distance configurations. AF, amplify and forward; DF, decode and forward.

BER performance gets even worse when the distance "source–stage 1" is larger. Hence, it is recommended to have the stages ordered in an increasing distance fashion, from the source to the destination. It is also interesting to mention that the path-loss effect can always be compensated by increasing the transmit power of the transmitting nodes (source and relays).

5.7. Impact of imperfect channel state information

Because of the similarity of results for AF and DF relaying, we choose in Equation (1) $\nu^{(l)} = c_l$ for AF relaying and

$\nu^{(l)} = 1$ for DF relaying. When $\nu^{(l)} = 1$, CEE are independent of SNR whereas CEE depend on SNR if $\nu^{(l)} = c_l$ (or $\nu^{(l)} = c_l^{DF}$).

In Figure 8a, when CEE are on hop 1, the BER degrades slowly for ${e_{s,k}^{(1)}}^2 \leq 10\%$ and rapidly for ${e_{s,k}^{(1)}}^2 > 10\%$. This result agrees with the general rule used for MIMO systems, stating that for a point-to-point MIMO communication, CEE should be less than 10% for an accurate channel estimation [30]. When CEE are on hop 2, the BER degrades faster, allowing an estimation error tolerance of ${e_{k,k'}^{(2)}}^2 = 5\%, \forall k, k' = 1, 2$. Because CEE depend on c_1 and $c_1 < c_0$, then their impact is more important. For ${e_{k,1}^{(3)}}^2 \leq 10\%$,
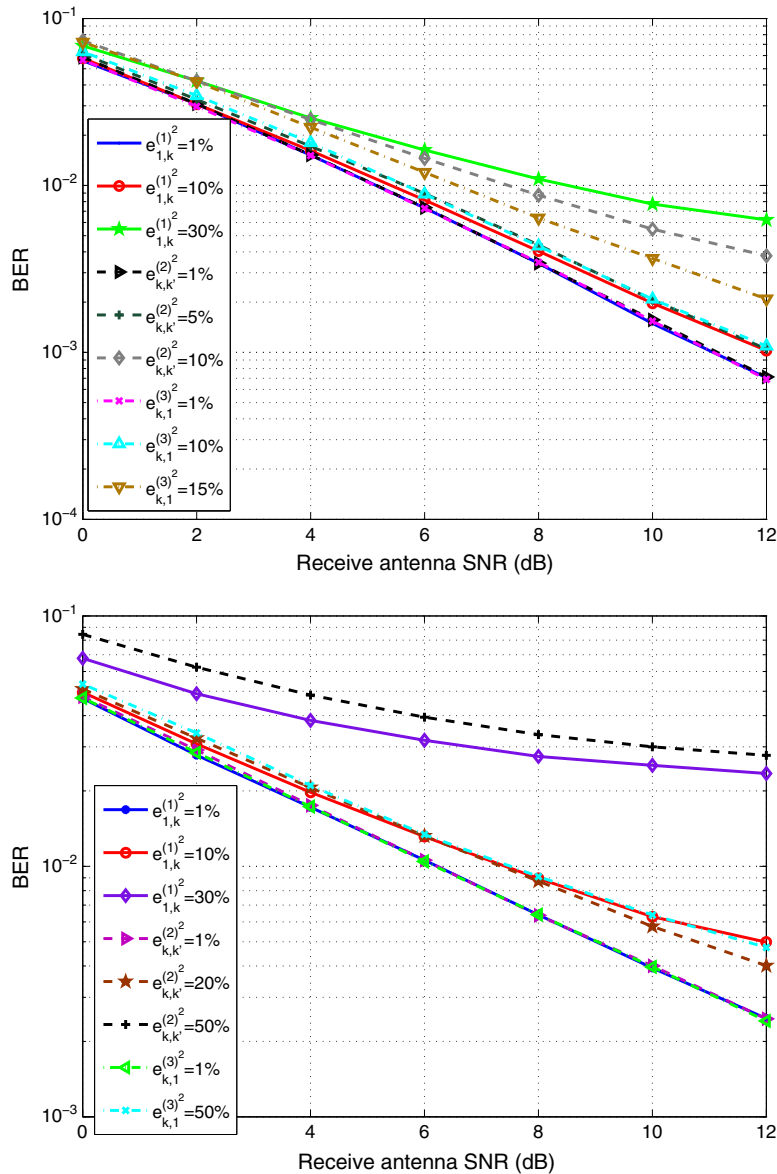
Figure 8. Bit error rate (BER) versus receive antenna signal-to-noise ratio (SNR) for three-hop wireless relay network with imperfect channel state information ($K_1 = K_2 = M_3 = 2$). AF, amplify and forward; DF, decode and forward.

the BER behaves similarly to the case of $e_{1,k}^{(1)^2} \leq 10\%$ even though $c_2 < c_0$. This enhancement of the BER is due to the presence of a MIMO destination.

