

On the Performance of Polarimetric Target Detection Algorithms

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Abstract

This paper presents an analysis of the performance of six polarimetric target detection algorithms. The detection performance of the optimal polarimetric detector (OPD), the identity-likelihood-ratio-test (ILRT), the polarimetric whitening filter (PWF), the single-polarimetric-channel detector, the span detector, and the power maximization synthesis (PMS) detector is compared. Results are presented for both random and deterministic targets in the presence of complex-Gaussian clutter. The results of our studies indicate that the PWF and the ILRT typically achieve near optimal performance. Each remaining detection algorithm typically yields performance that is degraded compared to the performance of the OPD, the PWF, and the ILRT.

Introduction

Several algorithms have been proposed for detecting targets in ground clutter using fully polarimetric, synthetic aperture radar imagery. In order to perform the detection process, the three channels of polarimetric data must be reduced to a single decision criterion. This transformation should be performed in such a way that targets are more easily discriminated from clutter. The optimal method of processing the data (in terms of obtaining the maximum detection probability for a specified probability of false alarm) is given by the solution of the likelihood-ratio-test. The resulting detector, derived in Reference [1], is called the Optimal Polarimetric Detector (OPD). Other simplified detectors have also been developed, including the polarimetric whitening filter (PWF) [2], the span detector, the power maximization synthesis (PMS) detector [3], and more recently, the identity-likelihood-ratio-test (ILRT) [4].

A significant trade-off among these detection algorithms is the amount of statistical information required by the algorithm versus the performance of the algorithm. For example, the span detector requires no statistical information about the target or clutter. In contrast, the optimal polarimetric detector requires knowledge of the polarization covariance matrices of both the target and the clutter as well as the mean of the target. Intuitively, knowledge of the target and clutter statistics should improve target detection performance. However, in practice, the target statistics may be unknown, and the clutter statistics may need to be adaptively estimated "on-the-fly" (at a significant computational cost). In order to evaluate the trade-off between detection performance, available a-priori information, and computational complexity,

it is necessary to quantify the improvement in performance that results from statistical knowledge of the target and clutter environment. In this paper, we compare the detection performance of a variety of polarimetric and non-polarimetric detection algorithms, while paying particular attention to the relationship between the amount of statistical knowledge required by an algorithm and the corresponding performance of the algorithm.

Polarimetric Clutter Model

We model the radar return from terrain-clutter as a complex-Gaussian random vector. The radar measurement vector consists of three complex elements, HH, HV, and VV, and we write:

$$\underline{X} = \begin{pmatrix} \text{HH}_i + j \text{HH}_q \\ \text{HV}_i + j \text{HV}_q \\ \text{VV}_i + j \text{VV}_q \end{pmatrix} \quad (1)$$

where, for example, HH_i and HH_q are the inphase and quadrature components of the HH polarimetric channel. The elements HH, HV, and VV of the vector \underline{X} are assumed to be jointly complex-Gaussian; therefore, the vector \underline{X} has a probability density function (PDF) of the form

$$f(\underline{X}) = \frac{1}{\pi^3 |\Sigma_c|} \exp\left(-\underline{X}^\dagger \Sigma_c^{-1} \underline{X}\right) \quad (2)$$

where $\Sigma_c = E\{\underline{X}\underline{X}^\dagger\}$ is the covariance of the complex polarimetric vector, \underline{X} , and \dagger denotes the complex conjugate transpose. Also, the clutter data is assumed to have a zero mean ($E\{\underline{X}\} = 0$). The complete characterization of the jointly Gaussian complex HH, HV, and VV returns is given by the covariance matrix which (in the linear polarization basis) is assumed to have the form

$$\Sigma_c = \sigma_{\text{HH}c} \begin{pmatrix} 1 & 0 & \rho_c \sqrt{\gamma_c} \\ 0 & \epsilon_c & 0 \\ \rho_c^* \sqrt{\gamma_c} & 0 & \gamma_c \end{pmatrix} \quad (3)$$

where

$$\begin{aligned}
\sigma_{\text{HH}_c} &= E\{|HH|^2\} \\
\varepsilon_c &= \frac{E\{|HV|^2\}}{E\{|HH|^2\}} \\
\gamma_c &= \frac{E\{|VV|^2\}}{E\{|HH|^2\}} \\
\rho_c &= \frac{E\{HH VV^*\}}{\sqrt{E\{|HH|^2\} E\{|VV|^2\}}}
\end{aligned} \tag{4}$$

Thus, the clutter polarization covariance is specified by the four parameters $(\sigma_{\text{HH}_c}, \varepsilon_c, \gamma_c, \rho_c)$.

