aperture D/F, the diameter of the phase disk should be less than $2\lambda F/D$, and preferably from 60 to 90 per cent. of this value (range $A-C$). In all cases part of the light is thrown outside the image of the mirror, forming broad diffraction rings round it. Their appearance gives a sensitive criterion for the exact centring of the phase disk on the star-image.

ON THE PHASE-CONTRAST TEST OF F. ZERNIKE.

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(Communicated by Professor F. J. M. Stratton)

Summary.—The Zernike test has the great advantage over the knife-edge test, as a null test for mirror systems, that it shows the absolute error, and not the differential of the absolute error in a particular direction. This makes the process of correcting irregular errors by local figuring much easier. A method of carrying out the test, which will detect 0·1 fringe of error, is described, and the practical limitations of this method are discussed, with special reference to the testing of large reflectors.

Acknowledgment.—When I learnt, in a private communication in 1933, of the principle of the Zernike test, I immediately resolved to apply the method to testing mirrors which I am figuring. I communicated the substance of my results to Professor Zernike, and the present paper is published with his approval.

§ 1. *The Nature of the Test*.—In the form of Zernike test which I have used, an artificial star of small diameter is placed at one of the stigmatic points of a mirror system, and at the other stigmatic point there is placed the small circular phase-retarding disk, which I shall refer to as a Zernike disk, or, for short, a Z-disk. The star and Z-disk are both preferably somewhat smaller than the Airy disks of the system at the points at which they are respectively placed. The eye of the observer is placed close behind the Z-disk, and he looks either directly at the mirror under test, or at its image in any auxiliary mirrors which may come between it and him. He then sees interference fringes formed between the wave leaving the mirror and the supplementary wave to which its passage through the Z-disk gives rise. If the Z-disk is not larger than the Airy disk produced by the system, that part of the supplementary wave lying within the cone of light proceeding from the mirror will be substantially spherical, so that since the wave-front leaving the mirror system departs from sphericity only by the errors of the mirror under test, the fringe system seen (in a given colour) will be substantially the Newton-ring system which would be seen if a true-figure test-plate were laid on, or very close to, the mirror under test. The colour-sequence of the fringes will be determined by the chromatic dispersion of phase retardation of the Z-disk, and can be ascertained from the colour changes which occur when the Z-disk is moved out of focus, and when it is decentred.

§ 2. *Making Zernike Disks.*—In setting up the test, the making of suitable Z-disks was the first problem. Their diameter must be comparable with that of the Airy disk (·01 mm. for an F/8 image): their thickness must be a small number of wave-lengths in order that the Zernike fringes shall show a good colour contrast. Clearly the best thickness will be comparable in magnitude with the best thickness for a phase-retarding knife-edge, and material giving good dispersion for such a knife-edge will also give good dispersion for a Z-disk. I therefore first made phase-retarding knife-edges of collodion, Canada balsam and resin. The resin films—pressed between a microscope slide and cover-glass—gave the strongest colour contrast: the best thickness was that at which interference fringes in the air film at the edge of the resin just ceased to be visible at normal incidence in white light. I then made phase-retarding strips by pressing resin fibres, for I had noticed that exceedingly fine fibres can be drawn from a blend of resin with a few per cent. of turpentine which I had used for polishing tools. The procedure is to dip two rods into the molten resin: touch them together and jerk them apart. Fibres ·01 mm. diameter can be drawn in this way easily, and I succeeded in drawing one ·002 mm. diameter. When laid on a slide and capped with a cover-glass, the fibres could be pressed into strips at room temperature. It is advisable, before pressing the fibre, to let down a spring clip gently on to one edge of the cover-glass, and then to tack the other edge to the slide with balsam, to prevent lateral motion during the pressing, which is done under the microscope, with a metal rod, and repeated all along the fibre, until Newton fringes are seen (in white light) while the pressure is applied. The fibre does not stick to the cover-glass, so that it is not torn if the cover-glass springs up when the pressure is relaxed. I found it necessary to re-press some fibres a few days after they were made, but after that they no longer tended to return to their original cylindrical shape. Obviously the next step was to make spheres of resin and press them into disks. Spheres of all sizes from ·05 mm. down to ·001 mm. or less are precipitated when a 5 per cent. solution of resin in acetone is poured into twenty times its volume of water. (The addition of turpentine is only necessary for drawing fibres.) Those that remain in suspension for 12 hours may be rejected as too small: those that settle in one hour are too big. I have tried distributing the globules over a slide with a needle: the difficulty is that they tend to stick to each other in chains, and also to stick to the needle, which one must "twang" to release them. By the time I had parted the chains, I had smeared and cracked many of the globules. However, even the fragments became reasonably circular on pressing. I find it better to distribute them in water. The procedure is: Dilute the suspension until it looks only faintly milky; pour a few drops on to a slide: shake off the excess, and allow the remaining film to dry before putting the cover-glass on. If you put the cover-glass on while the film is wet, the surface tension of the water pulls the globules into chains as it dries. The cover-glass is then tacked with balsam and pressed, under the microscope. Precautions should be taken to keep out dust while assembling the slide, and it is advisable first of all to obtain Newton rings between slide and cover-glass—in doing so one crushes foreign matter which the cleaning process has failed to remove. In this way one produces a slide containing perhaps 100 Z-disks, ranging from ·02 mm. to ·002 mm. diameter, many of them of the right order of thickness, and reasonably circular.

