ON THE POSSIBILITY OF EPISTEMIC LOGIC

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Can there be a logic of 'knowledge'—an epistemic logic? To be sure, some logicians such as E. J. Lemmon, G. H. Von Wright, and Jaakko Hintikka have claimed to produce such a logic.[1] Max Hocutt, in an article "Is Epistemic Logic Possible?", has contended, however, that such a claim is groundless.[2] Since it is Hintikka who has made the most thorough attempt at such a logic we will give a brief exposition of his system as given in *Knowledge and Belief* and then we will consider the cogency of Hocutt's objections. Hintikka sees epistemic logic as a branch of *modal* logic. The semantical approach to modal logic, more particularly the semantics of 'possible worlds' of which Hintikka was one of the first and foremost advocates, can therefore be applied to epistemic logic. Hintikka does not develop his system axiomatically but by asking ". . . what conditions the truth of a set of statements imposes on the world, or (equivalently) what kinds of 'possible worlds' there must be in order for a set of statements to be consistent."[3] On such an approach the semantical conditions are not given for all possible worlds 'absolutely' but only for all possible worlds *relative to* or *alternative to* a given world.

> The basic idea is that every attribution of a 'propositional attitude' such as knowing to a person can be paraphrased by speaking of the totality of possible worlds compatible with the presence of this attitude in the person in question (at the time about which we are talking). For instance, to say that p is known by **a** to be true is nothing more—or less—than to say that p is true in all of the possible worlds compatible with a's knowing what he in fact knows.[4]

If we let W_i $(i \geq 1)$ represent possible worlds where W_1 represents the possible world that is actualized, R represent the 2-place relation of epistemic alternativeness, and $K_a p$ represent '**a** knows that p', then this condition can be stated more formally:

(K*) *If $K_a p \epsilon W_1$, then $p \epsilon W_i$ for all i such that $W_1 R W_i$.*

Since the actual world is of course possible the condition (K*) yields $p \epsilon W_1$ when $K_a p \epsilon W_1$. Indeed, if we allow more generally that the relation of alternativeness is reflexive (i.e. that $W_i R W_i$ for all i) we have:

(K)   If $K_a p \in W_i$, then $p \in W_i$.

This condition, which we can symbolize $(K_a p \supset p)$, has been considered sound by all proponents of epistemic logic. Intuitively, it corresponds to the claim that whatever is known has to be true.[5] Even Hocutt admits that it is legitimate though one theorem, he says, does not a logic make.[6] (Formally, it corresponds to the axiom schema of $\square$-elimination, $\square A \supset A$, common to the various modal systems.)

An interesting feature of Hintikka's version of epistemic logic is that for $K_a p$ to be true not only must p be true in all possible worlds compatible with the given one but $K_a p$ must also be true in these worlds. This follows from his claim that 'to know' is equivalent to 'knowing that one knows'. More formally:

(KK)   If $K_a p \in W_i$, then $K_a p \in W_k$ for all k such that $W_i R W_k$.[7]

In more formal terms this means that the relation of alternativeness is transitive, i.e., if $W_i R W_j$ and $W_j R W_k$, then $W_i R W_k$.[8] In terms of modal logic Hintikka's system is thus an analogue to Lewis's system S4. That is, corresponding to the principle $\square A \supset \square\square A$ for alethic modalities, we have:

(KK) $K_a p \supset K_a K_a p$.

This thesis, usually called the 'KK-thesis', is supported not by an appeal to introspection but by a conceptual analysis of 'knowing'.[9] Hintikka finds support for his claim that this is the *basic* meaning of 'know' in such ancient philosophers as Plato and Aristotle as well as in more recent analyses of knowledge as a claim for which we have 'conclusive evidence' or for which we 'have a right to be sure'.[10]

In addition to the 'strong' operator K, Hintikka introduces a 'weak' operator P, where $P_a p$ is read 'It is possible for all **a** knows that p'. $P_a p$ is true if there is at least one possible world alternative to the one in which the statement is made in which p is true. More formally:

(P*)   If $P_a p \in W_1$, then there is at least one possible world, say W_k, such that $W_1 R W_k$ and $p \in W_k$.[11]

For Hintikka, this operator is interdefinable with the strong operator in the usual manner, i.e.,

$$P_a p =_{\text{def.}} \sim K_a \sim p.[12]$$

We thus have the following conditions:

($\sim$K)   If $\sim K_a p \in W_i$, then $P_a \sim p \in W_i$.
($\sim$P)   If $\sim P_a p \in W_i$, then $K_a \sim p \in W_i$.

