On the Persian Translation of Bhāskara's *Līlāvatī* by Abu'l Faiẓ Faiẓī at the Court of Akbar

Sreeramula Rajeswara Sarma[a,*], **Maryam Zamani**[b]

[a] Höhenstrasse 28, 40227 Düsseldorf, Germany
[b] Faculty of Mathematical Sciences, Guilan University, Rasht, Iran

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Abstract

At the court of Akbar, several Sanskrit texts were rendered into Persian; these included the epics *Mahābhārata* and *Rāmāyaṇa*, collections of fables and legends like the *Pañcatantra*, the *Siṃhāsana-dvātriṃśikā* and the *Kathāsaritsāgara*, and the historical work *Rājataraṅgiṇī*. Besides these, a Sanskrit mathematical text, the *Līlāvatī* of Bhāskarācārya was also translated into Persian by Akbar's Poet Laureate Faiẓī. While the Persian translations of the *Mahābhārata* and others have been critically examined in modern times, the Persian version of the *Līlāvatī* did not receive any scholarly attention, except in two minor cases. In 1816, John Taylor, in the preface to his translation of the *Līlāvatī* from the Sanskrit, opined that Faiẓī's Persian version omits certain sections of the *Līlāvatī*. In 1952, H. J. J. Winter and Arshad Mirza discussed a small fragment of the Persian version and translated 10 verses from it into English. Therefore, in this paper, an attempt is made for the first time to compare the Persian version with the Sanskrit original and to critically analyse the structure and style of the Persian version.

Key words: Ab*u*'l Faẓl, Akbar, Bhāskara, Bureau of Translation, Examples, Faiẓī, *Līlāvatī*, Rules.

1 Bureau of translation at the court of Akbar

The Mughal emperor Akbar (r. 1556–1605), desirous that the Muslim intelligentsia be made familiar with the classics of Hindu thought so that they have a better interaction with the Hindus, set up a bureau of translation (*maktabkhāna*) at his capital Fatehpur Sikri near Agra around 1575.[1] In this bureau several Sanskrit texts were rendered into Persian. These included the two great Sanskrit epics *Mahābhārata* and *Rāmāyaṇa*, collections of fables and legends like the *Pañcatantra*, the *Siṃhāsana-dvātriṃśikā* and the *Kathāsaritsāgara*, and the historical work *Rājataraṅgiṇī*.[2]

Among these Persian translations, that of the *Mahābhārata* occupies a pre-eminent position. Although it was named *Razm-nāma* (book of war), it was regarded by Akbar primarily as a book on statecraft. He commissioned the translation in 1582. It was done jointly by Naqīb

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* Email: SR@sarma.de.

[1]For an illustration of this translation bureau, see Adamjee & Truschke, 2015, p. 146, Fig. 5.2; Truschke, 2016, p. 106, Fig. 3.1.

[2]The following account is largely based on Truschke, 2016 and Ansari, 2019.

Khān, Mullā Shīri, ᶜAbd al-Qādir Badā'ūni and Sulṭān Thānīsarī, with the help of Sanskrit scholars Deva Miśra, Śatāvadhānī, Madhusūdana Miśra and Caturbhuja. In 1587 Abu'l Faẓl wrote an extensive Preface to this translation. Sometime later, Ab*u*'l Faẓl's elder brother Faiẓī prepared a verse translation of the first two books of this epic. The *Rāmāyaṇa* was similarly translated by a team consisting of Naqīb Khān, ᶜAbd al-Qādir Badā'ūnī and Sultān Thānīsarī.

The *Pañcatantra* was rendered into Pahlavi as early as the sixth century at the instance of the Sassanian ruler Anūshīrwān. Towards the middle of the eighth century this Pahlavi version was rendered into Arabic under the title *Kalila wa Dimna* at the Abbasid court. This Arabic rendering gave rise to many versions in modern Persian, notable of these being the *Anvār-i Suhaylī* (Light of Canopus) which was produced at Herat in 1504. Akbar sponsored at least two Persian versions of the *Pañcatantra*. Abu'l Faẓl composed the *ᶜIyārī Dānish* (Touchstone of Intellect) which is based on the *Anvār-i Suhaylī*. Muṣṭāfa Khliqdād ᶜAbbāsī authored *Pañcākhyānāh* (Five Stories) on the basis of a Jain version of the *Pañcatantra* entitled *Pañcākhyāna*. Likewise two Persian retellings were produced of the *Siṃhāsana-dvātriṃśikā*. Badā'ūnī claims to have made a translation under the name *Nāma-i Khirad Afẓā*. Chaturbhuja Das authored a separate adaptation under the title *Shāhnāma* (Book of Kings).

The *Kathāsaritsāgara* was composed by Somadeva in the eleventh century in Kashmir. It was partly rendered into Persian at the court of Zayn al-ᶜĀbidīn (r. 1420–1470) of Kashmir. At Akbar's orders, Badā'ūnī revised and completed this Persian translation. The *Rājataraṅgiṇī*, a chronicle of the kings of Kashmir by Kalhaṇa (12th century) was also rendered into Persian at the court of Zayn al-ᶜĀbidīn. Shāh Muḥammad Shāhābādī retranslated it and presented it to Akbar on his first visit to Kashmir in 1589. Akbar ordered Badā'ūnī to polish the language; accordingly, Badā'ūnī revised the Persian translation in 1591.

Many of these Persian renderings, especially those of the *Mahabhārata* and *Rāmāyaṇa* were lavishly illustrated by the painters at Akbar's atelier.[3] Quite different from these in content and style is Bhāskara's *Līlāvatī* on arithmetic and geometry which too was rendered into Persian. It may be noted that no manuscripts of this work were illustrated at Akbar's court.[4]

At Akbar's court there were not many scholars proficient in both the source language Sanskrit and the target language Persian.[5] How then were these translations made? According to ᶜAbd al-Qādir Badā'ūnī, the task was accomplished by teams of scholars, some proficient in Sanskrit and others in Persian. They did the work in three stages. First, the Hindu or Jain scholars prepared a paraphrase of the Sanskrit text in the local vernacular. In the second stage, this paraphrase was rendered into Persian by one of the several Muslim assistants. The Hindus and Jains who prepared the paraphrase were known as *mu'barān* (interpreters) and the Muslims who rendered the paraphrase into Persian were styled *mutarajimān* (translators). Finally, the Persian rendering was polished and put into elegant prose and verse by one of the more accomplished scholars, who signed it as his work. What resulted in this process cannot be termed an exact translation but rather a Persian paraphrase into which often the mediator's explanatory sentences also crept in (Hodivala 1939, pp. 564–566). According to Truschke, however, this is not the case with all the texts; the translation of the *Mahābhārata*, she argues, is more faithful to the original (Truschke 2016, p. 107).

2 Translation of the Līlāvatī into Persian

2.1 The original Sanskrit text

The *Līlāvatī* was composed by the renowned mathematician and astronomer Bhāskara II (born in 1114), who is respectfully referred to as Bhāskarācārya (Bhāskara + ācārya, i.e. 'Bhāskara, the revered teacher'). He authored his magnum opus *Siddhānta-śiromaṇi* at the age of 36 in 1150. The *Līlāvatī* on arithmetic and geometry and the *Bī-*

[3]There is extensive literature on the Mughal miniature paintings which illustrate these translations; see especially Adamjee and Truschke, 2015.

[4]However, a late manuscript of the Persian translation of the *Līlāvatī*, copied by Pandit Dayārām in 1857 at Lahore, carries an illustration of *Līlāvatī* and the bridegroom seated on either side of a highly stylized water clock; see Fig. 1 below. The manuscript is preserved in the Salar Jung Museum, Hyderabad.