§ 3. *The Focussing Carriage and "Star."*—The slide of Z-disks is mounted on a focussing carriage which is a geometrical slide running on bicycle balls, and fitted with three micrometer screws, controlling the vertical and horizontal and focal position of the slide. The carriage is clamped to a levelling table, so that the focussing track can be pointed exactly parallel to the optic axis. As "star" I use a pinhole, on which is focussed through a right-angle prism the filament of a 16-watt gas-filled lamp of the type used in railway signalling. The image is the same size as the filament, the diameter of which exceeds the diameter—·013 mm.—of the largest pinhole I have used. The filament is coiled in a fairly close helix, so that even with an uncorrected condensing lens the star gives a practically uniform cone of light up to F/8. The intensity is sufficient for testing with two reflections from silver plus two from unsilvered glass at normal incidence. The smallest pinhole I have used is ·005-mm. diameter (just small enough for testing at F/4): it gives enough light for testing with one reflection from unsilvered glass, but not with two. The ·005-mm. pinhole was made by resting the point of a mounted needle, which had been honed on an Arkansas hone, on a sheet of aluminium foil ("tinfoil") lying on a glass plate. Only a few per cent. of piercings are successful, most producing slots at least two widths long. Even the best piercing did not approximate to a circle more closely than does a square. One can also find natural holes of this order of smallness in "tinfoil."

§ 4. *The Test Procedure.*—The slide of Z-disks is set several mm. out of focus, and the observer, with his eye close behind the image, looks at the illuminated mirror. As he moves the centring screws, so as to traverse the slide across the cone of light, he will see circular patches, surrounded by rings, moving across the mirror. Each of these is the system of interference fringes created by a Z-disk: there is no difficulty in finding them, because the slide contains very many Z-disks. One is centred, and the focussing track is levelled and adjusted in azimuth so that the fringe system stays central when the Z-disk is taken through focus. It will be noticed that the Z-disks are not all identical, the fringe systems which they give differing in the colour of the central patch, which may be almost any colour, weak or strong, or neutral tint, or simply brighter than the general level of intensity of the rest of the mirror. They differ also in the number of rings which can be discerned surrounding the central patch.

In making the Zernike test for the first time, one is naturally tempted to select, out of the many Z-disks whose fringes can be seen when the slide is out of focus, that one which gives the most vividly coloured fringes. But consider what the conditions are for the production of brilliant fringes some distance out of focus. They are (*a*) that the chromatic dispersion of phase retardation of the Z-disk shall be good, (*b*) that the supplementary wave created by the passage of light through the Z-disk should be comparable in amplitude with the wave leaving the mirror; this can only be so if an appreciable part of the light leaving the mirror traverses the Z-disk—that is, if the diameter of the Z-disk is an appreciable fraction of the diameter of the cone of light at the plane of the Z-disk; this implies again, if the Z-disk is some distance out of focus, that its diameter should *not* be as small as that of the Airy disk formed at focus. Now the only circumstances under which we can be sure that the sphericity of the supplementary wave created by the Z-disk will be inappreciably affected by the errors of the mirror, and at the same time that the amplitude of the supplementary wave will be comparable with that of the wave leaving the mirror—so that the mirror errors shall show up in strong interference contrast—are that the Z-disk should be comparable in size with the Airy disk, and that it should be situated thereon, very nearly at focus. Such a Z-disk will necessarily give a weak supplementary wave when some distance out of focus, and its interference fringes will then be heavily diluted with uncoloured light. We must therefore reject those Z-disks which show brilliant colours some distance off focus, and choose one which shows very faint and ghostly fringes.

It is convenient to specify the distance of the Z-disk from focus in terms of the apparent error of the mirror in interference fringes (half wave-lengths), temporarily assuming "true figure" to be that which would focus where the Z-disk happens to be: the merit of this specification is that the fringe system given by a Z-disk depends essentially on its number of fringes off focus, whatever may be the focal ratio of the image. [To convert mm. axial distance off focus into fringes off focus, divide 250 by the square of the focal number of the image: the quotient is the number of fringes off focus per mm. off focus—thus F/16 gives one fringe per mm. This assumes $\lambda = 5 \times 10^{-5}$ cm.]

We can now apply a second check that the Z-disk is small enough, for if we put it a *small* number, *n*, of fringes inside or outside focus, it ought to show just *n* concentric fringes on the mirror (unless the mirror errors exceed one fringe). If we make *n* large, we shall not see *n* fringes for two reasons: first, because the source is not monochromatic, and the scale of the fringes depends on wave-length; and, second, because of the finite size of the source, which destroys the visibility of the higher-order fringes. One can just see the second fringe with a ·013-mm. source and an F/8 image (for which ·013 mm. is really rather too large). With an F/16 image about 4 fringes are visible.

At the same time, we can check that the Z-disk is sufficiently circular. If the fringe system is circular both inside and outside focus, then the Z-disk is circular, and the mirror has no astigmatism. An elliptical fringe system can be due to astigmatism or to an elliptical Z-disk. But if the ellipticity is due to astigmatism, the major axis of the ellipse will be perpendicular to the focal line to which the Z-disk is nearest, so that if the ellipse is elongated vertically inside focus it will be elongated horizontally outside focus: on the other hand, if the ellipticity is due to the Z-disk, the major axis of the ellipse will have the same direction as the major axis of the Z-disk, and the fringe system will be elongated in the same direction both inside and outside focus. Finally, we have yet another check that the Z-disk is sufficiently small: if the Z-disk is smaller than the Airy disk of the mirror, the supplementary wave to which it gives rise will spread through a wider cone than does the light from the mirror, so that we shall see the mirror surrounded by a halo of light, the outer diameter of which may be 50–70 per cent. greater than the diameter of the mirror. The intensity of the halo is proportional to that of the light traversing the Z-disk, so that the halo is only visible when the Z-disk is within a fringe or two of focus, and if the mirror errors exceed a fringe or two (so that "focus" is indefinite) the halo may be too faint to be seen. A Z-disk that is too large gives a very narrow halo, unless it has inclusions or cracks: these may give a wide halo, but because of their irregularity they give a secondary interference pattern in the halo, making it patchy. A large Z-disk with a small circular concentric inclusion would pass the "halo" test of suitability, but would have been rejected on the previous tests. (I have not found a Z-disk of this type.) Some Z-disks produce a coloured halo—blue, green or red. This, then, is the colour of the supplementary wave which they are creating: they are, in fact, absorbing selectively. I think this is due to interference in the air film between the Z-disk and the cover-glass, for some Z-disks look coloured under the microscope. It is presumably wise to reject a Z-disk giving a deeply coloured halo, for a Z-disk which produces absorption alone (without phase shifting) will show errors of opposite signs in the same colour, giving the same colour-sequence inside and outside focus (such a Z-disk should be rejected), and presumably one which produces absorption plus phase shifting will not give so good a colour contrast between "high" and "low" errors as one producing phase shifting alone.