From a formal point of view some such rules are obviously indispensable to complement our earlier rules for P and K.[13] When the set of rules (K), (KK), (P*), ($\sim$K), and ($\sim$P)—the rule (K*) becomes redundant with the addition of (KK)—are added to the semantic tableau rules for propositional logic, we obtain Hintikka's epistemic system for propositional logic.[14]

The rules (K) and (P*) are non-controversial according to Hintikka. He argues at length, however, for (KK) and the two rules (~K) and (~P). The last two are especially problematical. For example, suppose that $K_a p$ and that p logically implies q, i.e., $\vdash p \supset q$. Yet, suppose also that $\sim K_a q$, i.e., that it is not the case that **a** knows q. By the condition (~K) this implies that $P_a \sim q$, i.e., that it is possible for all **a** knows or that it is compatible with everything **a** knows that q is not the case. But this is not compatible with **a**'s knowing p, i.e. we have $\sim P_a \sim q$. Thus, by *reductio ad absurdum*, we have $K_a q$. Thus, by Hintikka's rules we have the following principle:

$$If \vdash p \supset q, \ then \vdash K_a p \supset K_a q.^{15}$$

Hintikka admits, of course, that whenever **a** knows p and p logically implies q, it is not always the case—as a matter of fact—that **a** will also know q.

> It is true, in some sense, that if I utter
>
> > *'I don't know whether p'
>
> [which Hintikka would symbolize $\sim K_a p \cdot \sim K_a \sim p$ where **a** = I] then I am not altogether consistent unless it really is possible, for all that I know, that p fails to be the case. But this notion of consistency is a rather unusual one, for it makes it inconsistent for me to say (*) whenever p is a logical consequence of what I know. Now if this consequence-relation is a distant one, I may fail to know, in a perfectly good sense, that p is the case, for I may fail to see that p follows from what I know. Hence there seems to be a discrepancy between my rules and the way the verb 'to know' is actually used.[16]

Hintikka also accepts the weaker principle:

$$K_a(p \supset q) \supset (K_a p \supset K_a q).$$

Even this weaker condition may not be the case in real life for if one knows that $p \supset q$ and also knows that p one may fail to infer that q. (How many logic students on an examination fail to make an 'obvious' application of *modus ponens*?)

This feature of Hintikka's system has been—as one might readily suspect—the source of frequent objections.[17] It would appear, for example, that if a person knew that the axioms of a standard logical system were logical truths and that the rules of the system preserved logical truth, then such a person would know all logical consequences of these axioms, i.e. they would know *all* logical truths. Thus, Hintikka's system seems to be a system for "logically omniscient knowers".[18] Hocutt, for example, contends that Hintikka's approach ". . . eliminates actual knowers from the range of values of personal variables in epistemic statements. These epistemic formulas have no interpretation in terms of, and are strictly meaningless when applied to, actual knowers."[19] Elsewhere, he comments that: "If epistemic logic is a logic of LPK's [Hocutt speaks of 'logically perspicacious knowers' rather than 'logically omniscient knowers'] alone, then it isn't a logic for normal men. It therefore hasn't anything to do with

what we normally call 'knowledge', a thing possessed by normal men who are not always as perspicacious as might be desired.''[20] The key words in a passage such as the one last cited are 'alone' and 'anything'. If Hintikka's theory has *nothing* to do with the real world, it would seem pointless to call it a logic of knowledge. In a later article, Hintikka contended that such an objection—though not entirely unfair—involved a misinterpretation of *Knowledge and Belief*.[21] He had earlier conceded:

> Our results are not directly applicable to what is true or false in the actual world of ours. They tell us something definite about the truth and falsity of statements only in a world in which everybody follows the consequences of what he knows as far as they lead him. A sentence is self-sustaining if it is true in all such worlds, defensible if it is true in at least one such world, and so on. They are applicable to actual statements only in so far as our actual world approximates one of the 'most knowledgeable of possible worlds', as we may call them, or can be made to approximate one of them by calling people's attention to consequences of what they know.[22]

The result that a knower would know all logical truths would follow only if the notion of self-sustenance (validity) were interpreted as truth on every possible occasion (in every possible situation). Such an interpretation is ruled out, he claims, by the above passage where self-sustenance is interpreted as truth only in every possible world whose inhabitants all follow up the consequences of what they actually know.

> If this interpretation of the metalogical notion of self-sustenance is adopted, together with the parallel interpretation of other basic metalogical concepts, then there is no objection to saying that the sense of knowing which we are dealing with is essentially the ordinary sense of knowing.[23]

One may well wonder whether this shifting of the questionable features to the meta-linguistic level only side-steps the force of the above objections. To be told that the notions of 'self-sustenance' and 'defensibility' are *metalogical* notions still leaves us with the question of the applicability of such notions. One may also be puzzled as to whether, with this kind of interpretation in our metalanguage, the 'K' in our object language can still be said to characterize the 'ordinary' sense of knowing.[24] Hintikka admits, as we have seen, the limited applicability of his system. In the *strict* sense the theorems of epistemic logic hold only for an 'ideal' world—a world in which knowers do follow up the logical consequences of what they know. It does not follow, however, that such a system has—as Hocutt maintains—'nothing' to do with the actual world. In *Knowledge and Belief* Hintikka noted that in those cases where p 'obviously' implies q or where the chain of deductive inference connecting p and q is very short, if **a** knows p then **a** is likely to admit that he knows q.[25] In later articles, he cites the applicability of the system to those situations where '$p \supset q$' is a 'surface tautology'.[26] In such cases, when a person does not know q, we may question whether he really knows or understands the meaning of p rather than rejecting the principle: $p \supset q$. $\therefore K_a p \supset K_a q$.[27] Hintikka's system is in effect a theoretical ideal which has only partial exemplification in the 'real' world. To say, as does Hocutt, that such a system cannot be a 'logic'

because the theorems are not always true of the actual world is to place too stringent a restriction on the construction of logical systems. The problem here is similar to that raised by non-Euclidean geometries and many-valued logics. The general trend amongst logicians is to adopt a more 'liberal' definition of logic than that proposed by Hocutt.[28] For example, Robert Ackermann writes:[29]

> When philosophers discuss logic and develop logical systems, they are interested in an ideal kind of validity that is *not* exhibited in most of the arguments occurring in real life. The ideal serves as a standard that can be used in the critical assessment of actual belief structures.[30]

There thus seems to be no reason to deny the possibility of epistemic logic on the basis of its 'ideal' character.

This feature of Hintikka's system also bears on another point made by Hocutt, i.e. that such a system cannot be called 'epistemic' since its characterization of 'know' does not correspond to the ordinary use of the term.[31] Part of the difficulty here may be due to a certain lack of clarity in *Knowledge and Belief*. Some passages might appear to indicate that—in spite of protests to the contrary—Hintikka is doing 'ordinary language analysis'.[32] In later writings he makes it even clearer that his approach is *theoretical* rather than descriptive.