[5]The only exceptions, as far as we know, are Khān-i Khānān ᶜAbd al-Rahīm Khān and Bihāri Kṛṣṇadāsa. The former composed several poems in Sanskrit and also an astrological work entitled *Kheṭa-kautuka*, in which he incorporated Arabic and Persian terms in Sanskrit verses (cf. Chaudhury, 1954). Bihāri Kṛṣṇadāsa authored the *Pārasīka-prakāśa* to teach Persian language through the medium of Sanskrit (see Sarma, 1995; Truschke, 2012).

jagaṇita on algebra, which lay the necessary foundation for the study of the *Siddhānta-śiromaṇi*, must have been composed by him prior to 1150.

The *Līlāvatī* was extremely popular; there are extant today some 600 manuscript copies of this work. These manuscripts are written in all the different scripts of India, thus testifying to the fact that the work was studied in all the regions of India (Pingree 1981, pp. 299–326). The *Līlāvatī* attained its high reputation not only for its mathematical content, but more particularly for its highly ornate style. At the very beginning of the *Līlāvatī*, Bhāskara states that the three main features of his exposition are brevity (*saṃkṣiptatā*), clarity (*prasphuṭatā*) and elegance (*lālitya*).[6] The first two pertain to the rules, for Bhāskara aims to make the rules as concise and brief as possible and at the same time very clear and without any ambiguities. The third quality of elegance belongs to the examples; this is achieved by pleasing alliteration and the use of various figures of speech.[7]

2.2 Faiẓī, the translator of the Līlāvatī

Whosoever advised Akbar to have the *Līlāvatī* rendered into Persian could not have chosen a better Sanskrit text than the *Līlāvatī*. Not only was the chosen Sanskrit text the most renowned, but the person commissioned by Akbar to render it into Persian was equally distinguished. Shaykh Abu'l Faiẓ ibn Mubārak (1547–1595) is popularly known by his pen-name (*takhalluṣ*) Faiẓī; later he changed the pen-name to Fayyāẓī.[8] He was born at Agra and was educated by his father Shaykh Mubārak. Having heard of his exceptional talents, Akbar summoned him to his court in 1567 and made him the Poet Laureate (*malik al-shu*^{c}*arā*) in 1588.[9]

Faiẓī was an outstanding scholar and an acclaimed poet. He wrote an exegesis on the *Qur'ān* in Persian, employing only the undotted letters of the alphabet. While other members of the court prepared a prose translation of the *Mahābhārata*, he created a verse translation of the first two books (Truschke 2016, pp. 133–137). The episode of Nala and Damayantī of the *Mahābhārata* inspired him to compose a *Maṣnavī* in 4000 verses under the title *Nal wa Dāman*. Badā'ūnī states that 'verily it is a *Masnavī*, the like of which for the last 300 years since Mīr Khusrū no poet has composed.'[10]

Faiẓī was ordered by Akbar to translate the *Līlāvatī* into Persian, which task he accomplished 1587, 'by taking' as he says, "the help of the knowledge of the experts of this science, especially the astronomers of the Deccan." Although we know the names of several Hindu and Jain scholars at the court of Akbar, it is not possible to ascertain who the astronomers of the Deccan were who assisted Faiẓī.[11]

Abu'l Faẓl refers to Faiẓī 's translation of the *Līlāvatī* in these words: "The Lílawatí, which is one of the most excellent works written by Indian Mathematician on Arithmetic, lost its Hindú veil, and received a Persian garb from the hand of my elder brother, Shaikh 'Abdul Faiz i Faizí."[12]

2.3 Faiẓī's preface and conclusion

Faiẓī declares that his translation consists of a preface (*muqaddama*), a number of rules (*chand ẓābiṭa*) and a conclusion (*khātima*). In the preface, Faiẓī declares that the *Līlāvatī* is "reputed, among the unique works of arithmetic and mensuration, for its fluency and elegance of its style." He goes on to say:

> Indeed the book is a wonderful volume of writing, a unique narration. If the Greek observers of the movements of stars were to use it as a protective

[6] *Līlāvatī* 1:

> *prītiṃ bhaktajanasya yo janayate vighnaṃ vinighnan smṛtaḥ*
> *taṃ vṛndārakavṛndavanditapadaṃ natvā mataṅgānanam |*
> *pāṭīṃ sadgaṇitasya vacmi caturaprītipradāṃ prasphuṭāṃ*
> *saṃkṣiptākṣara-komalāmalapadair lālitya-līlāvatīm ||*

[7] On the poetic beauty of the *Līlāvatī*, see Filliozat, 2019, pp. 40–55; on the mathematical content and the aesthetic quality, see Ramasubramanian et al, 2019, pp. 59–101.

[8] Blochmann, I, p. 540, n.1: "*Faiz* is an Arabic word meaning 'abundance;' *Faizí* would mean a man who has abundance or given abundantly. *Fayyáz* is the intensive form of *Faizí*, giving super abundantly. *Fayyájí* is originally an abstract noun, 'the act of giving superabundantly,' and then became a title."

[9] On the life and work of Faiẓī, see Blochmann, I, pp. 490–491; Ansari 2019, pp. 380–381.

[10] Badā'ūnī II, p. 411.

[11] Abu'l Faẓl enumerates the names of some 140 persons as 'the learned men of the time' (*Ā'īn-i Akbarī*, I, pp. 537–547); of these 32 are Hindus or Jains. But very little is known about their accomplishments.

[12] Abu'l Faẓl, *Ā'īn-i Akbarī*, I, p. 105.

Figure 1 Līlāvatī and the bridegroom with a water clock in the middle (From Kocchar & Narlikar 1995, plate C1)

> band on their arms, it would be just; and if the Persian experts of astronomical tables were to tie it as a talisman upon their heads, it would be appropriate. It is like a bouquet of flowers from the garden of science and knowledge (*guldasta ī ast az bahāristān i ḥikmat wa kārdānī*), a work of art from the picture gallery of the precious and unique aspects of reality (*kārnāma ī ast az nigāristān i badāyi^c wa nawādir i ma^cānī-yi intiẓām*).[13]

After this encomium, Faiẓī narrates a legend to explain why Bhāskara wrote the book of mathematics called *Līlāvatī* or rather why Bhāskara gave to the book of dry mathematics the name *Līlāvatī* which literally means 'a graceful woman'. The reason, he goes on to narrate, was that *Līlāvatiī* was the daughter of Bhāskara. For her marriage, Bhāskara fixed an astrologically propitious moment. But accidentally the water clock set up to determine the exact time of the marriage did not function and therefore the marriage could not take place. To console the daughter, who would spend the rest of life as a spinster, Bhāskara told her: "I shall write a book titled after your name, which will long endure in the world, for a good name is like a second life for one and confers immortal life upon the seed."