We can now consider the colours developed by the Z-disk on the mirror. When the Z-disk is, say, 3 fringes out of focus, the fringes will be faint and neutral tinted. But as it is brought nearer focus, colours will develop if it is of the right thickness. It will in general be found that the central colour of the fringe system is not the same inside and outside focus: for example, we may get blue and yellow, or green and red, depending on the thickness of the Z-disk: this is the effect that a Z-disk which retards the phase of some colours by $m \pm \frac{1}{4}$ periods produces. If the phase retardation is $\frac{1}{4}$ period for all colours, the central inside and outside focus colours will be black and white respectively: such a Z-disk can be used for the test, but I prefer to work with one giving a contrast of tint rather than of amplitude. Select then a Z-disk showing a good contrast between its inside- and outside-focus colours, but, above all, do not select a Z-disk on account of its good colours unless the preceding tests have shown that it is sufficiently small. I cannot too strongly emphasise this point. If one uses a Z-disk that is too large, it will create a supplementary wave containing a part of the short-period error and nearly the whole of the long-period error of the mirror, so that while it will show up local irregularities nearly as well as a smaller Z-disk, it will fail to show up long-period errors—simple spherical aberration, and more especially the two errors of longest period, astigmatism and error of absolute focal position. A test with a Z-disk that is too large is, in fact, exactly like testing a flat with a straight-edge that sags under its own weight: the local humps show up, but slow curvature does not. Therefore, even if one is not interested in determining the absolute focal position, it is essential to verify that it can be determined with precision—that a change of, say, $\frac{1}{2}$ fringe in the focal position does change the fringe system by $\frac{1}{2}$ fringe, as a proof that the Z-disk is small enough.

The foregoing considerations, then, provide a means of ascertaining from the appearances seen on the test itself whether a given Z-disk is a good one or not, so that while it is convenient to inspect the Z-disks with a microscope during their manufacture, it is by no means essential to pick out a good one with a microscope, and to ensure that that Z-disk, and no other, is used for the test.

§ 5. *The Interpretation of the Test.*—If the errors do not exceed $\frac{1}{4}$ to $\frac{1}{2}$ fringe, the interpretation of the test presents no difficulty, for by putting the Z-disk a fringe or so out of focus, one can superpose a general spherical error larger than the residual errors without making the colours too faint to identify, and one can even superpose an error of tilt by decentring the Z-disk when it is slightly out of focus: this is equivalent to the process of lifting one edge of a contact interference test-plate, to see how the colour-sequence runs; but there is the slight difference that if the Z-disk is inside focus we must imagine a hypothetical test-plate with its centre attached to the Z-disk to be making contact to the outside of the mirror-surface, this test-plate being slightly convex with respect to the mirror; so that as the Z-disk is decentred, it rolls the hypothetical test-plate over the mirror, and the fringe system moves in the same direction as the Z-disk, expanding as it approaches a hill and contracting when it gets to the top: contracting when it approaches a hollow and expanding when it gets to the bottom. If the Z-disk is outside focus, we must imagine the hypothetical test-plate to be slightly concave with respect to the mirror, and to be making contact to the inside of the mirror-surface from the back; so that as it is rolled over the back of the mirror-surface, the fringe system moves in the opposite direction to the Z-disk, and the pattern contracts when it approaches a hill, expanding when it gets to the top: expands when it approaches a hollow and contracts when it gets to the bottom.

If one decentres the Z-disk at focus, the visibility of the fringes fades very rapidly, as the Z-disk approaches the first zero of the Airy rings, and the supplementary wave is no longer created.

We have, further, the rule that the primary colour developed centrally inside focus will be the colour in which areas up to about $\frac{1}{4}$ fringe high will show when the Z-disk is at focus, and the primary outside focus colour will be that in which areas up to about $\frac{1}{4}$ fringe low will show.

Let us take as an example a test on a spherical speculum 12·5 cm. diameter, 200 cm. radius of curvature. When the chosen Z-disk was set 3 fringes inside focus, 3 fringes showed in faint nondescript colours, the third being so faint as to be only just distinguishable. When it was set $\frac{1}{2}$ fringe inside focus it produced a halo round the mirror, extending to about 18 cm. diameter. The fringe system on the mirror consisted of a blue central patch outside which the colour shaded through neutral tint into a tawny yellow, which in turn shaded into a very pale yellow near the edge of the mirror. The fringe system was not exactly circular—traces of the residual errors could, in fact, be seen "through" the $\frac{1}{2}$-fringe focussing error. When the Z-disk was set $\frac{1}{2}$ fringe outside focus, the central patch was deep tawny yellow, surrounded by neutral tint shading into blue-white, and to very pale yellow near the edge of the mirror: traces of residual error marred the precise regularity of the system.

When the Z-disk was set at focus neither the blue nor the deep tawny yellow were fully developed on any part of the mirror: there were, however, irregular patches where the neutral tint which covered most of the mirror shaded into a very weak blue or tawny yellow.