> A branch of logic, say epistemic logic, is best viewed as an *explanatory model* in terms of which certain aspects of the workings of our ordinary language can be understood. In some cases, this explanatory model may be thought of as bringing out the 'depth logic' which underlies the complex realities of our ordinary use of epistemic words ('know', 'believe', etc.) and in terms of which these complexities can be accounted for. It therefore does not represent a proposal to modify ordinary language but rather an attempt to understand it more fully. But this explanatory model does not simply reproduce what there is to be found in ordinary discourse. As the case is with theoretical models in general, it does not seem to be derivable from any number of observations concerning ordinary language. It has to be invented rather than discovered.[33]

Even this passage might still seem to indicate that ordinary usage is nevertheless the 'criterion' against which the adequacy of the theoretical analysis is to be tested. Elsewhere, however, he writes:

> In no case can the acceptability of the thesis [the *KK*-thesis] be decided by appeal to 'ordinary language'. It seems to me in fact that many analytical philosophers' methodological posture is far too timid. The ultimate court of appeal in deciding whether a logical principle governing some given concept is acceptable is not ordinary usage, however regimented, but rather whether the principle helps the concept in question to serve the purpose or purposes it in fact is calculated to serve in our conceptual repertoire, and whether these purposes are themselves worth our effort. By spelling out these purposes and by using them to evaluate various logical principles an analyst can perfectly well disagree with ordinary usage and even attempt to reform it.[34]

Hintikka has emphasized from the beginning—but especially in his later writings—that the sense of 'know' which he intended to capture with his logical system was the 'strong' sense, the philosopher's sense, of 'know'.[35] Hence, Hocutt's objection that such a system is not 'epistemic' because it

does not correspond to the sense of 'know' which is 'familiar to us' seems beside the point.[36]

Hocutt is, of course, aware that Hintikka would 'shift his ground' in the face of the earlier objections. According to him,

> As the most obvious way to respond to the fact that there are actual knowers who pose counterexamples to epistemic logic is to answer that there are possible knowers who do not, so the most obvious way to respond to the fact that there is a sense of 'know' for which its theorems turn out to be false is to counter that there is a sense of 'know' for which they do not.[37]

He then proceeds to criticize Hintikka's views—or what he takes to be his views—on 'virtual' knowledge as contrasted with 'active' knowledge. His objections, however, seem to be either unjustified or based upon a misunderstanding. He says that according to Hintikka "... people always *virtually* know the logical consequences of what they know although they may not *actively* know them".[38] This seems to attribute to *actual* knowers a kind of 'innate' knowledge of logical truths—perhaps after the model of the Platonic doctrine of recollection. Hintikka does at one point in *Knowledge and Belief* consider such an 'alternative interpretation' but he rejects it.[39] Hocutt also contends that such a defense of epistemic logic is 'circular'.

> It merely secures the theorems of epistemic logic by fiat: they are the truths about knowledge in the sense of 'know', whatever it may turn out to be, for which they are the truths about knowledge.[40]

Such an objection seems particularly unfair in the light of the depth and detail of Hintikka's discussion of the strong sense of 'know'.[41] Hocutt does admit that such a distinction exists but it corresponds, he insists, to Quine's distinction between referentially opaque and referentially transparent uses of the term 'know'. On the referentially transparent use the following principle is said to obtain:

$$*(p \equiv q) \equiv (K_a p \equiv K_a q).$$

One can easily see that on this interpretation the 'K' of epistemic logic would become redundant and Hintikka's system would reduce to ordinary propositional logic.[42] Hintikka would not agree, however, that his strong sense of 'know' is the referentially transparent sense and the principle (*) is of course not a theorem of his system. He says explicitly that the context of K_a is referentially opaque.[43] Whenever $K_a p$ is true, then so is p; it does not follow, however, that $K_a q$ will be true when q is substituted for p where q also has the value true. Indeed, for Hintikka, the transparent sense is said to be definable in terms of the basic (opaque) sense symbolized by K.[44] The above objection therefore rests on a mistaken identification of Hintikka's strong sense of 'know' with a referentially transparent sense.