It has been shown that this story of Bhāskara's daughter does not occur in any Sanskrit source, that it occurs for the first time in Faiẓī 's Persian translation, and that the story as narrated by Faiẓī is quite improbable (Sarma, 2019, pp. 23–39). Faiẓī's conclusion reads as follows:

> The translation of the book and explanation of arithmetic are completed at the beginning of spring when plenty of new things are created in the world and thousands of plants grow in gardens. The graces of the Lord are uncountable; the rains of Naisān[14] are countless, as also the pages of the book. Nightingales sing like children learning arithmetic; birds repeat themselves as freshmen do in mental calculation. It is hoped that the scientists in the service of the King, especially skilled mathematicians and astronomers may modify this [translation]. I close here by praying for the longevity of the Sovereign.
>
> The King by whom the Universal Wisdom is illuminated,
> Whose high ascending thought has gone beyond [our] thoughts.
> May everyone celebrate forever,
> This New Day, this New Month, and this New Year.[15]

2.4 Manuscripts and printed editions of the Persian version

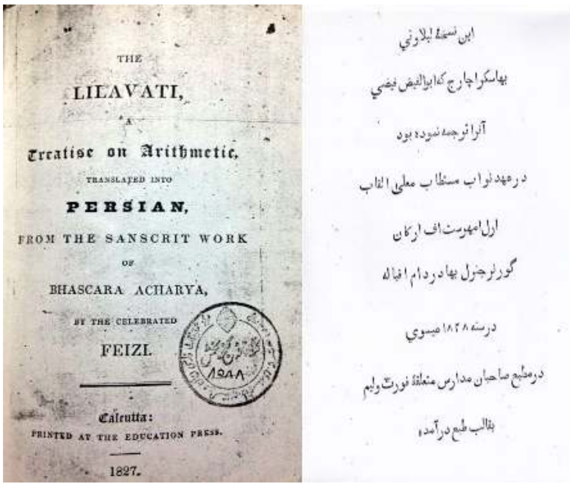
Some 37 manuscripts of the Persian version of the *Līlāvatī* are said to be extant in various collections in the Indian subcontinent and several more outside (Ansari, 2019, pp. 381–382). It was printed for the first time at Calcutta in 1827 under the auspices of the East-India Company presumably for use in the College of the Fort William, where the servants of the Company were trained in local languages.

The cover page of this first edition carries the following title in English: *Lilavati, a Treatise on Arithmetic translated into Persian from the Sanskrit Work of Bhascara Acharya by the celebrated Feizi.* The title page reads as follows in Persian: *īn nuskhi yi Līlāvatī i* | *Bhāskar Ācharj ki Abu'l Faẓl Faiẓī* | *ān rā tarjumi nimūdi būd* | *dar ^cahad i nuwwāb i mustaṭāb i mu ^callā alqāb* | *Earl Amherst āf*

[13] Translation by Professor Irfan Habib, here slightly modified; for the full translation of the Preface by Professor Habib, see, Sarma 2019, Appendix I on pp. 33–35.

[14] Naisān is the first month in the Syrian calendar and corresponds to April.

[15] All translations from the Persian and Sanskrit are by the authors, unless otherwise specified.


THE
LILAVATI,
A
Treatise on Arithmetic,
TRANSLATED INTO
PERSIAN,
FROM THE SANSCRIT WORK
OF
BHASCARA ACHARYA,
BY THE CELEBRATED
FEIZI.
Calcutta:
PRINTED AT THE EDUCATION PRESS.
1827.

این نسخهٔ لیلاوتی
بهاسکر اچارج که ابوالفیض فیضی
آنرا ترجمه نموده بود
در عهد نواب مستطاب معلی القاب
ارل امهرست اف ارکان
گورنر جنرل بهادر دام اقباله
در سنه ۱۸۲۷ عیسوی
در مطبع صاحبان مدارس متعلقهٔ فورت ولیم
بقالب طبع درآمده

Figure 2 The Calcutta edition of 1827; the cover page on the left, the title page on the right.

*Arkān | Gavarnar Janaral bahādur dām iqbāla | dar sani yi 1827 ʻīsawī | dar maṭbai*ᶜ *yi ṣāḥibān i madāris muta*ᶜ*liqi yi Fort William | bi qālib i ṭaba*ᶜ *dar āmdi* (This text of *Līlāvatī* by Bhāskar Ācharj, which Abu'l Faẓl Faiẓī translated, was printed at the time of the Right Honourable Lord (?), Earl Amherst of Arakan, the valiant Governor General — may his good fortune last forever — in the year 1827 CE at the Education Press in the Fort William).[16]

It is interesting to note that while the Persian translation was commissioned by the Mughal Emperor Akbar, its *editio princeps* was printed 240 years later under the authority of the Governor General of the East-India Company, which succeeded the Mughal Empire.

In the present paper we use this Calcutta edition of 1827 as well as a manuscript from the Bodleian Library at Oxford (MS. Ind. last. Pers.105). These two sources have some minor differences. But there are considerable differences between the Persian translation and the Sanskrit original which will be discussed in the following pages.

3 Structure of the Persian version

Having discussed Faiẓī's Preface and Conclusion, we now turn to the translation of the text itself. John Taylor of the East India Company's Bombay Medical Establishment, who translated the *Līlāvatī* from Sanskrit into English in 1816, makes the following remarks about Faiẓī's Persian rendering:

> By the direction of the emperor Acbar, whose liberal promotion of literature and science added glory to his conquests, it [i.e., the *Līlāvatī*] was translated into Persian in 1587 by Fyzi. ...It [i.e., the translation] is, however, often very obscure, and in several places there are considerable omissions, especially towards the end of the arithmetic, and in the geometrical operations which immediately precede the chapter on circles. The chapters on indeterminate problems and on transpositions are altogether omitted. Besides, the style is not only much more diffuse than what necessarily arises from the difference of the Persian and Sanscrit idioms, but the manner also of delivering the rules, and of detailing the operations, generally varies in a very considerable degree from that of the original text. This, indeed, is so remarkable as to induce a suspicion, that Fyzi contended himself with writing down the verbal explanation afforded by his assistants.[17]

3.1 Omissions in the Persian version

The Sanskrit original consists of 272 verses, which are arranged in 21 sections, each section containing several rules (*karaṇa-sūtra*) and examples (*uddeśaka* or *udāharaṇa*) to illustrate the rules; the verses, however, are numbered continuously from 1 to 272.[18] There are also short prose passages which introduce the rules and

[16]Another edition of the work was published in Lucknow in 1854. There is a third edition by Shaykh Zafar ᶜAlī in which no date and place of publication are mentioned.

[17]Taylor, 1816, Introduction, p. 2. Winter and Mirza, 1952 discuss an incomplete manuscript of the Persian version dated 1729 which is preserved in the John Rylands Library at Manchester (MS 699) and give complete translations of the following ten examples: 70, 71, 76, 80, 82, 86, 89, 95, 98 and 100. Apparently they have at their disposal Colebrooke's translation of the *Līlāvatī*, but they do not discuss how far the Persian version is faithful to the Sanskrit original and where it deviates from the Sanskrit.

[18]In his English translation, Colebrooke rearranged the 21 sections into 13 chapters and each chapter into several sections. While the printed editions of the Sanskrit text contain 272 verses, Colebrooke's English translation has 277. This difference arose because of certain anomalies in the numbering of the verses both in the Sanskrit original as well as in the English translation. Sometimes half verses which stand alone are numbered as full verses in Sanskrit manuscripts; for example, in the Sanskrit text, 16 and 95 are half verses, but Colebrooke did not give separate numbers to them. On the other hand, in some

statements called *nyāsa* where the numerical quantities mentioned in the examples are laid out. The Persian rendering is entirely in prose and makes no distinction between the different parts of the original. The verse numbers as in the original are not mentioned at all, but the different parts are separated sometimes by the term *ẓābiṭa* (rule), some other times by phrases such as "the other method is ...", or "the second / third way is ..."