From the appearances $\frac{1}{2}$ fringe inside and outside focus, I interpret the colour-sequence for this Z-disk when at focus as follows :—

Very pale yellow to deep tawny yellow ...	$\frac{1}{2}$ to $\frac{1}{4}$ fringe low.
Deep tawny yellow to neutral tint ...	$\frac{1}{4}$,, 0 ,, ,,
Neutral tint to blue	0 ,, $\frac{1}{4}$,, high.
Blue to very pale yellow	$\frac{1}{4}$,, $\frac{1}{2}$,, ,,

The patches, where with the Z-disk at focus the neutral tint changes to weak blue or tawny yellow, I interpret as being about $\frac{1}{8}$ fringe high or low. I have done a Michelson test on this mirror, and did not find more than about $\pm 0{\cdot}1$ fringe error. I had, of course, seen the existence of these errors on the knife-edge test, and indeed had reduced them to their present value by local figuring, testing with vertical and horizontal knife-edges in succession. But the interpretation of irregular error from a pair of knife-edge tests is by no means easy, especially when one is nearing the limit of sensitivity of the test, and I was not sufficiently certain of the interpretation to continue the local figuring. The great value of the Zernike test is not that it may be somewhat more sensitive, nor even that it is free from the disadvantage of producing logarithmically infinite illumination at the edge of the mirror (the "Rayleigh ring" of the knife-edge test), but that no matter how complicated the errors may be, the colours indicate the errors themselves, and not the slope of error in some particular direction.

To check the Zernike test on a system whose error could be easily measured by other methods, I made a mirror of the alloy Cu_4Sn, 7·5 cm. diameter, 75 cm. radius of curvature, with $\pm \frac{1}{4}$ fringe of oblate error, the centre and edge being $\frac{1}{4}$ fringe high and the intermediate zone $\frac{1}{4}$ fringe low (as determined by zonal knife-edge measurements). When the Z-disk was set at the mean focal position, it showed a patch of primary inside focus colour (blue) at the centre, and a ring of blue round the edge, with a ring of yellow (primary outside focus colour) in between. This mirror has since provided a remarkable demonstration of the development of irregular error: after a few weeks the knife-edge shadows showed mottled patches superposed on the spherical aberration: clearly the mirror had become irregular, but the precise shape of the individual irregularities was extremely difficult to deduce from a pair of knife-edge tests. The Zernike test now shows patches, about 4 mm. diameter with sharply defined edges, contrasting markedly in colour with their surroundings. When one so focusses the Z-disk that the surroundings show primary low colour, the patches in general show primary high colour: they must be about $\frac{1}{2}$ fringe high. Apparently blocks of crystals in the casting have slipped. The casting was not a good one: it has flaws which seem to penetrate right through it, and I would not have used it for any other than a "pathological" mirror. I have not seen this effect with any other metal mirror.

§ 6. *Interpretation of the Test with Large Errors.*—When the mirror has several fringes error, the visibility of the fringes with a small Z-disk may be reduced, owing to the spread of the image, to substantially nil, and it may be necessary, in order to see fringes at all, to use a Z-disk so large that that part of the supplementary wave which it creates from the contribution to the incident wave arising from a given small portion of the mirror is not spread over the solid angle fed by the whole mirror, but has appreciable amplitude only in the neighbourhood of that part of the incident wave that produced it. The fringes seen on any part of the mirror then approximate to those which would be seen on that part if the remoter parts of the mirror did not exist, and an area shows the primary inside or outside focus colours (which, were the Z-disk small, we would interpret as meaning up to $\frac{1}{4}$ fringe high or low respectively), not because it is high or low with respect to the true figure, but because the Z-disk is inside or outside the focus of the area in question. The primary colours appear, in fact, at those parts of the mirror which make internal or external contact with any parallel to the true figure. Thus, so far as removal of the errors by figuring is concerned, we may safely polish any zone which shows primary inside focus colour when the Z-disk is set in the desired focal position, but it will not necessarily be possible to attain the true figure without polishing some zone that shows primary outside focus colours at the beginning of the figuring process, for some zone, although above the true figure, may be surrounded by a zone which is higher still.

Secondary colours—orders of interference higher than the first, which are usually of nondescript tint—will surround the primary colours, and will move with them when the Z-disk is decentred, and it is possible for the secondary fringes associated with the primary inside focus colour (interferences of the +1st, +2nd . . . +*n*th order) to move through the fringes associated with the primary outside focus colour (interferences of the −1st, −2nd . . . −*n*th order) when the Z-disk is decentred: we may expect visibility to be lost rapidly in the process.

As an example of this let us consider a Zernike test which I made on a slightly astigmatic prolate spheroid 11·2 cm. diameter, 231 cm. radius of curvature, before I realised the importance of choosing the smallest Z-disks with which the Zernike fringes are visible.

Starting well inside focus (for all zones of the mirror), I chose a Z-disk giving a fringe system consisting of a blue centre, bordered by neutral tint and then pink, followed by nondescript-coloured fringes of second and