Hocutt next considers in more detail Hintikka's characterization of 'defensibility'—the term that Hintikka uses in place of 'consistency'. According to Hocutt:

Consistency has, of course, to do with truth, and therefore the terminological shift signals the surrender of the notion of truth, as Hintikka informs us that the rules of his epistemic logic 'are not concerned with the *truth* of statements at all; they merely tell us that certain adjunctions preserve the defensibility of sets of sentences' (p. 32).[45]

Hocutt also cites another passage where Hintikka admits that the elements of an 'indefensible' set, say $\{K_a p, \vdash p \supset q, \sim K_a p\}$, 'may even be true simultaneously' (p. 31) as further evidence that Hintikka himself realizes that his theorems are 'false'.[46] We will see later that there is a legitimate basis for some of Hocutt's concerns here. Hintikka does seem at times to vacillate between a 'performatory' sense of 'know' to which the attribution of truth would not be appropriate and a 'statement' sense in which $K_a p$ could be said to be true. Most of his objections here, however, seem to revert to his earlier criticism that Hintikka's system cannot be called a 'logic' because some of its theorems are not 'true'.

This restricted conception of logical truth is also reflected in the objections he directs against Hintikka's proof technique. Hocutt contends that Hintikka's 'reductive' technique for showing the indefensibility of formulas involves a reduction of the formulas to their propositional content and, hence, a reduction of epistemic logic to propositional logic.[47] In applying the technique one is to 'Throw away the troublesome operators 'P_a' and 'K_a' so that you can get at what matters logically'.[48] Thus, consider what happens when we wish to show that $K_a p \cdot K_a q \supset K_a(p \cdot q)$ is a theorem. For Hintikka, this involves showing that the set $\{K_a p \cdot K_a q, \sim K_a(p \cdot q)\}$ is indefensible. The proof proceeds as follows:

1. $K_a p \cdot K_a q \,\epsilon\, W_1$ hypothesis
2. $\sim K_a(p \cdot q) \,\epsilon\, W_1$ hypothesis
3. $K_a p \,\epsilon\, W_1$ 1, ·
4. $K_a q \,\epsilon\, W_1$ 1, ·
5. $P_a \sim (p \cdot q) \,\epsilon\, W_1$ 2, (~K)
6. $\sim(p \cdot q) \,\epsilon\, W_2$ where W_2 is some world alternative to W_1 5, (P*)
7. $K_a p \,\epsilon\, W_2$ 3, (KK*)
8. $K_a q \,\epsilon\, W_2$ 4, (KK*)
9. $p \,\epsilon\, W_2$ 7, (K)
10. $q \,\epsilon\, W_2$ 8, (K)

At this point we see that we have either

11. $\sim p \,\epsilon\, W_2$ 6, ~·

or

12. $\sim q \,\epsilon\, W_2$ 6, ~·

On either case we have a contradiction, i.e. either $\{p, \sim p\} \,\epsilon\, W_2$ or $\{q, \sim q\} \,\epsilon\, W_2$. Hence the set is indefensible and $K_a p \cdot K_a q \supset K_a(p \cdot q)$ is a theorem. According to Hocutt, "*this procedure makes the indefensibility of a set of epistemic sentences to consist in the inconsistency of their propositional contents.*"[50] By this 'eliminative' approach the set $\{K_a p, K_a q, \sim K_a(p \cdot q)\}$ is

said to reduce to $\{p, q, \sim(p \cdot q)\}$. Hocutt says this approach will not work with regard to the principle $K_a p \supset K_a K_a p$. (Note that in our proof above we could have gone directly from steps 3 and 4 to 9 and 10 by the rule (K*).) If it did, then the claim that the set $\{K_a p, \sim K_a K_a p\}$ and its equivalent set $\{K_a p, P_a \sim K_a p\}$ are indefensible would amount to the claim that the set $\{p, \sim K_a p\}$ is inconsistent and, hence, $p \supset K_a p$—which Hocutt reads 'Everything true is known by **a**—would be a theorem.[51] And yet, if one 'eliminates' the second K operator, the set would reduce to $\{p, \sim p\}$ which is inconsistent. For example, let us 'extend' Hintikka's proof that $\{K_a p, \sim K_a K_a p\}$ is indefensible.[52]

1. $K_a p \in W_1$ assumption
2. $\sim K_a K_a p \in W_1$ assumption
3. $P_a \sim K_a p \in W_1$ 2, (~K)
4. $\sim K_a p \in W_2$ where W_2 is an alternative to W_1 3, (P*)
5. $K_a p \in W_2$ 1, (KK*).