In Figure 8b, the BER drops fast as $e_{1,k}^{(1)^2}$ increases for CEE on hop 1. When CEE are at the second hop, BER degrades slowly for $e_{k,k'}^{(2)^2} \leq 20\%$ and rapidly for $e_{k,k'}^{(2)^2} > 20\%$. When errors happen at the third hop, the BER decreases slightly for increasing $e_{k,1}^{(3)^2}$ from 1% to 50%.

We conclude that the error tolerated on the channel estimation for a three-hop WRN using AF relaying is not higher than 15% (SNR-dependent CEE), whereas a

threshold of 50% is obtained using DF relaying (SNR-independent CEE). Without loss of generality, the threshold on the tolerated CEE using DF relaying is found to be always higher than that of AF relaying (further results in [19,20]).

6. CONCLUSION

In this paper, we have investigated the error performance of the multi-hop multi-antenna WRNs using distributed STBC at the relay nodes. We derived the PEP and the BER for such networks. We showed that the diversity depends on the number of antennas at each stage for AF relaying,

and it is controlled by the minimal number of antennas at the relays of all stages for DF relaying. Moreover, we evaluated the network error performance for several parameters such as the distributed STBC used at the stages, the number of relays at each stage, the number of stages, the distances between the stages, and the CSI. Then, we provided observations and guidelines about the design of the WRN in order to improve transmission reliability.

APPENDIX A: CALCULATION OF C_L

c_l is calculated with respect to the power constraint on each stage. For any i_lth antenna of node k_l at stage l ($l = 1, \ldots, L-1$, $k_l = 1, \ldots, K_l$, $i_l = 1, \ldots, M_{k_l}^{(l)}$), we have

$$E\left\{\mathbf{r}_{k_l,i_l}^{(l)\{1\}}\left(\mathbf{r}_{k_l,i_l}^{(l)\{1\}}\right)^*\right\} = \sum_{k'=1}^{K_{l-1}} M_{k'}^{(l-1)}\sigma_{k',k}^{(l)}{}^2 E\left\{\mathbf{X}_{k'}^{(l-1)}\left(\mathbf{X}_{k'}^{(l-1)}\right)^*\right\} + TN_0 = \left(\frac{P}{LM^{(l-1)}}\sum_{k'=1}^{K_{l-1}} M_{k'}^{(l-1)}\sigma_{k',k}^{(l)}{}^2 + N_0\right)T$$

(A1)

Thus, the energy of the transmitted signal by the i_lth antenna of this relay, $E\left\{\mathbf{x}_{k_l,i_l}^{(l)}\left(\mathbf{x}_{k_l,i_l}^{(l)}\right)^*\right\}$, is given by

$$E\left\{\mathbf{x}_{k_l,i_l}^{(l)}\left(\mathbf{x}_{k_l,i_l}^{(l)}\right)^*\right\} = \frac{PT}{LM^{(l)}}$$

(A2)

$$\Leftrightarrow c_l^2\left(\frac{P}{LM^{(l-1)}}\sum_{k'=1}^{K_{l-1}} M_{k'}^{(l-1)}\sigma_{k',k}^{(l)}{}^2 + N_0\right)T = \frac{PT}{LM^{(l)}}$$

(A3)

$$\Leftrightarrow c_l = \sqrt{\frac{\frac{P}{LM^{(l)}}}{\frac{P}{LM^{(l-1)}}\sum_{k'=1}^{K_{l-1}} M_{k'}^{(l-1)}\sigma_{k',k}^{(l)}{}^2 + N_0}}$$

(A4)

We assume that $\sigma_{k',k}^{(l)}{}^2 = \sigma^{(l)2}$, $\forall k' = 1, \ldots, K_{l-1}$ and $k = 1, \ldots, K_l$. Thus, c_l is expressed by

$$c_l = \sqrt{\frac{P}{PM^{(l)}\sigma^{(l)2} + LM^{(l)}N_0}}$$

(A5)