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higher orders. The blue patch increased in size as the Z-disk was centred, showing that the centre had a shorter focal length—in fact, that the mirror was prolate. On approaching mean focus the blue patch expanded, until when it was 3–4 cm. diameter it began to develop a pink centre: clearly the Z-disk was now just outside the focus of the central zone. There were then three bright and two dark fringes surrounding the outer pink border of the blue patch: the whole pattern was quite circular. Thus it seemed that the aspherical error of this mirror must be simply $2\frac{1}{2}$ fringes (of some visible wave-length) of prolate departure from the sphere of closest fit to the centre. On moving the Z-disk farther out, the pink centre expanded to 4 cm. diameter and a dark border developed between it and the blue: it also acquired a yellowish tint in the middle. Up to this point I argued that I was seeing interference fringes between a spherical supplementary wave and the prolate wave from the mirror, and on this basis I expected that as the Z-disk was moved farther out, so that the centre became increasingly "low," high-order "low" fringes would begin to develop from the middle of its now yellowish-pink centre. But this was not what happened: the dark border separating the pink (low) centre from the blue (high) ring surrounding it split into two concentric dark borders, the outer one belonging to the blue ring and expanding with it, and the inner one belonging to the pink, which now began to *contract*; and in the annular space between these dark borders higher-order fringes arose in pairs, expanding towards the expanding blue ring and contracting towards the contracting pink centre, as the Z-disk was moved farther out. Finally, when the blue ring had expanded to the edge of the mirror, the pink centre had contracted to 1 cm. diameter, and between it and the blue there were five concentric nondescript-coloured rings which only appeared really well defined when the Z-disk was very exactly centred, and lost visibility rapidly on decentring. On careful inspection one could see that the effect of decentring was to split these rings into two sets, one belonging to the blue, and moving with it and with the Z-disk, and the other belonging to the pink, moving with it and against the Z-disk. Both sets of rings at the same time split up into pairs of opposing crescents. Clearly one could no longer argue that the supplementary wave was spherical, and the final fringe system would hardly bear a more precise interpretation than that the focal length of the centre was shorter than that of the outer zone, and that the prolateness amounted to several fringes. I then knife-edge tested the mirror zonally: its prolateness with respect to the sphere of closest fit to the centre was 8×10^{-5} cm.: this agrees with the $2\frac{1}{2}$ fringes seen on the Zernike test if we assume these nondescript-coloured fringes to refer to $\lambda = 6{\cdot}4 \times 10^{-5}$ cm. – red light. The Zernike test, then, in spite of the use of too large a Z-disk, had shown roughly the right amount of prolate error. But a test with two knife-edges showed nearly one fringe of astigmatism, and this I had not seen with the Z-disk. Clearly that could only have been because the Z-disk was too large or not circular. I therefore repeated the Zernike test with many different Z-disks, with the following result: Every Z-disk that showed really brilliant colours when some distance inside focus gave a substantially circular fringe system at mean focus, thus failing to detect the astigmatism. Every Z-disk which showed faint colours till it was within a few fringes of mean focus and correctly centred, gave a fringe system showing the right amount of astigmatism: when such a Z-disk was set just outside focus for the central zone of the mirror, so that the centre showed primary low colour, the high ring surrounding the centre had not circular symmetry, but was composed of two crescents, the tips of which just touched one another on the low astigmatic axis; outside this high ring were nondescript-coloured (low) fringes, elliptical in shape, to the extent of just one fringe at the edge of the mirror.

I obtained a fringe system complementary to the above, showing a ring of primary low colour, surrounded internally and externally by several orders of high fringes, by testing a spherical mirror on a test null for a prolate ellipsoid. The star was placed slightly to the right of the far focus, and a pair of small perpendicular flats were placed so that their line of intersection passed through the near focus, thus returning the light to the mirror so as to form the final image slightly to the left of the far focus, doubling all the mirror errors save those antisymmetrical with respect to a plane through the focus perpendicular to both flats. (One can double *all* the errors by using not two perpendicular flats but three—the "optical cube"—with their corner at the near focus. Mr. B. K. Johnson has pointed out to me that it is essential to decentre the star in such a way that all the light traverses the three flats in the same order. This is possible, and when it is done no lines due to the junctions of the three flats will be seen on the mirror. Extreme accuracy in their perpendicularity is then not essential.) I noted in this experiment a fact apparently in conflict with current opinion: I have seen it stated that although prolate error ("turned-down edge") in a converging wave-front produces an image looking fuzzy inside and sharp with internal diffraction rings outside focus, the converse does not hold for a wave-front with oblate error ("turned-up edge"). Now my spherical mirror was producing oblate error in the final wave-front, and it gave an image looking fuzzy outside and sharp with internal diffraction rings inside focus.

§ 7. *Astigmatism.*—It was of the first importance to verify that, provided one resisted the temptation to use a large Z-disk on account of its attractive appearance when out of focus, astigmatism could and would always be detected. I therefore set up the 12·5 cm. speculum previously discussed, so that the image was displaced horizontally by 4·5 cm. from the star: this introduces $\frac{1}{2}$ fringe of astigmatism (in $\lambda = 5 \times 10^{-5}$ cm.). As focus was approached from inside, the Zernike fringes became increasingly elliptical, until at the vertical focal line the fringe system consisted of a horizontal blue band (high) bordered above and below by neutral tint shading to deep tawny yellow, which became rather lighter at the top and bottom of the mirror. Moving the Z-disk $\frac{1}{2}$ fringe farther out in focus brought it to the horizontal focal line, when the fringe system became a vertical band of deep tawny yellow, bordered on either side by neutral tint shading to blue, which became rather whiter towards the sides of the mirror. All Z-disks which satisfied the criterion of smallness—colours very faint when 3 fringes out of focus—showed the astigmatism in correspondingly contrasting colours.

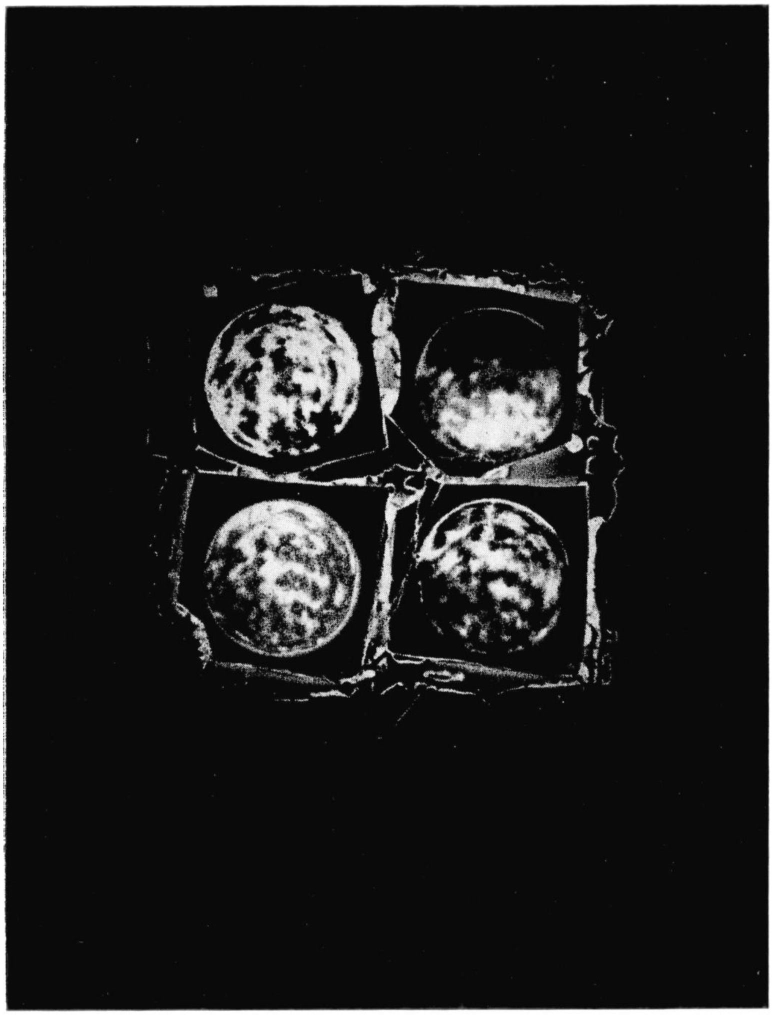
I also had occasion to test a glass disk of 11 cm. diameter, 93 cm. radius of curvature, which showed a little more than $\frac{1}{2}$ fringe of astigmatism plus about $\frac{1}{4}$ fringe of oblate error: on displacing the image 1·8 cm. in the appropriate direction from the star—which cancels 0·6 fringe of astigmatism —I obtained a circular fringe system showing the oblate error alone. I am satisfied that the only circumstances under which astigmatism is liable to escape detection are when it is masked by a still larger spherical or zonal aberration, and, even then, only if an unnecessarily large Z-disk is used.