Moreover, of 272 verses of the original, some 65 verses are not translated into Persian. In particular, the last two sections consisting of 31 verses are totally left out. These two sections are entitled *Kuṭṭaka* and *Aṅkapāśa*. *Kuṭṭaka*, rendered literally into English as "pulveriser", deals with the solution of indeterminate equations by the method of continued fractions; this procedure is employed in astronomical texts for deriving the number of revolutions a planet makes in a long period called *Mahāyuga*. The last section *Aṅkapāśa*, literally "net of numbers", deals with permutations of numbers. Besides these, a large continuous group of 23 verses (178–200) dealing with areas of irregular figures in geometry is also omitted. One may explain these omissions by saying that these three groups deal with topics of a very narrow specialization and therefore are not of much interest for a general reader. Finally, 11 verses are omitted here and there in different chapters; these are as follows:

1, 9 Salutations to Gaṇeśa

113 Example on the permutation of the syllables in the *Gāyatrī* metre of Sanskrit prosody

130-131 Rule for calculating the number of varieties in a metre of Sanskrit prosody.

132 Example related to the *Anuṣṭubh* metre of Sanskrit prosody.

138 Rule for determining the length of the hypotenuse in a right triangle, when the lengths of the two sides are given, by calculating the square-root of their sum.

152 Example of a lotus stalk bending in water

158 Example to find the value of the two sides of a right triangle, when their difference and the length of the hypotenuse are given.

166 Example, given the values of the two sides and the base of an irregular triangle, to find its area etc.

231 Maxim: the price of bricks or stones is determined by the softness or hardness of the material and by the agreement with the workmen.

Verses 113 and 130–132 deal with the arrangement of syllables in metres of Sanskrit prosody and therefore were not of interest for those who did not know Sanskrit. Verses 138, 152, 158, 166 and 231 may have been considered too specialized.

Verses 1 and 9 are in praise of Hindu god Gaṇeśa; these are omitted in the Persian version which was meant primarily for Muslim readers. However, two verses referring to Hindu gods and Hindu pilgrimages are included in the Persian version. The first verse translates thus:

> Form a group of flawless lotus flowers, he worshipped the Three-eyed [Śiva], Hari and the Sun respectively with a third, a fifth and a sixth part; one fourth [was likewise offered] to Pārvatī. With the remaining six lotuses, he [worshipped] his venerable teacher. Tell quickly the number of all his lotuses.[19]

For metrical reasons, Bhāskara employs in this verse *Trinayana* for Śiva and *Āryā* for *Pārvatī*. The Persian version uses the more common synonyms Mahādeva, Kṛṣṇa and Pārvatī and adds also the solution:

> Someone has a bouquet of lotus flowers; he offered a third part to Mahādeva, a fifth to Krishna, a sixth

places, the Sanskrit text does not give a separate number to half verses, but Colebrooke does so; for example, that which is numbered 214 in Sanskrit is actually a unit of one and a half verses, to which Colebrooke gives the two numbers 217 and 218.

For general convenience, we follow Colebrooke's numbers in this paper. When we refer to, say verse 73 in the Sanskrit original, we write "*Līlāvatī* 73"; when we refer to Colebrooke's translation of the same verse, we write "Colebrooke 70". But when we write just a number without any qualification, it indicates the number assigned by Colebrooke in his translation.

[19] *Līlāvatī* 53 (Colebrooke 52):
amalakamalarāśes tryaṃśa-pañcāṃśa-ṣaṣṭhais
trinayana-hari-sūryā yena turyeṇa cāryā |
gurupadam atha ṣaḍbhiḥ pūjitaṃ śeṣapadmaiḥ
sakalakamalasaṃkhyā kṣipram akhyāhi tasya ||

> part of them to Sun, and presented a quarter of them to Pārvatī. He offered the six remaining lotuses to the teacher. According to the procedure explained [in the rule], the sum of flowers is twelve.

The next verse in the *Līlāvatī* is as follows:

> A pilgrim gave away half of his money at Prayāga; two-ninths of the remainder at Kāśī; one-fourth of the remaining for the tax on the road; six-tenths of what remained at Gayā; he returned home with the remaining sixty-three niṣkas. Tell quickly the original amount of his money, if you have understood the method of reduction of fractions of residues (*śeṣajāti*).[20]

The Persian version reads as follows:

> A pilgrim carried a certain amount of gold; he gave half of his gold coins to Brahmins at Prayāga, spent two-ninths of the reminder at Kāśī; he also gave away a quarter of the remainder as religious charity (*zakāt*); spent six-tenths of the remaining amount at Gayā. There remained sixty-three. What was the [original] amount of gold coins?

3.2 Additions made in the Persian version

John Taylor, as we have seen, complains about the many omissions in Persian rendering, but he does not mention that several additions were also made in the Persian version.

3.2.1 Metrology

The first section of the *Līlāvatī*, entitled *paribhāṣā* (definitions), contains tables of measurement: of monetary units, weights and linear measures (Colebrooke 2–8). One would expect that the Persian version would give here contemporary measures, but it does not. It merely transliterates the Sanskrit units of money, weight and distance. The Sanskrit original does not mention the units of time, stating that these can be known from the common usage (*śeṣāḥ kālādi-paribhāṣā lokaprasiddhyā*). But the Persian version adds them, having culled these from other Sanskrit sources. The traditional division of time in India has been the following:

10 *guru-akṣaras*	= 1 *prāṇa*	(4 seconds)
6 *prāṇas*	= 1 *pala*	(24 seconds)
60 *palas*	= 1 *ghaṭī*	(24 minutes)
60 *ghaṭīs*	= 1 nychthemeron	(24 hours)

This is rendered into Persian as follows:

> On measures of time: the time span in which we can pronounce a word having two letters like 'ka' and 'ta' ten times without hesitation, is called a *prana*; six *pranas* are equal to one *pala*; sixty *palas* are equal to a *gahri*; and sixty *gahris* are equivalent to the duration of a day and a night.

Here the smallest unit is *guru-akṣara*, time taken to pronounce a long syllable; this is explained as the "time to pronounce 'ka' and 'ta' without hesitation". The next unit is *prāṇa* 'respiration' meaning the average time taken for breathing once in and once out. This unit is rendered as *prana* in Persian without any explanation. The last two unit *pala* and *ghaṭī* are retained as such in Persian, with slight phonetic modification, because Muslims have adopted these units of time, at least since the time of Babur.

3.2.2 Decuple terms

Metrology is followed by a list of decuple terms up to *Parārdha* in two verses (10–11). These two verses are rendered into Persian thus:

> There are *'āḥād* (unity), *ᶜushrāt* (tens), *mi'āt* (hundreds), *'ūlūf* (thousands); *ᶜushrāt 'ūlūf* (ten thousands) is called *ayuta; mi'āt 'ūlūf* (hundred thousands) is *laksha*; *ᶜushrāt mi'āt 'ūlūf* (ten hundred-thousands) *prayuta; mi'āt mi'āt 'ūlūf* (hundred hundred-thousands) *koti* or *kror*, and so on indefinitely, and each upper place is ten times higher than the lower place. Ten *kotis* is an *arbuda*; ten *arbudas* is an *abja*; ten *abjas* is a *kharva*; ten *kharvas* is a *nikharva*; ten *nikharvas* is a *mahāpadam*; ten

[20] *Līlāvatī* 54 (Colebrooke 53):

svārdhaṃ prādāt prayāge nava-lavayugalaṃ
yo 'vaśeṣāc ca kāśyāṃ
śeṣāṅghriḥ śulkahetoḥ pathi daśamalavān
ṣaṭ ca śeṣād gayāyām |
siṣṭā niṣka-triṣaṣṭir nijagṛham anayā tīrthapānthaḥ prayātas
tasya dravyapramāṇaṃ vada yadi bhavatā śeṣajātiḥ śrutāsti ||