REFERENCES

1. Gesbert D, Shafi M, Shan Shiu D, Smith P, Naguib A. From theory to practice: an overview of MIMO space–time coded wireless systems. *IEEE Journal on Selected Areas in Communication* 2003; **21**(3): 281–302.
2. Tarokh V, Seshadri N, Calderbank A. Space–time codes for high data rate wireless communication: performance criterion and code construction. *IEEE Transactions on Information Theory* 1998; **44**(2): 744–765.
3. Sendonaris A, Erkip E, Aazhang B. Increasing uplink capacity via user cooperation diversity. In *Proceedings of the IEEE International Symposium on Information Theory*, Aug 1998; 156.
4. Sendonaris A, Erkip E, Aazhang B. User cooperation diversity. Part I: system description. Part II: implementation aspects and performance analysis. *IEEE Transactions on Commununications* 2003; **51**(11): 1927–1948.
5. Laneman J, Wornell G. Distributed space–time-coded protocols for exploiting cooperative diversity in wireless networks. *IEEE Transactions on Information Theory* 2003; **49**(10): 2415–2425.
6. Laneman J, Tse D, Wornell G. Cooperative diversity in wireless networks: efficient protocols and outage behavior. *IEEE Transactions on Information Theory* 2004; **50**(12): 3062–3080.
7. Van der Meulen EC. Three-terminal communication channels. *Advances in Applied Probability* 1971; **3**: 120–154.
8. Cover T, Gamal A. Capacity theorems for the relay channel. *IEEE Transactions on Information Theory* 1979; **25**(5): 572–584.
9. Jing Y, Hassibi B. Distributed space–time coding in wireless relay networks. *IEEE Transactions on Wireless Communications* 2006; **5**(12): 3524–3536.
10. Jing Y, Jafarkhani H. Using orthogonal and quasi-orthogonal designs in wireless relay networks. *IEEE Transactions on Information Theory* 2007; **53**(11): 4106–4118.
11. Oggier F, Hassibi B. A coding scheme for wireless networks with multiple antenna nodes and no channel information. In *Proceedings of the International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Vol. 3, Apr 2007; 413–416.
12. Jing Y, Hassibi B. Cooperative diversity in wireless relay networks with multiple-antenna nodes. In *Proceedings of the International Symposium on Information Theory (ISIT)*, Sept 2005; 815–819.
13. Yang S, Belfiore J-C. Distributed space–time codes for the multi-hop channel. In *Proceedings of the International Workshop on Wireless Networks: Communication, Cooperation and Competition (WNC3'07)*, Limassol, Cyprus, April 2007.

14. Yang S, Belfiore J-C. Diversity of MIMO multihop relay channels—part I: amplify-and-forward. *ArXiv e-prints* Apr 2007.

15. Oggier F, Hassibi B. Code design for multihop wireless relay networks. *EURASIP Journal on Advances in Signal Processing* 2008; **2008**: 1–12.

16. Ghosh A, Ratasuk R, Mondal B, Mangalvedhe N, Thomas T. LTE-advanced: next-generation wireless broadband technology [invited paper]. *IEEE Transactions on Wireless Communications* 2010; **17**(3): 10–22.

17. Jaafar W, Ajib W, Haccoun D. On the performance evaluation of distributed STBC in multi-hop wireless relay networks. In *Proceedings of the European Wireless Conference*, Apr 2010; 223–230.

18. Yoo T, Goldsmith A. Capacity of fading MIMO channels with channel estimation error. *IEEE International Conference on Communications* 2004; **2**(20–24): 808–813.

19. Jaafar W, Ajib W, Haccoun D. Performance evaluation of distributed STBC in wireless relay networks with imperfect CSI. In *Proceedings of the International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Sept 2009; 2335–2339.

20. Jaafar W, Ajib W, Haccoun D. Impact of CSI on the performance of multi-hop wireless relay networks. In *Proceedings of the 72nd IEEE Vehicular Technology Conference (VTC-Fall)*, Sept 2010; 1–5.

21. Herhold P, Zimmermann E, Fettweis G. Cooperative multi-hop transmission in wireless networks. *Computer Networks* 2005; **49**(3): 299–324.

22. Jakllari G, Krishnamurthy SV, Faloutsos M, Krishnamurthy PV, Ercetin O. A framework for distributed spatio-temporal communications in mobile ad hoc networks. In *Proceedings of the 25th IEEE International Conference on Computer Communications (INFOCOM)*, April 2006; 1–13.

23. Jakllari G, Krishnamurthy SV, Faloutsos M, Krishnamurthy PV, Ercetin O. A cross-layer framework for exploiting virtual MISO links in mobile ad hoc networks. *IEEE Transactions on Mobile Computing* 2007; **6**(6): 579–594.

24. Yackoski J, Zhang L, Gui B, Shen C-C, Cimini L. Realistic evaluation of cooperative relaying networks using decentralized distributed space–time block coding. In *Workshop Proceedings of the IEEE International Conference on Communications (ICC)*, May 2008; 342–348.

25. Ramanathan R. Challenges: a radically new architecture for next generation mobile ad hoc networks. In *Proceedings of the 11th Annual International Conference on Mobile Computing and Networking (MobiCom)*, 2005; 132–139.

26. Rappaport T. *Wireless Communications: Principles and Practice*. Prentice-Hall, Inc.: Upper Saddle River, New Jersey, 1996.