§ 8. *Irregular Errors.*—I have been trying to figure aspherical surfaces by strictly local working—the optical analogy of the engineer's process of scraping a flat—the point being that this is presumably the only method possible when the desired figure is very deeply aspherical (say by several mm.), and is not a figure of revolution. This process necessarily produces a highly irregular surface, "true to figure" in the same sense that a cobbled pavement can be said to be "flat," and the whole interest attaches to just what fraction of a wave-length it is practicable to reduce the "cobbles." If the testing is done with vertical and horizontal knife-edges used in succession, the interpretation of the test is very difficult indeed. Zonal measurement with both knife-edges and graphical integration along two co-ordinates is out of the question owing to cumulative errors, and the only procedure possible seems to be to look at both knife-edge shadows in succession, and consider them as views of the errors in bas-relief, lit from perpendicular directions, which, as is known, is not strictly permissible. Perhaps that is why one is liable to make mistakes in the process, which is extremely tedious and exhausting in practice.

On the other hand, with the Zernike test indicating the absolute errors, it seemed that highly localised figuring would form a practicable process, and to form some idea of its possibilities I figured a decentred ellipsoid in speculum metal, 12·5 cm. diameter, having the near focus 102 cm. and the far focus 395 cm. from its centre, the near focus being distant 12·5 cm. from the line joining the far focus to the centre of the disk, so that the axis of revolution passes clear of the disk by about 6 cm. The desired figure departs by some 20 fringes from the nearest sphere, half the asphericity being astigmatic and half comatic. I put in most of the astigmatism—at the cost of introducing a large irregular error at the outset—by fine grinding. When the mirror had been polished (with a 2·5 cm. polisher) I tested it with the star at the far focus and the Z-disk at the near focus. The errors were so large (of the order of 10 fringes) that it was not possible to see the Zernike fringes over more than a small portion of the mirror at a time. I therefore had to test with a rather large Z-disk, placed either inside or outside focus for all zones of the mirror, and move the Z-disk laterally, so as to roll the "hypothetical test-plate" over all parts of the mirror in turn (p. 389), noting where the fringe systems became smallest (with the Z-disk inside focus)— these places being the tops of the hills. Polishers were used of various sizes, slightly smaller than the areas on which it was desired to work. At the end of three days the Zernike fringes spread over the whole mirror, only two or three orders of interference being present. It took a further six days, using sometimes a pitch polisher 1 cm. diameter and sometimes the finger-tip, to reduce the errors until only the two primary colours showed (except at the extreme edges)—indicating errors of the order of $\pm \frac{1}{4}$ fringe. The procedure was to set the Z-disk at the mean focus, and paint dots of rouge over all the areas showing primary high colour. (The dots of rouge, reflecting practically nothing, showed up in the same colour as the halo—practically the same colour as the star.) Figuring was then carried out over the rouged areas for periods of the order of 5 minutes, and the mirror was retested. I do not think the process would have taken so long had I realised at the outset the importance of choosing a Z-disk which only develops its colours strongly when very near mean focus. Not realising this, I failed to keep the mirror free from astigmatism, and in the final removal of the astigmatism I produced a fresh set of irregular errors.

I stopped figuring when, except at the extreme edge, only the primary colours showed, and tried to photograph the Zernike fringes. This proved extremely difficult, for 20 minutes' exposure was necessary, and the apparatus did not remain in adjustment for so long, which is hardly surprising, seeing that a movement of the Z-disk of ·01 mm. made the colours practically vanish. However, I succeeded in taking a photograph which gives a rough idea of the distribution of the errors: this is shown in fig. 1 (*a*), together with photographs with a vertical knife-edge coming in from the right at mean focus (*b*), and with a horizontal knife-edge coming upwards at mean focus (*c*), and also $\frac{1}{4}$ mm. outside focus (*d*). The primary inside focus colour (blue) has photographed as light, and the outside focus colour (yellow-brown) as dark. Thus the light areas in the Zernike photograph 1 (*a*) are high, and the dark low by about $\frac{1}{4}$ fringe. Inspection of the two knife-edge photographs, (*b*) and (*c*), will give some idea of the difficulty of deciding where to figure such a surface from knife-edge tests alone. But it should be remembered that in practice one cannot photograph the two knife-edge tests before each figuring operation, so that normally one cannot have the two knife-edge tests side by side, and glance rapidly from one to the other.