Polarimetric Target Models

At the radar frequencies of interest, targets maybe considered to be made up of a spatially distributed collection of simple polarimetric scatterers (e.g. dihedrals and trihedrals). Depending upon the resolution of the radar and the size of the target, these point scattering elements may be imaged individually (using a high-resolution polarimetric SAR radar) or may be unresolved, that is, combined coherently, resulting in a multi-scatterer model (using a medium or low-resolution polarimetric SAR radar). In this paper we investigate both of these cases.

The target models for these cases are described as follows:

(1) For the medium or low-resolution polarimetric SAR radar, the target yields a multi-scatterer return, which is assumed to have a jointly complex-Gaussian PDF, and is independent of the clutter return. The measured target-plus-clutter return is given (by superposition) as

$$\underline{X}_{t+c} = \underline{X}_t + \underline{X}_c \tag{5}$$

This implies the measured target-plus-clutter return is zero-mean, complex-Gaussian with covariance

$$\Sigma_{t+c} = \Sigma_t + \Sigma_c \tag{6}$$

The target covariance is assumed to have the same general structure as the clutter covariance and is also specified by the four parameters $(\sigma_{\text{HH}_t}, \varepsilon_t, \gamma_t, \rho_t)$. These random target and clutter models were proposed previously in References [1,2,4].

(2) For the high-resolution polarimetric SAR radar, a target is modeled as a deterministic, individually resolved unitary scatterer. The return due to a unitary scatterer is given by [4]

$$\underline{\bar{X}} = \alpha \begin{bmatrix} \cos x \\ e^{jy} \sin x \\ -e^{j2y} \cos x \end{bmatrix} \tag{7}$$

Two special cases of unitary scatterers that we shall consider are the trihedral and the dihedral. For the trihedral, $x = 0$ and $y = \frac{\pi}{2}$. For the dihedral, $x = 2\theta$ and $y = 0$, where θ specifies the orientation of the dihedral. In both cases, α specifies the amplitude of the return.

The measured target-plus-clutter return consists of the unitary scatterer with an additive random clutter component.

$$\underline{X}_{t+c} = \underline{\bar{X}}_t + \underline{X}_c \tag{8}$$

Furthermore, the target-plus-clutter covariance is equal to the clutter-only covariance, since the target component is deterministic.

$$\Sigma_{t+c} = \Sigma_c \tag{9}$$

Target Detection Algorithms

Algorithms for optimal and suboptimal processing of polarimetric radar data have been derived and studies have been performed to predict detection performance achievable using various amounts of polarimetric information (see References [1, 2, 4]). In this section, we give a brief description of the algorithms considered in this paper.

(1) Optimal Polarimetric Detector (OPD)

The optimal polarimetric detector (OPD) is simply the polarimetric likelihood-ratio-test, derived under the assumption of jointly complex Gaussian statistics. The OPD is the quadratic

$$\underline{X}^\dagger \Sigma_c^{-1} \underline{X} - (\underline{X} - \underline{\bar{X}}_t)^\dagger (\Sigma_t + \Sigma_c)^{-1} (\underline{X} - \underline{\bar{X}}_t) > T \tag{10}$$

where $\underline{\bar{X}}_t$ is the target mean, Σ_t and Σ_c are the target and clutter polarization covariances described earlier and T is the detection threshold. Note that this detector requires *a priori* knowledge of the target mean and the target and clutter covariances; thus, it is difficult to implement.