Table 1 Sanskrit and their corresponding Persian terms for *eka, daśa, śata, ..., parārdha.*

	Sanskrit	Persian
1	*eka*	*'āḥād*
10^1	*daśa*	*ᶜushrāt* (tens)
10^2	*śata*	*mi'āt* (hundreds)
10^3	*sahasra*	*'ūlūf* (thousands)
10^4	*ayuta*	*ᶜushrāt 'ūlūf* (ten thousands)
10^5	*lakṣa*	*mi'āt 'ūlūf* (hundred thousands)
10^6	*prayuta*	*ᶜushrāt mi'āt 'ūlūf* (ten hundred-thousands)
10^7	*koṭi*	mi'āt mi'āt 'ūlūf (hundred hundred-thousands)
10^8	*arbuda*	*arbuda*
10^9	*abja*	*abja*
10^{10}	*kharva*	*kharva*
10^{11}	*nikharva*	*nikharva*
10^{12}	*mahāpadma*	*mahāpadam*
10^{13}	*śaṅku*	*sanku*
10^{14}	*jaladhi*	*jaladhī*
10^{15}	*antya*	*antya*
10^{16}	*madhya*	*madhya*
10^{17}	*parārdha*	*parārdha*

> *mahāpadams* is a *sanku*; ten *sankus* is a *jaladhi*; ten *jaladhīs* is an *antya*; ten *antyas* is a *madhya*, and ten madhyas is a *parārdha*.

The Sanskrit terms for *eka, daśa, śata* and *sahasra* are translated by the corresponding terms in Persian, but thereafter the terms up to *koṭi* are explained as multiples of tens, hundreds and thousands; beyond that the Sanskrit terms are used with slight phonetic variations, as shown in the Table 1.

3.2.3 Elaborate solutions

The Persian translation generally provides the solutions of the examples or problems. In the earlier parts, the solutions are given step by step, as is done in some of the Sanskrit commentaries, but often much more elaborately. We shall discuss three such cases of elaborate solutions.

Case one

The very first example given for addition and subtraction (verse 13) is rendered in bare essentials,[21] but what is particularly interesting is that the solution of the problem is explained in great detail, both in proper order (*krama*), i.e., adding the unities first, tens next and so on, and in the reverse order (*utkrama*), i.e., adding the hundreds first, tens next and unities last, as is suggested in the rule itself. This is how the Persian rendering reads:

> For example, if we want to add 2, 5, 32, 193, 18, 10, and 100 and then we want to subtract the sum obtained from 10,000.
>
> We add two with five the result is seven, seven with two is nine, nine with three is twelve, and twelve with eight is twenty. The ones are done. We put zero under ones, and keep in mind the two of twenty. After that we take up the tens. We add the aforementioned two with three the result is five, five with nine is fourteen, fourteen with one is fifteen, fifteen with one is sixteen, and then we write down six to the left of zero. Also, for tens we keep the one in mind because of the rule of the order of tens, and begin to add the order of hundreds. Afterwards we add the one of the order of hundreds with that one kept in mind, the result is two. Two with one is three, we write down 3 to the left of six, the result is 360.
>
> If we want to do the addition in the reverse (*bar ᶜaks*) order (*tartīb*), we add one from the order of hundreds with the second one, the result will be two, we write down two under the last digit of hundreds. We add three from the order of tens with nine, the result is twelve, twelve with one is thirteen, and thirteen with one is fourteen. We write down four to the right of two, and we add the one [of the order of hundreds] to two that we registered already, the result is three, and then we take up the order of ones. We put two, five, two, three, and eight together, the result is twenty. We write down zero to the right of four, and then we add two of the order of tens to four, the result is six; we have completed the task; the final figure is 360.
>
> If we want to subtract 360 from 10,000 in proper

[21] For the Persian rendering of this verse, see 4.1 below.

order (*tartīb*) or in reverse order (*bī tartīb*) the answer is 9640.

Case two

The second case is the addition (*jam*^{c}) and subtraction (*tafrīgh*) of fractions (*kusūr*). The example given in the *Līlāvatī* actually consists of three problems; in Colebrooke's translation (35), it reads as follows:

> [i] How much is a quarter added to its third part, with a quarter of the sum? [ii] and how much are two-thirds, lessened by one-eighth of them, and then diminished by three-sevenths of the residue? [iii] Tell me, likewise, how much half less its eighth part, added to nine-sevenths of the residue, if thou be skilled, dear woman, in fractional increase and decrease?[22]

The Persian rendering:

> [i] If we want to add a quarter to its third part and the half of the sum. At first, we write down 1, then put under it four, under it 1, under it $\overline{3}$, under it 1, under it $\overline{2}$, like this:

1 4
1 3
1 2

> We multiply $\overline{3}$ by 4, we get $\overline{12}$, we add that 1 to above 3 to 3, we get $\overline{4}$, we multiply that 1 above all by $\overline{4}$, and the product is $\frac{4}{12}$, then we multiply 2 by $\overline{12}$, we got $\overline{24}$, we add the 1 above 2 to 2, we get 3, we multiply $\overline{4}$ by 3, we get 12, so, the result is $\frac{12}{24}$.

In modern notation:

$$\begin{aligned}\frac{1}{4}+\frac{1}{3\times 4}+\frac{1}{2}\left(\frac{1}{4}+\frac{1}{3\times 4}\right) &= \frac{4}{12}+\frac{1}{2}\times\frac{4}{12}\\ &= \frac{4}{12}\times\frac{3}{2}\\ &= \frac{12}{24}\\ &= \frac{1}{2}\end{aligned}$$

> [ii] We want to decrease two-thirds by its one-eighth part, and from the remaining to subtract three-seventh part of them. We write down 2, and put under it $\overline{3}$, under it 1, under it $\overline{8}$, under it 3, under it $\overline{7}$, like this:

2 3
1 8
3 7

> We multiply 8 by 3, the result is $\overline{24}$, we subtract from 8 the 1 above it, we get $\overline{7}$, then we multiply 2 above all by $\overline{7}$, we get $\overline{14}$. The result is $\frac{12}{24}$, like this:

14 24
3 7

> Then we multiply $\overline{7}$ by $\overline{24}$, we get $\overline{168}$, we subtract 3 above 7 from $\overline{7}$, we get $\overline{4}$ then, we multiply $\overline{14}$ by $\overline{4}$, we get $\overline{56}$, the result is $\frac{56}{168}$, and that is one third.

In a modern notation:

$$\begin{aligned}\frac{2}{3}-\frac{2}{3}\times\frac{1}{8}-\frac{3}{7}\left(\frac{2}{3}-\frac{2}{3}\times\frac{1}{8}\right) &= \frac{14}{24}-\frac{3}{7}\times\frac{14}{24}\\ &= \frac{14}{24}\times\frac{4}{7}\\ &= \frac{1}{3}\end{aligned}$$

[22] *Līlāvatī* 36:

aṅghriḥ svatryaṃśayuktaḥ sa nijadalayutaḥ
kīdṛśaḥ kīdṛśau dvau
tryaṃśau svāṣṭāṃśahīnau tadanu ca rahitau
tau tribhiḥ saptabhāgaiḥ |
ardhaṃ svāṣṭāṃśa-hīnaṃ navabhir atha yutaṃ
saptamāṃśaiḥ svakīyaiḥ
kīdṛk syādbrūhi vetsi tvam iha yadi sakhe
'ṃśānubandhāpavāhau ||

It is surprising that here and elsewhere Colebrooke renders *sakhe*, which is masculine vocative, as "dear women"; the feminine vocative would be "*sakhi*" and not "*sakhe*". See also Colebrooke 37.