Steps 4 and 5 violate Hintikka's principle that a 'possible world'—or more formally what he calls a 'model set'—cannot contain a formula and its negation. Hence, the set is indefensible. Yet, if we continue the proof:

6. $P_a \sim p \in W_2$ 4, (~K)
7. $\sim p \in W_3$ where W_3 is an alternative to W_2 6, (P*)
8. $K_a p \in W_3$ 5, (KK*)
9. $p \in W_3$ 8, (K)

Steps 7 and 9 show that the set $\{p, \sim p\} \in W_3$. Thus, the set can be reduced to an inconsistent set in terms of its propositional contents. This does not mean, however, that we have 'reduced' epistemic logic to propositional logic any more than the fact that the application of the semantic tableau technique to predicate logic involves the 'elimination' of the quantifiers—under various restrictions—and the reduction of the formula to inconsistent sets of the form $\{P_{a2}, \sim P_{a2}\}$, for example, means that predicate logic is 'reduced' to propositional logic.[53] Nor does it mean, as Hocutt claims, that Hintikka has confused epistemic sentences with their propositional contents.[54] Hocutt neglects to consider here the fact that the semantical technique involves the assignment of truth-values in more than one possible world and that the inconsistency may occur in some other possible world than the original one. (Though this means also that the original set cannot be true then in the 'actual' world.) Hocutt, because he mistakenly concludes that this 'reductive' technique cannot be applied to $K_a p \supset K_a K_a p$, which he also misinterprets—at least from Hintikka's point of view—as the 'Cartesian doctrine of the self-illumination of the knowing mind', also mistakenly infers that Hintikka has *two* epistemic logics and that he is using the operator K equivocally.[55]

Hocutt's final round of objections turn upon what he calls the 'performatory' character of knowledge claims. According to Hocutt:

For Hintikka, a sentence is an act of making a statement; the statement is what is

made. Indefensibility is a property, not of the statement made, but of the making of it. It is a point which Hintikka also puts by saying that indefensibility is 'of a performatory character' (p. 77), using Austin's term to refer to the performance of 'uttering'.[56]

On this interpretation, Hintikka's book is said to become a set of helpful comments about 'self-defeating speech acts' but it is not then correct to call it 'logic'.[57]

There are several points to be noted here. First, the above distinction noted by Hocutt is not Hintikka's distinction between 'statement' and 'sentence'. According to Hintikka, ". . . a statement is the act of uttering, writing, or otherwise expressing a declarative sentence. A sentence is the form of words which is uttered or written when a statement is made."[58] Secondly, the passage from Hintikka cited above is from chapter IV in which he is discussing 'Moore's problem', i.e., the status of the expression 'p but I do not believe that p'.[59] Hintikka introduces a 'doxastic' logic or a logic of 'belief' complementary to his epistemic logic. The strong operator $B_a p$ is, of course, read 'a believes that p'; the weak operator $C_a p$ is read 'It is compatible with everything a believes that p'. Principles (C*), (~B), (~C), (B*), and (BB*) comparable to the epistemic principles are said to hold for doxastic logic.[60] The corresponding principle (B), i.e., $B_a p \supset p$, however, does not hold. The 'possible worlds' here represent doxastic alternatives, possible states of affairs compatible with what an individual believes. The relation between these and the epistemic alternatives is not a simple one.[61] Yet, we can say that every doxastic alternative is also an epistemic alternative. For Hintikka, this can be formulated as