For the deterministic target-in-clutter case (such as a trihedral or dihedral), the OPD reduces to a linear detector of the form

$$\text{Re} \left\{ \underline{X}^\dagger \Sigma_c^{-1} \underline{\bar{X}}_t \right\} > T \tag{11}$$

This linear detector is simply the matched filter for detecting a known polarimetric target in correlated (non-white) clutter.

(2) Identity-Likelihood-Ratio-Test (ILRT)

An alternative to the OPD was proposed by DeGraff [4] -- he substituted a scaled identity matrix for the target covariance in the equation defining the OPD and assumed $\bar{\underline{X}}_t = 0$. The resulting detector, called the identity-likelihood-ratio-test (ILRT), is given by

$$\underline{X}^\dagger \left[\Sigma_c^{-1} - \left(\frac{1}{4} E\{\text{span}(\underline{X}_t)\} I + \Sigma_c \right)^{-1} \right] \underline{X} > T \quad (12)$$

where I is the 3 by 3 identity matrix.

The ILRT requires knowledge of the clutter covariance matrix and the target-to-clutter ratio (in the form of $\frac{1}{4} E\{\text{span}(\underline{X}_t)\}$). However, it has the advantage that the mean and covariance of the target are not required precisely. Furthermore, the effect of target mismatch (substituting $\frac{1}{4} E\{\text{span}(\underline{X}_t)\} I$ for Σ_t) is not well understood.

(3) Polarimetric Whitening Filter (PWF)

Using a different approach, a simple quadratic detector was derived in Reference [2]. This algorithm requires *a priori* knowledge of the clutter polarization covariance only. This detector, the polarimetric whitening filter (PWF), is given by the quadratic

$$\underline{X}^\dagger \Sigma_c^{-1} \underline{X} > T \quad (13)$$

The PWF has been shown to minimize the "speckle" or standard deviation-to-mean ratio of the clutter background in a SAR image, thereby enhancing the ability to detect targets. Results were reported for an armored target in clutter which show that the detection performance of the PWF is almost identical to that of the OPD, indicating that knowledge of the target covariance is not essential.

(4) Single Channel Detector $\left(|\text{HH}|^2 \right)$

The simplest possible processor is the use of a single-polarimetric-channel. In this case, the detector simply compares the magnitude squared of the HH channel to the detection threshold, T, as indicated below.

$$|\text{HH}|^2 > T \quad (14)$$

(5) Span Detector

The span detector is a widely used processor which is a weighted non-coherent sum of all three polarimetric channels. The algorithm is given by

$$|\text{HH}|^2 + 2 |\text{HV}|^2 + |\text{VV}|^2 > T \quad (15)$$

The span provides an improvement over a single-channel $|\text{HH}|^2$ measurement because it uses information from all three channels. However, it does not require the knowledge of the target or clutter statistics.

(6) Power Maximization Synthesis (PMS)

Power maximization synthesis (PMS) has been proposed as an improvement to the span detector [3, 4]. The PMS detector is specified by

$$\frac{1}{2} \left(|\text{HH}|^2 + 2 |\text{HV}|^2 + |\text{VV}|^2 \right) + \sqrt{\left(|\text{HH}|^2 - |\text{VV}|^2 \right)^2 + 4 |\text{HH}^* \text{HV} + \text{VV} \text{HV}^*|^2} > T \quad (16)$$

As with the span detector, the PMS detector is a function of the components of the measurement vector and makes no use of *a priori* target or clutter statistics.

Algorithm Performance Comparisons

This section summarizes typical detection performance results obtained for the polarimetric detectors described above. Results are presented for the two target-in-clutter scenarios studied -- the random (multi-scatterer) target-in-clutter and the deterministic (dihedral or trihedral) target-in-clutter.