§ 9. "*Accidental*" *Z-disks*.—A clean microscope slide, exposed to the atmosphere, will soon accumulate on its surface particles of foreign matter which will work as Z-disks. Most of them are opaque, and so give a fringe system in the same colours inside and outside focus, but an appreciable number are transparent, and work as phase-retarding disks. In fact, it is possible to do quite a satisfactory Zernike test with an "accidental" Z-disk. One can also make Z-disks some of which work quite well by simply touching a clean microscope slide with the finger-tip, though only a small proportion of the natural-grease particles thus imprinted on the slide are sufficiently round. It is best to touch the slide very lightly for this purpose. I have also done Zernike tests with a drop of milk and water between a cover-glass and slide: the mixture should be so dilute as to be only faintly milky. Milk contains fat globules of all sizes from ·02 mm. to less than ·001 mm. diameter: those that one succeeds in finding with an F/8 image produce excellently coloured fringes. There are also Z-disks present in one's eye: most people, when making knife-edge tests, will have seen small circular patches, b d



a c

Fig. 1.

(*a*) Zernike photograph : light areas high ; dark areas low.
(*b*) Vertical knife-edge, coming in from the right.
(*c*) Horizontal knife-edge, coming upwards at mean focus.
(*d*) ,, ,, ,, ,, $\frac{1}{4}$ mm. outside mean focus. Image cone, F/8.

C. R. Burch, The Phase-contrast Test of F. Zernike.

sometimes brilliantly coloured, surrounded by rings, appearing on the mirror and moving when the eye is moved. Observers have recognised that it is possible to use the pupil of the eye as a knife-edge, and so to see the errors of a mirror by holding one's eye in the right place with respect to the image. It occurred to me that it should be possible to make a Zernike test, using one of the natural Z-disks present in the eye, and I tried this on the prolate spheroidal mirror previously discussed, using an obviously rather large Z-disk which for an hour at least was a persistent feature of my right eye. As might be expected, I found it extremely difficult to hold my head—supported on both hands—steady enough, but I was able to see the green "high" patch created by this Z-disk when inside focus expand into a crescent with its concave side towards the centre ; it moved rapidly round the centre as I endeavoured to centre it, which I could not do for long enough to catch more than a fleeting glimpse of a green "high" ring. However, from the crescent shape of the patch when the Z-disk was not central, it was obvious that the mirror was prolate.

§ 10. *Zernike Test using an Actual Star.*—By courtesy of Professor F. J. M. Stratton I was able to witness an attempt made by Dr. R. O. Redman to do a Zernike test on the 36-inch Newtonian telescope (by Dr. Common) of the Cambridge Solar Physics Observatory, using an actual star as source. The Zernike fringes were in continual rapid motion, owing to atmospheric irregularity, so that it was only possible to see them when the Z-disk was some distance out of focus, and we must undoubtedly have been using Z-disks far larger than should be used for testing this F/3·7 mirror at focus. The out-of-focus fringe systems were elliptical, with considerable eccentricity, the major axis of each fringe system changing direction by 90° when the Z-disk was so decentred as to carry the fringe system from one side of the mirror to the other: thus the mirror is astigmatic, and the astigmatism is not regular. This was known to be the case.

Very perfect seeing conditions would be necessary to detect errors of less than one fringe in this way.

§ 11. *The Limitations of Testing with a Z-disk.*—The Zernike test becomes more difficult to carry out when the focal ratio of the image is large (*i.e.* F number small) for two reasons: first, because the diameter of the Z-disk must decrease *pari passu* with that of the Airy disk, and it is difficult to make very small Z-disks of the thickness which shows nicely coloured fringes; and, second, because a corresponding decrease is necessary in the diameter of the geometrical image of the star, so that the intensity of the light leaving the mirror-surface is decreased. If the star is not sufficiently small, the delicacy of the test is reduced in one of two ways : if the star is so fed with light as to be in effect self-luminous (*i.e.* fed by a fully corrected very wide angle condensing system, focussed on the pinhole), then the fringes are blurred, because the fringe systems formed by the independently lit points of the star-surface do not coincide. If the star is fed with wave-fronts whose radii of curvature when they reach the pinhole are large compared with the diameter of the pinhole, then any individual wave-train will not spread with equal amplitude over the whole mirror, but will have appreciable amplitude only over a restricted part of it, and will thus form an Airy disk of the same order of diameter as the pinhole. To obtain a sufficiently strong supplementary wave it will be necessary to use a correspondingly large Z-disk, and so the fringe-system formed on any part of the mirror will depend only on the figure of the mirror near to the part in question, so that long-period errors will not be shown up to their full extent: astigmatism will not be apparent, and absolute focal position will not be determinable with precision. The same difficulty arises with the knife-edge test: if the delicacy of the test (expressed in fringes) is to be maintained as the focal ratio is increased, the geometrical image of the star must be correspondingly reduced. With either test, if we use a star giving a geometrical image a given fraction of the width of the Airy disk, and look not directly at the mirror but at the image thereof in some optical system, the brilliance of the mirror's image depends only on its angular diameter and on the intrinsic brilliance of the star, and not on the focal ratio of the mirror itself. In fact, we can increase the apparent surface brightness of the mirror to any degree not exceeding the intrinsic brilliance of the star, if we are prepared to look at it through a reducing telescope minimifying it sufficiently; and if the telescope objective is perfectly corrected, not as a telescope, but as a microscope objective, we may conveniently place the Z-disk not at the Airy disk formed by the mirror, but at the magnified image of the Airy disk which the telescope can be arranged to form, so that we can work with a larger and more easily made Z-disk. One would, however, always prefer—other things being equal—to avoid the reduction in angular diameter of the mirror's image, and also the necessity for a perfectly corrected microscope objective in the telescope.