> [iii] We will mention another example (*mithāl*) of addition (*jam*^{c}) and subtraction (*nughṣān*) mixed: if we want to subtract from a half, its eighth part, and add to the remaining its nine-seventh part. At first, we write down 1, and put under it $\overline{2}$, under it 1, under it $\overline{8}$, under it 9, under it $\overline{7}$, in this manner:
>
> | |
> |---|
> | 1
2 |
> | 1
8 |
> | 9
7 |
>
> We multiply $\overline{8}$ by $\overline{2}$ from fraction above, we get $\overline{16}$, we subtract 1 from 8, we get $\overline{7}$, we multiply the first registered 1 by 7, it becomes 7, the result is $\frac{7}{16}$, like this:
>
> | |
> |---|
> | 7
16 |
> | 9
7 |
>
> Then we multiply $\overline{16}$ by $\overline{7}$, we get $\overline{112}$. We add $\overline{9}$ to $\overline{7}$, we get $\overline{16}$, then we multiply the $\overline{7}$ registered above all by $\overline{16}$, we get $\overline{112}$, the result is $\frac{112}{112}$, and that is one.

In modern notation:

$$\frac{1}{2} - \left(\frac{1}{2} \times \frac{1}{8}\right) + \frac{9}{7}\left(\frac{7}{16}\right) = \frac{7}{16} + \frac{9}{7} \times \frac{7}{16} = \frac{112}{112} = 1.$$

Case three

The third case pertains to the summation of arithmetic series. *Līlāvatī* 117 (Colebrooke 115) introduces two technical terms *saṃkalita* and *saṃkalitaikya*. The former denotes the sum of natural numbers; *saṃkalitaikya* is the sum of the *saṃkalitas*. This rule lays down that the sum of the natural numbers from 1 to n is $\frac{(n+1)n}{2} = N$ and that the sum of such sums from 1 to n is $\frac{(n+2)N}{3}$.

This rule is followed by the example 118 (Colebrooke 116) which asks for the sums of the natural numbers from one to nine, i.e., 1, 1+2, 1+2+3,...separately at each stage (*pṛthak*), and the sums of the series generated above, i.e., 1, {1 + (1+2)}, {1+(1+2) + (1+2+3)}, ..., also separately at each stage. Here "separately at each stage" means that we are required to tell the sums and the "sums of the sums" at 2, at 3, at 4 and so on up to 9. The numerical statement given below the example (*nyāsa*) provides the answers for the two questions.

> *ekādīnāṃ navāntānāṃ pṛthak saṃkalitāni me |*
> *teṣāṃ saṃkalitaikyāni pracakṣva gaṇaka drutam* ||118||
>
> | | | | | | | | | | |
> |---|---|---|---|---|---|---|---|---|---|
> | *nyāsaḥ* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
> | *saṃkalitāni* | 1 | 3 | 6 | 10 | 15 | 21 | 28 | 36 | 45 |
> | *eṣām aikyāni*: | 1 | 4 | 10 | 20 | 35 | 56 | 84 | 120 | 165 |
>
> Oh mathematician, tell me quickly the sum of natural numbers (*saṃkalita*) from one up to nine and also the sums of the [above mentioned] sums (*saṃkalitaikya*), separately [at each stage].

Persian version

This example is rendered as follows in the Persian Version, where an illustrative example is added at the end.

> For example, we begin with 1 then put them down one by one thus 1, 2, 3, 4 ...
>
> If we want to know sum (*jam*^{c}) of [of all numbers up to] 4, we add 1 to it, the result is 5, we multiply 5 by half of 4, which is 2, and the result is 10. So the sum of [all numbers up to] 4 is 10. Likewise for 6, we add 1 to it, the result is 7, we multiply 7 by 3, which is half of 6, the result is 21; and the sum up to 9 is 45.
>
> If we want to know the summed sum (*jam*^{c} *i majmū*^{c}) of numbers from 1 to 3, we add 2 to 3, the result is 5, we multiply it by 6 which is the sum of 3, the result is 30, we divide 30 by 3, the quotient is 10. So the summed sum from 1 to 3 is 10. It is 20 for the summed sum of 4, because we add 2 to 4, the result is 6, we multiply it by 10 which is the sum of 4, and result is 60, then we divide it by 3, the result is 20.
>
> If we want to know the summed sum of all numbers from 1 to 9, we add 2 to 9, the sum is 11, we

multiply 11 by 45, which is the sum of 9, the result is 495, we divided it by 3, and the quotient is 146.

[This can be illustrated by the following example]. If a man gives somebody for 9 days in the following manner. He gifts him one [thing] on the first day, two [things] on the second day, three [things] on the third day, thus up to ninth day, the sum of gifts is 6 on the third day, 10 on the fourth day, 15 on the fifth day, and 45 on the ninth day.

If he gives on the first day 1 and on the second day 3, that is the sum of given things in two days, on the third day 6, that is the sum of all things given in three days, on the fourth day 10, I mean the sum of them in four days, and so on, the sum of gifts is 165 on the ninth day.

3.2.4 Other additions

Bodleian library manuscript

The Bodleian Library manuscript carries the following additional example in the section on the addition of fractions immediately after Colebrooke 30; it does not occur in the printed edition, nor in the Sanskrit original

The sum of these fractions is

$$\frac{2}{5}+\frac{1}{4}+\frac{1}{2}+\frac{1}{3}=\frac{24}{120}+\frac{30}{120}+\frac{60}{120}+\frac{40}{120}=\frac{154}{120}$$

Persian version

Immediately after Colebrooke 237, the Persian version carries two strange rules which are not found in the original text. These may have been taken from some manuscript of the *Līlāvatī*, but not rendered carefully into Persian. The first of these rules is for finding the time of the day from the shadow length of a gnomon (*mīl*) of 3 digits. The rule reads as follows:

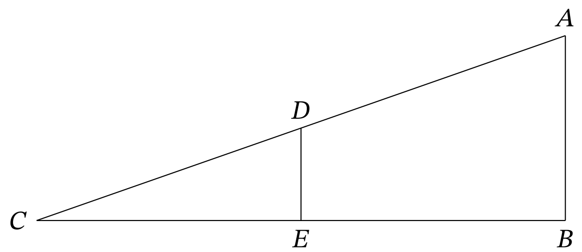
Finding the hour of the day:

There are some methods, one of them is this. Take a rod (*mīl*) of 3 (*sih*) digits height and insert it on a flat ground without any inclination; then we measure the rod's shadow by digits, we take it and add 3 to it, which is the rod's length. Then we multiply it by a number as the product gets more than 60, 60 is the hour's factor. If the product be less than 60, then we multiply the difference between it and 60 by 60 (60–x). Then we divide the result by (60–x), the quotient is *pal*, and the number less than 60 is equal to (*taḥghīgh*) the hour (*sā'āt*).

There is apparently some lacuna in this statement, as well as certain inconsistencies. It is surprising that the height of the gnomon is stated to be 3 digits; traditionally it is either 7 feet (Arabic *qadam*, plural *aqdām*) or 12 digits or fingers (Arabic *aṣba*^{c}, plural *aṣābi*^{c}). Secondly, *pal* or *pala* is the one-sixtieth part of the *ghaṭī*, the standard unit of time in traditional India. It is not clear why it is mentioned along with the "hour".

The second rule is for determining the height of a tall building or tree with the help of a gnomon.