For this study, the target-to-clutter ratio is defined as the ratio of the expected span of the target-only return to the expected span of the clutter-only return.

$$T/C = \frac{E\{\text{span}(\underline{X}_t)\}}{E\{\text{span}(\underline{X}_c)\}} \quad (17)$$

In the multi-scatterer target case, the target-to-clutter ratio is given by

$$T_M/C = \frac{\sigma_{\text{HH}_t} (1 + 2\varepsilon_t + \gamma_t)}{\sigma_{\text{HH}_c} (1 + 2\varepsilon_c + \gamma_c)} \quad (18)$$

In the deterministic target case, the target-to-clutter ratio is given by

$$T_d/C = \frac{2\alpha^2}{\sigma_{\text{HH}_c} (1 + 2\varepsilon_c + \gamma_c)} \quad (19)$$

Typical performance results for the random target-in-clutter case are presented first. Figures 1, 2, 3, and 4 compare the detection performance obtained for the polarimetric detectors under consideration. Each figure corresponds to a particular target with a specified target-to-clutter ratio. Figures 1 and 2 show the performance results for Target 1 (an armored target) in meadow clutter, with T/C ratios of 6 dB and 10 dB, respectively. Figures 3 and 4 show the performance results for Target 2 (a truck) in meadow clutter, with target-to-clutter ratios of 6 dB and 10 dB, respectively. The target and clutter polarization covariances for these studies were obtained empirically. The values of the covariance statistics are given in Table 1.

	ϵ	γ	$\rho\sqrt{\gamma}$
Target 1	0.19	1.0	0.28
Target 2	0.02	1.1	0.83
Clutter	0.18	1.1	0.63

Table 1: Polarimetric Parameters of Targets and Clutter

The performance curves shown in Figures 1-4 tend to cluster into three groups. The OPD, the PWF, and the ILRT yield similar performance. The span and PMS detectors are similar to each other in performance, but have degraded performance in comparison with the OPD, PWF, and ILRT. Use of a single-polarimetric-channel detector ($|HH|^2$) is least effective. An exception to these generalizations occurs in Figure 3 (the truck with a 6 dB T/C ratio). In this case, the ILRT performs significantly poorer than the OPD and the PWF.

Another measure of detection performance is the log standard deviation (σ) dB of the clutter and target-plus-clutter detection statistics. For the same target-to-clutter ratio, the detector whose detection statistic has the smallest target-plus-clutter and clutter-only standard deviations typically yields the best detection performance. A comparison of the log standard deviations of each polarimetric detector is given in Table 2. In a typical case,

$$\begin{aligned}
 (\sigma_{HH})_{dB} &> (\sigma_{span})_{dB} \approx (\sigma_{PMS})_{dB} > (\sigma_{ILRT})_{dB} \\
 &\approx (\sigma_{PWF})_{dB} \approx (\sigma_{OPD})_{dB}
 \end{aligned}$$

This comparison substantiates the comparisons made above -- specifically, the OPD, the PWF, and the ILRT achieve comparable performance; their performance is better than the other algorithms. The span and PMS detectors achieve comparable performance. The single-polarimetric-channel ($|HH|^2$) achieves the worst performance.

	Clutter Only	Target 1 6 dB	Target 1 10 dB	Target 2 6 dB	Target 2 10 dB
$ HH ^2$	5.57	5.56	5.56	5.57	5.57
Span	3.15	2.93	2.92	3.76	3.89
PMS	3.07	2.93	2.92	3.52	3.63
ILRT	2.72	2.84	2.82	2.82	2.99
PWF	2.70	2.77	2.79	2.95	3.07
OPD	2.71	2.81	2.80	3.21	3.21

Table 2: Log Standard Deviation of Detection Statistics

Next, we present typical performance results for the deterministic target-in-clutter case. Specifically, we show the detection performance achieved by the polarimetric detectors in the case of a single unitary scatterer in meadow clutter. Four such cases are considered: dihedrals oriented at 0° , 22.5° , and 45° , respectively, and a trihedral. The target-to-clutter ratio for each case is chosen to be 3 dB as defined in equation 17.

Figures 5-7 compare the detection performance achieved by the polarimetric detectors for the dihedral cases. In each case, the PWF and ILRT detectors exhibit performance which is near that of the OPD detector. The PMS detector yields performance similar to that of the span detector; both are significantly degraded from the optimal. The single-polarimetric-channel detector ($|HH|^2$) is least effective for the dihedral cases. Note that as the dihedral orientation angle approaches 45° , the $|HH|$ -detector performance degrades significantly. At 45° , there is no dihedral target return in the HH-channel; thus, for this case $P_D = P_{FA}$ (see Figures 5-7).