(KB) *If $K_a q \epsilon W_i$, then $B_a K_a q \epsilon W_i$*

or more simply as:

$$\text{If } K_a q \epsilon W_i, \text{ then } B_a q \epsilon W_i.\text{[62]}$$

Intuitively, this expresses the principle that whatever a person knows he also believes. The solution that Hintikka offers to 'Moore's paradox' is that although the formula $(p \cdot {\sim}B_a p)$ is defensible, the formula $B_a(p \cdot {\sim}B_a p)$ is not, i.e., the original formula is doxastically indefensible. Thus the main point of the paradox is said to lie in the fact that $(p \cdot {\sim}B_a p)$ is necessarily *unbelievable* by the speaker **a**.[63]

> And this unbelievability of theirs [i.e. doxastically indefensible statements] is of a logical character; it can be seen from the very form of words the speaker is using (provided that we know how he is referring to himself). Doxastically indefensible statements are therefore *self-defeating*.[64]

Hintikka admits that such statements are characterized in terms of speech acts and that their absurdity is of a performatory character.[65]

> The notion of doxastic indefensibility was not defined, unlike the other notions we have defined so far, for *sentences* (or for sets of sentences) as much as for sets of *statements*. It does not depend solely on the forms of words uttered; it also depends on the speaker (or writer) and on the ways in which he is referring to himself. It can be defined for sets of sentences only by making it relative to a name or pronoun occurring in the sentences in question.[66]

One should note carefully here the expression 'unlike the other notions we have defined so far'. Thus, although Hintikka allows that in the strict sense it is 'statements' rather than 'sentences' that ought to be said to be true,[67] the systems of epistemic and doxastic logic can be developed for *sentences*. In the "Prolegomena" to his book he says:

> To begin with, it is advisable to confine our attention to those properties of classes of statements which depend solely on the sentences they exemplify. Thus we shall in the first few chapters study the consistency of sets of statements only in so far as it turns on the forms of the words used.[68]

This will, he says, greatly simplify matters and frequently allow one to disregard the distinction between statements and sentences. The notion of consistency can thus be defined for sentences in addition to sets of statements.

> Of course, on such a definition the consistency of a set of sentences will only mean that, whenever this set of sentences is uttered (on one and the same occasion by one and the same speaker or writer, addressed to one and the same person, and so forth), then the resulting set of statements is consistent in so far as one can tell without knowing who the speaker is, when the statements were made, or any other facts about them except the forms of words they exemplify.[69]

Hocutt's comments on this point appear to turn on an 'over-emphasis' upon Hintikka's remarks on doxastic indefensibility and a misunderstanding of his distinction between sentences and statements.

Nevertheless, we must admit that Hintikka *does* at times seem to treat knowledge claims as 'performatory'—or, at least, 'quasi-performatory'—in character. For example, he says, regarding the KK-thesis, that: "It is not based on psychological or quasi-psychological evidence. If you want to see in the equivalence a reflection of a more interesting truth you may try the quasi-performatory character of 'I know' statements (*cf.* section 3.8) rather than the transparency of our minds."[70] There is another passage in *Knowledge and Belief* that also seems to bear on this point. We have already noted that in seeking contemporary philosophical support for the sense of 'know' which he was trying to symbolize that Hintikka cited Ayer's view that to say 'I know' is to say 'I have a right to be sure'. Yet, he then goes on to say: "We must realize, however, that having this right need not mean that one's grounds are so strong that they logically imply that what one claims to know is true."[71] This is to allow that $K_a p$ could be true while p is false. How can this be reconciled with his earlier comment that we ". . . . cannot be said really to know what is not the case"?[72] We should point out that Hintikka later came to admit that he should not have appealed to a position such as Ayer's to support his KK-thesis.[73] *If* one knows in the strong sense, then one *cannot* be mistaken. This might seem to limit our knowledge claims, the p's and q's that fall within the scope of our epistemic operators, to only logical or mathematical truths. He also cites, however, Norman Malcolm's discussion of the knowledge claim that there is an ink-bottle before him as one that could be interpreted as knowledge in the strong sense. The circumstances that could go against such a claim can be

and are ruled out. Indeed, Malcolm says that there is *nothing* that could be counted as evidence against the claim.[74]   Hintikka admits to being uncertain about Malcolm's intent but