Determination of the height:
[a] If we want to know the height of a high object, a tree or a mountain, we measure its shadow in feet and keep it in mind. Then we measure our shadow in the same way [in feet]. We divide the object's shadow length by our shadow length, we multiply the quotient by 7. If there is no remainder, the result is the height of the object.
[b] If there is any remainder, we multiply it by 60, divide it by our shadow length, multiply the quotient by 7, then divide it by 60, we add its quotient to the first quotient. It is the height of the object.
[c] If there is any remainder which is more than [height of] the object, we subtract a half from the sum. If it is equal to or less than it, we do not make the subtraction.



[a] In the figure above, let *AB* be the high object and *BC* its shadow. *DE* represents the height of a man; it is 7 feet, and *EC* is his shadow. The value 7 is significant because the height of a man is supposed to be 7 times the length of his own feet. *ABC* and *DEC* are similar right triangles. Therefore,

$$AB : DE :: BC : EC$$
$$AB = \frac{BC}{EC} \times DE = 7\frac{BC}{EC}$$

Supposing BC is 60 and EC is 20, the height of AB will be $\frac{60}{20} \times 7 = 21$ feet.

Or, let BC be 50 and EC 20, then the height of AB will be $\frac{50}{20} \times 7 = \frac{5}{2} \times 7 = \frac{35}{2} = 17\frac{1}{2}$ feet. Thus, after dividing BC with EC, if there is a remainder, this too can be divided by EC and the value of AB would be a compound fraction. It is not clear what purpose the stipulations [b] and [c] serve.

4 Style of the Persian version

4.1 Examples

Bhāskara infuses elegance in his examples by couching them in different poetic metres, by alliteration, by different figures of speech and, above all, by choosing charming motifs for illustration. Poetic metres and alliteration cannot be reproduced in another language, but figures of speech can be rendered in other languages to some extent. This too was not done in the Persian rendering. Bhāskara frequently includes addresses, some to feminine pupils and some to masculine pupils, sometimes with teasing riders. The very first example, which pertains to addition and subtraction, reads as follows in Sanskrit:

> *aye bale līlāvati matimati brūhi sahitān*
> *dvi-pañca-dvātriṃśat trinavati-śatāṣṭādaśa daśa* |
> *śatopetān etān ayutaviyutāṃś cāpi vada me*
> *yadi vyakte yukti-vyavakalana-marge 'si kuśalā* ||[23]

> Dear intelligent *Līlāvatī*, if thou be skilled in addition and subtraction, tell me the sum of two, five, thirty-two, a hundred and ninety-three, eighteen, ten and a hundred added together; and the remainder, when their sum is subtracted from ten thousand. (Colebrooke's translation, 13).

In the Persian version, the address and the rider are omitted, the problem is posed in bare essentials as follows:

> For example, if we want to add 2, 5, 32, 193, 18, 10, and 100 and then we want to subtract the sum mentioned from 10,000.[24]

4.2 Technical terminology

In his Preface, Faiẓī states the following about the technical terms: "In the case of some Hindī (i.e., Sanskrit) terms whose equivalents were not found in other books dealing with this science, these were retained in their Hindī garb, and so explained that the language be not found difficult for a reader of Persian." That is to say, in the case of technical terms, where there are no exact equivalents in Persian, Sanskrit terms are retained in the Persian translation, but always with an explanation so that the connotation of the term is intelligible to the readers of the translation. But this maxim has not always been maintained; often there are Sanskrit technical terms without any explanation or with inadequate explanation. Even the names of the units of measurement occurring in the examples have not been replaced consistently by contemporary terms, as will be shown below in 4.2.2.

4.2.1 Rule of Three

This may be illustrated with the following example. In Sanskrit arithmetic, *Trairāśika* holds an important place (Sarma, 2002). In the middle ages, it was hailed as the "Golden Rule". Al-Bīrūnī even wrote a separate monograph on it entitled *Rāshikāt al-Hind* (Kusuba, 2014, pp. 469–485; Yazdi, 2010). It is usually known as the "Rule of Three" in English or "Dreisatz" in German. It deals with problems like the following: if 3 apples cost 7 Euros, what is the price of 5 apples?

Sanskrit texts usually teach a mechanical method of solution: writing down the three given terms in a linear sequence (A → B → C) and then, proceeding in the reverse direction, multiply the last term with the middle term and divide their product by the first term ($C \times \frac{B}{A} = D$). Accordingly the solution of the apple problem is to write down the three terms 3 7 5 and then $5 \times \frac{7}{3} = 11.666$.

[23] *Līlāvatī* 13.

[24] John Taylor's English rendering is also limited to the bare essentials; this is how he translates the same verse (p. 6): "Tell me the sum of two, five, thirty-two, one hundred and ninety-three, eighteen, ten and one hundred ? Also what is the result if this sum be subtracted from ten thousand ?

These four terms are technically known as A = *pramāṇa*, B = *phala*, C = *icchā*, D = *icchā-phala*. With this background, we shall cite the rule from the Sanskrit *Līlāvatī*:

> *Pramāṇa* and *icchā* are of the same denomination; they should be placed at the beginning and at the end [of the row]. *Phala* is of a different denomination; it must be placed in the middle [of the two terms]. That [*phala*] multiplied by *icchā* and divided by the first term (*pramāṇa*) will be the *icchā-phala*. In the inverse case, the procedure is the reverse.[25]

The Persian translation is as follows:

> The rule for three quantities: For instance, an assessed quantity is called *phala*. Second, the price is called *pramāna*. Third, the amount of money for purchasing is called *ichhā*. There are also rules for four or five quantities.
>
> I will explain the first one. That method is: *pramāna* and *ichhā* must be of the same kind in two places, such as *Dramma*. *Phala* stands between them in its denomination. Then, *phala* is multiplied by the *icchā*, the result is divided by *pramāna*; the quotient is the of assessed quantity for money.

Here the same Sanskrit technical terms are retained in the Persian translation, but their explanation is totally off the mark. The assessed quantity (3 apples) is not *phala*, but *pramāṇa*. The price (7 Euros) paid for the assessed quantity is not *pramāna*, but *phala*. Third, the *icchā* is not money for purchasing, but the amount or the number of objects one desires to buy (5 apples).

4.2.2 Inverse Rule of Three

We saw that the Rule of Three is employed when *A* and *B* (i.e., 3 apples and 7 Euros) are directly proportionate. But there are cases where *A* and *B* could be inversely proportionate. At the end of the rule above, it has been stated that, in such cases, the reverse procedure (*vyasta-vidhi*) should be adopted. This is elaborated in two verses in the *Līlāvatī*:

> Now the rule for the Inverse Rule of Three:
>
> When there is increase in *icchā* and diminution in *phala*, [or] diminution [in *icchā*] and increase [in *phala*], then the experts in computation should know that it is [a case of] Inverse Rule of Three.
>
> When the price of living beings [is determined according to] their age, when [the price of] gold [is determined according] to the weight (*taulya*) and touch (*varṇa*), or when heaps are subdivided, let the inverse rule of three terms be [employed].
>
> Example of living beings and their ages:
>
> If a sixteen years old female obtains thirty-two [*niṣkas* as her price], what [is the price of] twenty years old [female]? If a draught-ox carrying loads in the second year (*dvidhūrvaha*) fetches four *niṣkas* [as its price], then what [is the price of] a draught-ox which carried loads for six years (*dhūḥṣaṭkavaha*)[26]

The Persian translation abridges the statement considerably:

> The inverse method (*ṭarīgh*): if finding the age is necessary in the case of animals, or if we want to know the carat (‘*ayār*) in the case of gold, we will invert the rule. We take *ichhā* for *pramāna*, and *pramāna* for *ichhā*. So, first we multiply *phala* by *pramāna* and then we divide it by *ichhā*, then the quotient is *phala ichhā*.