Figure 8 illustrates the detection performance for the trihedral case. As indicated, optimal detection of a trihedral is more difficult than optimal detection of a dihedral (compare OPD curves shown in Figures 5-8). This occurs because clutter (which is predominantly odd-bounce) is statistically more similar to the trihedral (an odd-bounce reflector) than it is to the dihedral (an even-bounce reflector). Since detectors such as the PWF and IRLT attempt to exploit the differences in statistical properties of the targets versus the clutter, they have difficulty in discriminating the trihedral from clutter. The span and PMS detectors, however, achieve approximately the same performance against dihedrals and trihedrals.

Summary and Conclusions

The results presented in this paper correspond to the ideal case in which target and clutter statistics are known exactly. Unfortunately, such statistics are non-trivial to obtain in practice. It may not be possible to measure the necessary statistics of the desired targets *a priori*. Similarly, clutter statistics are difficult to obtain *a priori* because they vary spatially with the type of terrain and temporally due to weather and seasonal changes (see Reference [5]). However, clutter statistics may be estimated at processing time (i.e. "on-the-fly") with a considerable computational cost. (This is done by the adaptive polarimetric whitening filter which is described in Reference [2].) Consequently, it may be desirable to choose a

detection algorithm which requires as little statistical information as possible.

The results presented above indicate that significant improvement in detection performance may be obtained by using the clutter statistics. The additional information provided by the target statistics appears to yield only a modest improvement in the detection performance. Of course, such information would be crucial for target discrimination. The results of our study indicate that the PWF, which requires only the clutter covariance, typically approaches optimal performance. Furthermore, the ILRT does not perform significantly better than the PWF despite the fact that the ILRT uses more statistical information than the PWF. In some cases, the ILRT performs somewhat worse than the PWF (see Figures 3 and 7). The span, PMS, and $|HH|^2$ detectors which make no use of target and clutter statistics, are typically less effective for target detection than the PWF. This suggests that the PWF may provide the best trade-off between detection performance and the amount of statistical information required.

References

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- [4] S.R. De Graff, "SAR Image Enhancement via Adaptive Polarization Synthesis and Polarimetric Detection Performance," Polarimetric Technology Workshop, Redstone Arsenal, AL, August, 1988.
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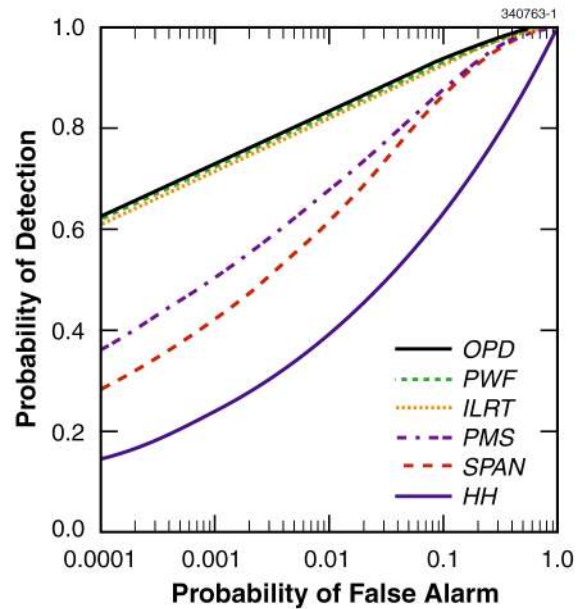


