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## On the Positive-Parity States with Anomalous $\alpha$-Decay Properties in ${ }^{12} \mathrm{C}$

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The excited positive-parity levels at 7.7 and 10.3 MeV in ${ }^{12} \mathrm{C}$ nucleus are famous for their anomalously large $\alpha$-decay widths. ${ }^{11}$ They have been considered to be typical states with $\alpha$-clustering, which is readily expected by Ikeda diagram, ${ }^{2)}$ and investigated theoretically. ${ }^{3)}$ But unfortunately experimental information is so rare that the spin of the 10.3 MeV level is not established as yet. ${ }^{4)}$ On the standpoint of $\alpha$-clustering expected, we have studied dynamical properties of $3 \alpha$-particle system microscopically by using the generator coordinate method (GCM). And we have obtained the excited positive-parity states, $0_{2}{ }^{+}, 2_{2}{ }^{+}, 0_{3}{ }^{+}, 2_{3}{ }^{+}, 4_{2}{ }^{+}$and $3_{1}{ }^{+}$(Ref. 5), hereafter referred to as I). The purpose of the present paper is to investigate $\alpha$-decay properties of these states and to clarify nuclear structure of excited states in this energy region of ${ }^{12} \mathrm{C}$.

In Fig. 1 an energy spectrum obtained by the GCM calculation I shows that the $2_{2}{ }^{+}$state is the next positive-parity state of the $0_{2}{ }^{+}$state. The $0_{3}{ }^{+}$state comes out about 4 MeV high above the $0_{2}{ }^{+}$state together with the degenerate $2_{3}{ }^{+}$state. This situation is not changed by any truncation of the basis wave functions for large inter-$\alpha$-distances. Compared with experiment


Fig. 1. Energy spectrum from I. Levels are classified into several bands. (B) and (C) show the excited positive-parity levels. See Ref. 5).
the energy difference between the $0_{2}{ }^{+}$and $2_{2}{ }^{+}$states is fairly good, if the $2_{2}{ }^{+}$state is assigned to the 10.3 MeV level. Then the $2^{+}$assignment is energetically favoured.

Further we analyse $\alpha$-decay widths of these states by using so-called "Separation Energy Method", ${ }^{6}$ ) in which inner $\alpha$-reduced width amplitudes $r y_{c \equiv(I, l)}(r)$ are smoothly connected to outer resonance tails $G_{\boldsymbol{l}}\left(k_{c} r\right)$ with given separation energies. $\theta_{\alpha, c}^{2}$ and $\Gamma_{\alpha}$ are estimated by the relations $\theta_{\alpha, c}^{2}(a)$ $=(a / 3)\left(a y_{c}(a)\right)^{2}$ and $\Gamma_{\alpha}=\sum_{c} \Gamma_{\alpha, c}=\sum_{c} 2 P_{c}$ (a) $\left(3 \hbar^{2} / 2 \mu a^{2}\right) \theta_{\alpha, c}^{2}(a)$, where $a$ is a channel radius and $P_{c} \equiv k_{c} a /\left(F_{l}{ }^{2}\left(k_{c} a\right)+G_{l}{ }^{2}\left(k_{c} a\right)\right)$. In the calculation of the $\alpha$-reduced width amplitudes, a Brink-Bloch type wave function ( $d=3.5 \mathrm{fm}$ ) is taken for the ${ }^{8} \mathrm{Be}$ core.

Calculated $\alpha$-decay widths are listed in the Table, together with the existing experimental data. We can see that the width of the $0_{2}{ }^{+}$state is reproduced excellently well and that a calculated $\alpha$-decay width of the $2_{2}{ }^{+}$state is fairly large. On the other hand, an $\alpha$-decay width of the $0_{3}{ }^{+}$ state is rather small and smaller than that of the $2_{2}{ }^{+}$state although it can decay into the ${ }^{8} \mathrm{Be}$ ground state with a partial angular momentum $\boldsymbol{l}=0$. Hence we suggest strongly that the 10.3 MeV broad level is the $2_{2}{ }^{+}$

Table I. $\alpha$-decay widths in eV for the $\mathrm{O}_{2}{ }^{+}$state and in keV for the others. Values in parenthesis are those when the renormalization correction is taking into account. ${ }^{6}$ ) Calculated $\theta_{\alpha}{ }^{2}$ value for the $0_{2}{ }^{+}$state is 0.75 at 5.8 fm , whereas 0.82 in exp. The excitation energy of the $0_{3}{ }^{+}$state (not observed yet) is assumed to be 11.7 MeV estimated by $\left.7.7 \mathrm{MeV}\left(0_{2}{ }^{+}\right)+4 \mathrm{MeV}\right)$.