27. Buehrer R, Kumar N. The impact of channel estimation error on space–time block codes. *IEEE 56th Vehicular Technolgogy Conference (VTC-Fall), Sept* 2002; **3**: 1921–1925.

28. Taricco G, Biglieri E. Exact pairwise error probability of space–time codes. *IEEE Transactions on Information Theory* 2002; **48**(2): 510–513.

29. Da Costa D, Aissa S. Dual-hop decode-and-forward relaying systems with relay selection and maximal-ratio schemes. *Electronics Letters* 2009; **45**(9): 460–461. [Online]. Available: http://link.aip.org/link/?ELL/45/460/1.

30. Paulraj DGA, Nabar R. *Introduction to Space–Time Wireless Communications*. Cambridge University Press: Cambridge, 2006.

AUTHORS' BIOGRAPHIES

Wael Jaafar has received the BEng degree from the École Supérieure des Communications de Tunis (High Institute of Communications, SUPCOM), Tunis, in 2007 and the MASc degree in Electrical Engineering from the École Polytechnique de Montréal (Montreal Polytechnic), Montreal, in 2009. Since 2010, he is a PhD candidate at the Electrical Engineering Department of the École Polytechnique de Montréal. He ranked in top 10% in BEng studies and received excellency scholarships during MASc and PhD studies. He was a research intern at the Department of Computer Sciences, Université du Québec à Montréal, QC, Canada between February 2007 and September 2007. His research interests include multiple-input-multiple-output communications, wireless communication networks, cooperative communications, and cognitive radio networks.

Wessam Ajib received an Engineer diploma from Institut National Polytechnique de Grenoble, France (INPG-ENSPG) in Physical Instruments, in 1996, a DEA (Diplôme d'Études Approfondies) from École Nationale Supérieure des Télécommunications (ENST), Paris, France in 1997 in Digital Communication Systems, and a PhD degree in Computer Sciences and Computer Networks from ENST, Paris, France, in 2001. He had been an architect and radio network designer at Nortel Networks, Ottawa, ON, Canada between October 2000 and June 2004. He had conducted many projects and introduced different innovative solutions for Universal Mobile

Telecommunications System, the third generation of wireless cellular networks. He had a post-doc fellowship at the Electrical Engineering Department of École Polytechnique de Montréal, QC, Canada between June 2004 and June 2005. Since June 2005, he had been with the Department of Computer Sciences, Université du Québec à Montréal, QC, Canada, where he is an assistant professor of computer networks. His research interests include wireless communications and wireless networks, multiple access and medium access control design, traffic scheduling error control coding, multiple-input-multiple-output systems, and cooperative communications. He is the author or co-author of many journal papers and conferences papers in these areas.

David Haccoun received his Engineer and BASc (magna cum laude) degrees in Engineering Physics from École Polytechnique de Montréal, Montréal, QC, Canada, his SM degree in Electrical Engineering from the Massachusetts Institute of Technology, Cambridge, and his PhD degree in Electrical Engineering from McGill University, Montréal, Canada. Since 1966, he has been with the Department of Electrical Engineering, École Polytechnique de Montréal, where he has been a Professor of Electrical Engineering since 1980. He was the founding Head of the Communication and Computer Section in 1980. He was a Visiting Research Professor with Concordia University, Montréal; INRIA Laboratories, Paris, France; École de Technologie Supérieure, Montréal; the University of Victoria, Victoria, BC, Canada; and the Advanced Study Institute, University of British Columbia, Vancouver, BC, Canada. From 1990 to 2002, he was a Researcher with the Canadian Institute for Telecommunications Research under the National Centers of Excellence of the Government of Canada. He is the holder of a US patent on an error-control technique and is a coauthor of the books 'The Communications Handbook' (Boca Raton, FL: CRC, 1997 and Piscataway, NJ: IEEE, 2001), 'The Encyclopedia of Telecommunications' (New York: Wiley, 2003), and 'Digital Communications by Satellite: Modulation, Multiple-Access and Coding' (New York: Wiley, 1981), a Japanese translation of which was published in 1984. His research interests include communication theory, theory and applications of error-control coding, wireless and mobile communications, and digital communication systems by satellite. He is the author or coauthor of over 300 journal papers and conference papers in these areas. Dr Haccoun is a Fellow of IEEE and a Fellow of the Engineering Institute of Canada. He is a Member of the Order of Engineers of Quebec, Canada; Sigma Xi; The New York Academy of Sciences; and the American Association for the Advancement of Sciences. He has been elected Member of the Board of Governors of the IEEE Vehicular Technology Society for 2009-2012, a Member of the Board of the Telecommunications Engineering Management Institute of Canada, and was a Member of the Board of the Communications Research Centre, Ottawa, ON, Canada. He was the General Co-chair of the IEEE Vehicular Technology Conference, Fall 2006 in Montréal.