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Victor A. Eremeyev, Franz Fischer

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# On the phase transitions in deformable solids

**Victor A. Eremeyev**[1,2,*] **and Franz D. Fischer** [3,**]

[1] Lehrstuhl für Technische Mechanik, Zentrum für Ingenieurwissenschaften, Martin-Luther-Universität Halle-Wittenberg, Kurt-Mothes-Str. 1, 06099 Halle (Saale), Germany

[2] South Scientific Center of RASci & South Federal University, Milchakova str., 8a, Rostov-on-Don, 344090, Russia

[3] Institute of Mechanics, Montanuniversität Leoben, Franz-Josef-Str. 18, 8700 Leoben, Austria

Modelling of phase transitions (PT) in deformable solids is one of the most important problems of continuum mechanics, physics of solids, and mechanics of materials. Its actuality is determined by the fact that the majority of materials used in modern engineering undergoes PT during manufacturing or exploitation. Examples of processes, where the phenomenon of PT plays an important role, are the stress-induced martensitic PT in alloys and steels, PT in piezoceramics, ferroelectrics, crystal growth, twinning, ice glacier formation, metal solidification or melting, PT in rock materials, PT in liquid crystals, etc. In particular, PT are the origin of the mechanisms of the prominent shape memory effect and pseudoelasticity, also known as superelasticity of shape memory alloys and polymers.

The phenomenon of PT is known since ancient times. For example, C. Huygens (1629–1695) discovered the twinning of the calcite crystals. Investigations of the thermodynamics of PT were initiated by original work of J. W. Gibbs "On the equilibrium of heterogeneous substances" (1876, 1878). Using the variational principles, known as Gibbs variational principles, he established the phase equilibrium condition on the interface separating the material phases. Another starting point of the future theory of PT is the paper "Über die Theorie der Eisbildung, insbesondere über die Eisbildung im Polarmeere" (1889, 1891) by J. Stefan on the ice formation. Since their works thousands of papers and monographs have been published on PT and related problems. The literature on PT in deformable solids is very extensive, see the monographs and collections of papers [1-13] among others.

Modelling of PT has various aspects. From the mathematical point of view PT have itsbase in such branches of Mathematics as the calculus of variations, variational inequalities, moving boundary problems, Stefan problem, theory of groups, nonconvex analysis, etc. Within the framework of Continuum Thermomechanics one deals with the description of interaction between the stress, strain, temperature, electric fields, etc. and PT. Or, in other words, we consider the boundary-value and initial-boundary-value problems taking into account PT. There exist also various approaches to the description of PT.

In the first approach one assumes explicit existence of a sharp phase interface being a sufficiently regular surface or curve separating different material phases. The position and motion of the phase interface itself are among the most discussed issues in the field. In the literature many one-dimensional, two-dimensional, and three-dimensional problems were analysed theoretically, numerically and experimentally, which adequately describe the behaviour of bodies made of materials undergoing PT. The compatibility conditions on the phase interface may be obtained by the variational approach, by using the integral balance laws, etc. In these considerations the Eshelby tensor appears, which is also known as the tensor of chemical potential or the energy-momentum tensor. In dynamics and/or quasistatic deformations one needs to formulate the kinetic equation describing the relation between the phase interface velocity and the jumps of some fields, representing thermodynamic forces. Note that the corresponding boundary-value problems are nonlinear ones, in general. This approach gives the detailed description of the stress and strain fields in the vicinity of equilibrium of a propagating phase interface. It is called as sharp interface model.

In the second approach models do not use the explicit introduction of the phase interfaces. For the description of PT here the special constitutive equations with additional "internal" variables are used. The examples are the phase concentration variable, order parameters, phase strain tensor, etc. Since such materials with PT as shape memory alloys demonstrate a quasi-plastic behavior, some of these models are similar to ones used in plasticity. Other models use the methods of

* Corresponding author E-mail: eremeyev.victor@gmail.com, Phone: +49(0) 345 55 284 36, Fax: +49(0) 345 55 273 59.

** E-mail: fischer@unileoben.ac.at, Phone: +43 3842 402 4007, Fax: +43 3842 460 48.

mechanics of composites to find the averaging material properties with PT. In contradiction to the sharp interface model this approach may be called macro-approach. However, for the satisfying description of the material behavior one needs to consider the micro-level of material structure (grains, lattices, etc.) and its evolution. So, these two approaches are closely connected with each other. Both approaches have, however, advantages as well as disadvantages.

This issue of the ZAMM includes several papers in the field of phase transformation models and related problems.

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