Figure 1. Detection performance for target 1 with a 6 dB T/C ratio

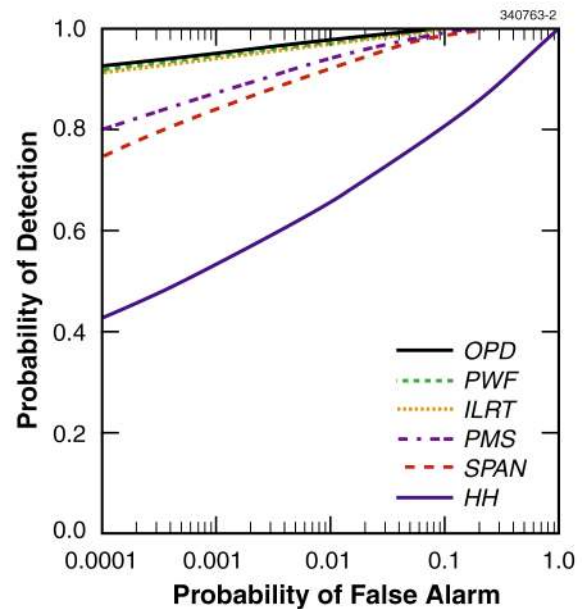


Figure 2. Detection performance for target 1 with a 10 dB T/C ratio

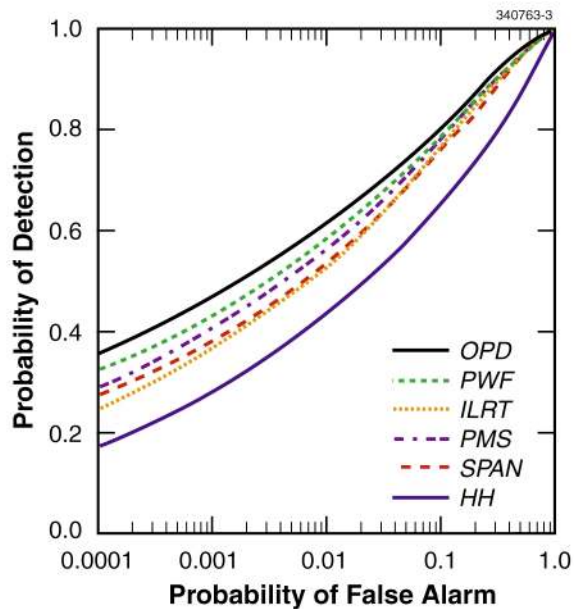


Figure 3. Detection performance for target 2 with a 6 dB T/C ratio

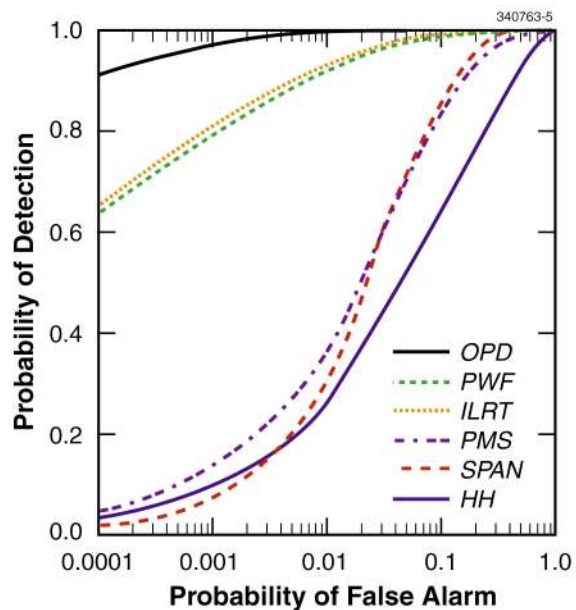


Figure 5. Detection performance for a dihedral oriented at 0° with a 3 dB T/C ratio