|  | $0_{2}{ }^{+}(7.66 \mathrm{MeV})$ | $2_{2}{ }^{+}(10.3)$ | $0_{3}{ }^{+}(10.3)^{\mathrm{a})}$ | $0_{3}{ }^{+}(11.7)$ | $2_{3}{ }^{+}(11.2)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\Gamma_{\text {exp. }}$ | $8.7 \pm 2.7(\mathrm{eV})$ | $3000 \pm 700 \mathrm{keV}$ |  |  | $430 \pm 80$ |
| $\Gamma_{\text {cal }}(a=5.8)$ | $7.9(14)$. | $1200(2600)$ |  |  |  |
| $\Gamma_{\text {cal }}(a=6.6)$ | $8.8(11)$. | $1000(1400)$ | $380(550)$ | $710(1000)$ | $73(87)$ |
| $\Gamma_{\text {cal. }}(a=7.4)$ |  |  | $490(560)$ | $1000(1200)$ | $81(85)$ |

a) Calculated according to the assumption that the excitation energy is 10.3 MeV , which gives the wave number $k_{c}$ in the penetration factor.


Fig. 2. $\alpha$-reduced width amplitudes $r y_{c}(r)$ and connected resonance tails $G_{l}(r)$. Brackets $C \equiv[I \times l]$ denote a spin of ${ }^{8}$ Be core $I$ and an orbital angular momentum of $\alpha$-particle $l$, identifying channels. Wave functions are normalized in whole space in bound state approximation.
state. The unexpected narrowness of the $\alpha$-decay width of the $0_{3}{ }^{+}$state in spite of its rather high excitation energy is easily understood by inspecting the $\alpha$-reduced width amplitudes. We can see in Fig. 2 that the states have distinct $\alpha$-clusterings. Their amplitudes, however, concentrate on proper channels respectively. Then we can classify the states into two groups; one includes the $0_{2}{ }^{+}$and $2_{2}{ }^{+}$states, the other does the $0_{3}{ }^{+}$and $2_{3}{ }^{+}$states. In the former an $\alpha$-particle interacts mainly with the ${ }^{8} \mathrm{Be}$ ground state, while in the latter it interacts with the first excited state of
${ }^{8} \mathrm{Be}$. Then we have a weak coupling picture of the rotational motion of ${ }^{8} \mathrm{Be}$ core and an $\alpha$-particle motion. It is worthwhile noticing here that from this analysis we can recognize that positive parity excited states with distinct $\alpha$-clustering are exhausted in this energy region by the states obtained. In decays of dominant components into respective members of the rotational band of ${ }^{8} \mathrm{Be}$, the widths show different tendencies between two groups due to different relative energy positions to the tops of the effective barriers. More precisely, in the $0_{2}{ }^{+}$and $2_{2}{ }^{+}$states, out-
ward extensions of the tail parts due to the $\alpha$-clusterings and increasing amplitudes in decaying channels cause the large $\alpha$ decay widths. On the other hand, the latter states can hardly decay into their proper channel where they have their dominant components. Thus their total $\alpha$ decay widths mainly depend on the small components involved which decay into the ${ }^{8}$ Be ground state, and result in being rather narrow.
It is concluded that we have had a simple overview on the nuclear structure of the excited positive-parity states with distinct $\alpha$-clustering and is strongly suggested that a spin of the broad 10.3 MeV level is to be $2^{+}$with the dominant ${ }^{8} \mathrm{Be}\left(0^{+}\right)$ $\otimes \boldsymbol{l}=2]_{J=2}$ component. As for the somewhat smaller prediction of its total width, it is worth-while mentioning a possibility that the broad $\mathrm{O}_{3}{ }^{+}$and $2_{3}{ }^{+}$states at somewhat higher excitation energies may con-
tribute to the extremely large width observed, overlapping with the $2_{2}{ }^{+}$state.

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Note Added: Recently Y. Fukushima and M. Kamimura also have studied the structure of ${ }^{12} \mathrm{C}$ nuclei by the Resonating Group Method (to be published in the Proceedings of the International Conference on Nuclear Structure, Tokyo, 1977).