I have tried a Zernike test without a reducing telescope on an unsilvered glass disk 11·25 cm. diameter, 31·5 cm. radius of curvature (imaging at F/2·8), using a star ·005 mm. diameter. I found plenty of Z-disks sufficiently small to show about the right number of fringes when set a few fringes inside or outside focus, but they nearly all showed "dark" and "light" as primary inside and outside focus colours: presumably all the really small Z-disks were too thin to develop colour-contrast. The largest "halo" that I was able to produce extended 2 cm. radially from the mirror: it was not quite regular. The fringe system at mean focus showed that the centre and edge of the mirror were about $\frac{1}{4}$ fringe low (which I confirmed with a knife-edge), and the system was astigmatic by about $\frac{3}{4}$ fringe. The astigmatic axes did not rotate when the mirror was rotated, so that this astigmatism must be ascribed to the decentring of the star from the optic axis, which I could not reduce below 2·5 mm.: the direction and sign of the astigmatism were correct for this. Now the astigmatism introduced by this amount of decentring is $\frac{5}{4}$ fringe (in $\lambda = 5 \times 10^{-5}$ cm.), so that although it is not clear to what wave-length the observed $\frac{3}{4}$ fringe of astigmatism should be referred, it would appear that I had failed to detect the whole of the astigmatism. The visibility of the fringes was only moderately good even at mean focus. However, it is gratifying to see that the method even with a star that is rather too large is still capable of moderate accuracy, at F/2·8.

I have used the method on an F/7 image, with a very good spherical mirror, and had no difficulty in finding Z-disks giving blue and yellow as primary colours in strong contrast when the Z-disk was set $\pm \frac{1}{4}$ fringe from focus.

I am not sure whether it would be possible to increase the intrinsic brilliance of the star sufficiently to test with a ·005 mm. star after two reflections from unsilvered glass, which are necessary in null-testing a paraboloid with the aid of a flat. I do not know how much I am losing in intensity, with the gas-filled lamp as source, by not correcting the condensing lens. Perhaps the gain in source-brilliancy of 10 or so, to be obtained by the use of an arc as source, together with the automatic removal, due to the increased size of the source, of loss due to aberrations in the condensing system, would be sufficient to permit testing with two reflections from unsilvered glass, without a reducing telescope, using a star small enough for a paraboloid such as that used at Meudon, of F/3. Paraboloids of the modest aperture of F/8 can certainly be Zernike-tested at the focus, using a star lit by a gas-filled lamp.

Professor Zernike has pointed out to me that one could increase the available light at least 100 times by using a slit instead of a pinhole, and a Z-strip instead of a Z-disk, and that the appearance of irregular error is very little altered by this change, which makes the test null for that part of the error which is not constant along a co-ordinate perpendicular to the strip. And indeed I find that when spherical aberration and astigmatism are absent, and the residual error is irregular, the substitution of a Z-strip for a Z-disk makes very little difference to the appearance at focus: it is quite difficult to tell, from the appearance of the fringe system (even though one has also seen it with a disk), in what direction the Z-strip lies. (I made at first the mistake of using too wide a strip (·03 mm.). It is difficult to make the resin fibres so narrow as not to exceed ·01 mm. after pressing, and I found it simpler to make strips by simply stroking the surface of a glass plate with a "cutanit" (molybdenum-titanium carbide) tool. If the pressure is not too great, no sign of fracture is visible under the microscope, the glass having either flowed or been planed away. These strips give "dark" and "light" as inside and outside focus colours.)

It may be, then, that a slit and a strip which can be rotated together through a right angle, for the detection of astigmatism, will be the form in which the Zernike test will be used in practice in the figuring of wide-aperture reflectors.

But I would always prefer, if enough light is available, to use a star and a Z-disk, so that the whole of the error can be seen at once; and I feel it is perhaps better to develop methods for null-testing big reflectors, not at the focus (for after all we can only test conic-sections null-fashion in that way, with the aid of a flat or spherical mirror, and the Ritchey-Chrétien curves, for example, do not belong to that class), but at the centre of apical curvature: it should not be impossible to devise systems of compensating mirrors of which neither the asphericity nor the diameter need be so great as that of the mirror under test, which will supply light having the correct asphericity to stigmatise after one reflection from the unsilvered surface: the intensity would then be high enough to permit the use of a Z-disk.

To take an example, is it easier to make a six-inch compensating mirror having a prescribed asphericity of about 5 fringes, or a three-foot flat, in order to null-test a three-foot paraboloid of F/3·7? And would it not be worth while to figure, though it might have to be with zonal testing, a small compensating mirror, in order to be able to apply during the figuring of a large Ritchey-Chrétien reflector this very beautiful and sensitive null interference test which Professor Zernike has placed at our disposal?

Imperial College of Science
and Technology:
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AN ANALYSIS OF THE RADCLIFFE PROPER MOTIONS IN THE SELECTED AREAS.

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For the past twenty-five years the Radcliffe Observatory has been engaged on determining the proper motions of faint stars in the Selected Areas. The catalogue containing the results for more than 32,000 stars is now in the press.* After omitting (*a*) the stars brighter than 9·0, (*b*) the faintest stars in each area, since their proper motions have been determined with less weight, and (*c*) those with annual proper motions larger than 0″·1, there remain some 25,000 stars, which have been subjected to analysis with the object of testing whether they show the same tendencies to systematic motion as the brighter stars. A detailed description of this work will appear in the volume already referred to; the present paper contains a brief account of the methods employed and some comments on the results obtained.

The material was divided into two parts, comprising the stars of magnitude 9·0 to 14·0 and those from 14·1 downwards, the magnitudes being on the international scale. These two groups, referred to hereafter as the brighter and fainter stars, contain 10,342 and 14,569 stars respectively, the mean limiting magnitude of the latter being 15·0. The areas have been grouped together, one area in declination $+75°$ with two at $+60°$, two at $+45°$ with two at $+30°$, and two at $+15°$ with two on the equator, thus forming thirty regions symmetrically distributed over the northern hemisphere. The proper motions as given in the catalogue are relative to the mean of the comparison stars used for each area; the number of these averaged 128, and their mean magnitude 13·7. They are thus sufficiently numerous and faint to form a base of reference that is unlikely to be sensibly affected by their peculiar motions, especially as no star with proper motion greater than about 0″·5 per annum has been used as a comparison star.

* *The Radcliffe Catalogue of Proper Motions in the Selected Areas* 1 *to* 115, 1934.