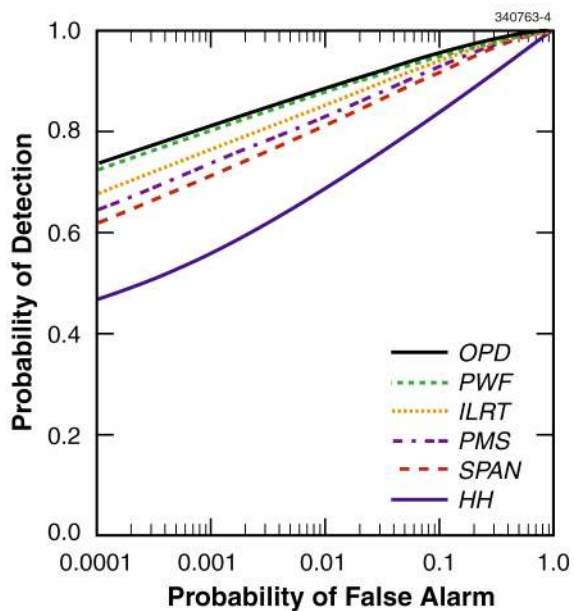


Figure 4. Detection performance for target 2 with a 10 dB T/C ratio

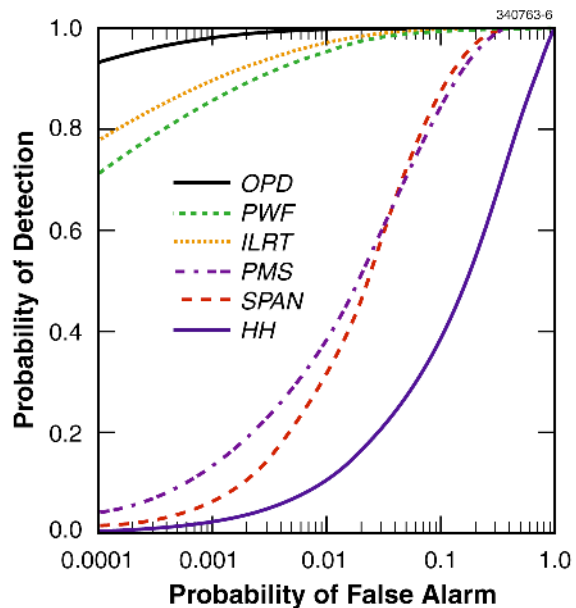


Figure 6. Detection performance for a dihedral oriented at 22.5° with a 3 dB T/C ratio

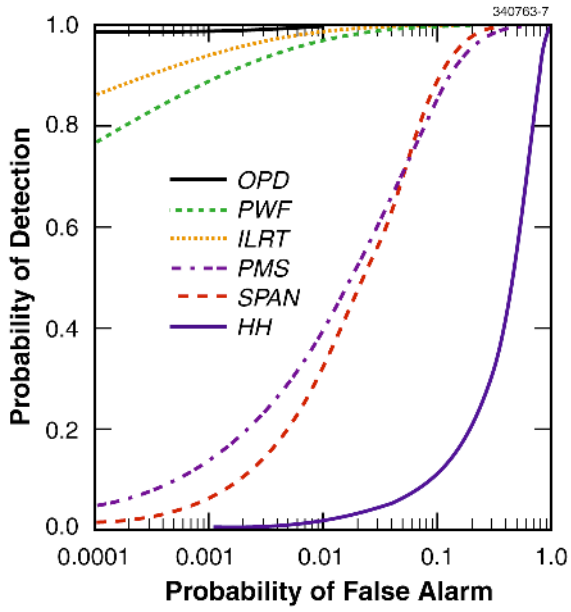


Figure 7. Detection performance for a dihedral oriented at 45° with a 3 dB T/C ratio

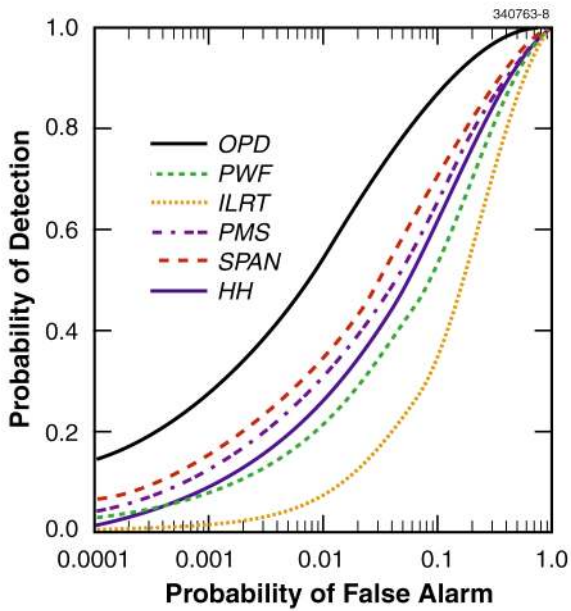


Figure 8. Detection performance for a trihedral with a 3 dB T/C ratio