[25] *Līlāvatī* 73 (Colebrooke 70):

pramāṇam icchā ca samānajātī
ādyantayos tat-phalam anyajāti |
madhye tad icchāhatam ādyahṛt syād
icchāphalaṃ vyastavidhir vilome ||

[26] Līlāvatī 77–79 (Colebrooke 74–76):

atha vyastatrairāśike karaṇasūtraṃ

icchāvṛddhau phale hrāso hrāse vṛddhiś ca jāyate |
vyastaṃ trairāśikaṃ tatra jñeyaṃ gaṇitakovidaiḥ | 77 ||

yatrecchāvṛddhau phale hrāso hrāse vā phalavṛddis tatra vyasta-trairāśikam| *tad yathā*

jīvānāṃ vayaso maulye taulye varṇasya haimane |
bhāgahāre ca rāśīnāṃ vyastaṃ trairāśikaṃ bhavet || 78 ||

jīva-vayo-mūlya udāharaṇam |

prāpnoti cet ṣoḍaśavatsarā strī
dvātriṃśataṃ viṃśativatsarā kim |
dvidhūrvaho niṣkacatuṣkam ukṣā
prāpnoti dhūḥṣaṭkavahas tadā kim || 79 ||

> If the price of sixteen years old female is 32 *ashrafī*, then what is the price of a twenty years old female? If the price of a draught-ox carrying loads in the second year is four *nishkas*, then what is the price of a draught-ox which carried loads for six years?

The original rule states that first *icchā* and *phala* are multiplied and then their product is divided by *pramāṇa*. Inverting this rule means first multiplying *pramāṇa* and *phala* and then dividing their product by *icchā*. Or to use, our earlier notation

$$\text{Rule of Three}: \frac{C \times B}{A} = D$$
$$\text{Inverse Rule of Three}: \frac{A \times B}{C} = D$$

But the Persian version, instead of inverting the rule, inverts the terminology itself ("We take *ichhā* for *pramāna*, and *pramāna* for *ichhā*").

It may be noted that in the first part of the example, the price of the sixteen years old female is given as 32 *niṣkas* in the Sanskrit original; in the Persian rendering, the name of the coin is changed appropriately to *ashrafī*, which is the name of a gold coin in Mughal India. But in the second part such a change is not made and the Sanskrit name is retained in the Persian version as well.

5 Conclusion

Thus the Persian rendering of the *Līlāvatī* is rather uneven. Faiẓī apparently composed just the flamboyant preface and the short conclusion, but left the rest to the others. The translation of the actual text was done by Sanskrit scholars explaining the substance in the local vernacular and the Muslim counterparts putting it down in Persian. However, the same set of Sanskrit and Persian scholars may not have worked on the entire text; it is more likely that different sets of experts may have tried their hands at different periods, which explains the unevenness in the translation. Our analysis confirms John Taylor's remarks cited earlier on the omissions in Persian rendering. But, more important, our analysis draws attention to the additions made to the text in the way of new examples, new rules and elaboration of the solutions of certain problems.

We conclude the discussion with a geometrical problem concerning right-angled triangles, namely *Līlāvatī* 152.

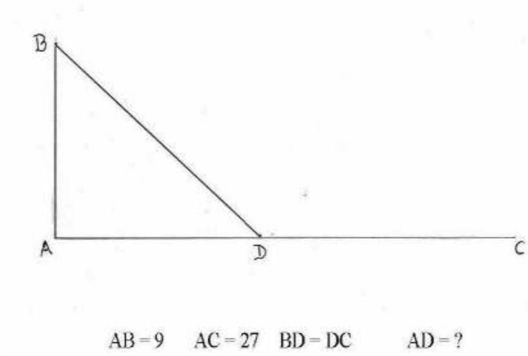


Figure 3

In Figure 3, *ABD* is a right triangle. *AB* measures 9 units, *AC* is said to be thrice as much, i.e., 27 units. If *BD* equals *DC*, what is the length of *AD*? This geometrical problem is presented in the following manner in the Sanskrit original:

> A pet peacock is perched on the top of a pillar which is 9 cubits high. At the foot of the pillar is the hole [of a snake]. The peacock noticed a snake at a distance thrice the height of the pillar. The snake was gliding towards its hole at the bottom of the pillar. The peacock swooped down on the snake, [rushing] in an oblique path. Tell quickly at how many cubits from the hole will they meet, both proceeding at the same speed.[27]

This is how it is rendered in Persian:

> There is a nine-*gaz* pillar with a snake's hole at the foot of it, a peacock is seated at its top. The peacock saw a snake at a distance of 27 *gaz*, coming

[27] *Līlāvatī* 152:

asti stambhatale bilaṃ tadupari krīḍāśikhaṇḍī sthitaḥ
stambhe hasta-navocchrite triguṇite stambhapramāṇāntare |
dṛṣṭvāhiṃ bilam āvrajantam apatat tiryak sa tasyopari
kṣipraṃ brūhi tayor bilāt kati karaiḥ sāmyena gatyor yutiḥ ||

Colebrooke's translation (150), surprisingly, omits the height of the pillar: "A snake's hole is at the foot of a pillar, and a peacock is perched on its summit. Seeing a snake, at the distance of thrice the pillar, gliding towards his hole, he pounces obliquely upon him. Say quickly at how many cubits from the snake's hole do they meet, both proceeding at the same speed."

> towards the pillar pounced from the top of the pillar to catch the snake. What is the amount of the peacock's movement in gaz?
>
> The square (*majdhūr*) of 9 is 81, when divided by 27, the quotient (*khārij i ghismat*) is 3; it is subtracted from 27; the remainder (*bāghī māndih*) is 24; its half is 12; that is *bhuja*; it is the distance from the hole to the place when the peacock caught the snake. We subtracted 12 from 27, the remainder is 15, which is *krana* (*karṇa*), from the top of the pillar to the meeting point. The length of the pillar is *kūti* (*koṭi*).
>
> It should be said that this is the translation of the method explained in the book *Līlāvatī*; however, it is an unwise assumption, because there is no rule for the speed of the snake's gliding and the peacock's pouncing.

The height of the pillar is given as 9 *hastas* or cubits in the original. The translator quite sensibly changed it to *gaz* which is a common linear measure in Mughal India. The original does not explain the solution, but the translation explains the different steps in the solution, which is apparently taken from a commentary on the *Līlāvatī*. However, the translation does not explain the Sanskrit terms used for the three sides of the right-angled triangle: *karṇa* is the hypotenuse, *bhuja* and *koṭi* are the two sides, the former is the base and the latter the vertical side.

However, it is quite amusing that the translator rebukes the author of the Sanskrit original for his unwise assumption. But the beauty of the *Līlāvatī* lies in such unwise assumptions. This example was obviously very popular; it is illustrated in some manuscripts as shown in Figure 4 and Figure 5.

Figure 4 From an unidentified manuscript of the *Līlāvatī* reproduced in an INSA brochure.

It is also represented in a dance performance "Bhaskaracharya's Leevahi Ganitham" by the students of the Rishi Valley School, during the international conference organized to commemorate the 900th birth anniversary of Bhāskārācārya in September 2014 at Vidya Prasarak Mandal, Thane.

Figure 5 Illustration from a manuscript of the *Līlāvatī*, dated 1650 (From Filliozat 2019, p. 51).

Figure 6 Dance performance of the peacock-snake problem (photo by S. R. Sarma).

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