> ... the most he can claim, surely, is that *if* I in fact know in the strong sense that the ink-bottle is there, *then* there will not be any possible evidence showing that I do not know it. If 'possible' here means 'possible as far as my knowledge goes', we have precisely my strong sense. More generally, we might perhaps say that *if* one knows in the strong sense that *p*, then it is the case that one will refuse (if acting rationally) to consider any experience compatible with what he in fact knows as evidence against one's knowing that *p*.[75]

If one is *justified* in making a knowledge claim in this sense, *then* it seems that what one knows is true. Hintikka agrees that this strong sense is 'irrelevant' for many important purposes for which we would like to apply the concept of knowledge.[76] He also agrees that it is in general 'unrealistic' to expect to obtain such knowledge.[77] Knowledge in this sense is of interest primarily to the *philosopher*.

> It may be important for philosophical purposes to be reminded that we do know many things in a sense so strong that our knowing them entitles us to disregard any possible counter-evidence. However, these purposes are not likely to be pertinent to the search for new knowledge (information) in science or everyday life.[78]

We thus return again to the general problem of the 'applicability' of Hintikka's system and the 'ideal' character of his analysis.[79]

We should point out that Hocutt finally comes to consider the possibility that epistemic logic might be interpreted as a 'normative science' — as ". . . telling us not what people *in fact* know but rather what they *ought* to know, given that they know something else."[80] No epistemic logician has, he says, endorsed such an approach though he sees Hintikka's and Lemmon's views as having 'normative connotations'.[81] A 'normative' interpretation of logic, i.e. that it prescribes norms for correct thought, is quite an improvement, he agrees, over a 'psychological' interpretation that views it as describing actual processes of thought. He then allows:

> Epistemic logic construed as consisting of descriptive statements about the necessities of actually existing knowledge is merely false, and is much more palatably construed as determining a set of canons for logically consistent knowledge.[82]

Yet, when he thus appears to be on the verge of conceding that perhaps Hintikka's system is in a sense 'logic', he turns instead to criticize such a normative interpretation. It suffers, he says, from three defects. First, the interpretation is said to need interpreting. I am not sure what this means. (Perhaps Hocutt intends to make a point akin to the third one that he cites.) Secondly, it is said to turn things around: "One shouldn't 'think' a contradiction because it would be illogical to do so; a contradiction isn't illogical because one shouldn't think it." Again, the wording is puzzling. Perhaps what Hocutt is saying is that what makes a contradiction a contradiction is not the fact that one should not think it but because it (the contradiction) is itself illogical. I do not see, however, why a normative

interpretation should require the reversal. Indeed, it seems it would involve just the opposite—i.e. that the normative interpretation would coincide with the point Hocutt is making. Thirdly, the normative interpretation is itself said to be 'quasi-psychologistic' unless interpreted as saying that 'truth' is normative for 'thought' as indicated by Peirce. That is, if one 'ought' to 'think' the conclusion of an argument, it is only because the premises are true and the argument is valid.[83] Yet this latter interpretation seems quite in agreement with what Hintikka is doing. He is not describing actual thought processes but rather logical conditions of an 'ideal world' in which it is *true* that knowers follow up the logical consequences of what they know and to which we as actual knowers *ought* to approximate as far as possible.

One may rightly have some misgivings about the applicability or the success of such an 'ideal analysis'. One might still raise questions as to how we can know that a knowledge claim is *justified* beyond all further need of evidence? Does one *know* that one's evidence is 'complete' or 'sufficient'? Or is it the case—as Hintikka might seem to indicate in his discussion of 'surface tautologies'—that in some cases we 'just know' that certain claims are true without evidence, i.e. do we have some kind of 'basic' knowledge? Hintikka's discussion of the KK-thesis and his attempt to distinguish his sense of 'know' from the 'true belief' sense would seem to place his view in the classical tradition of knowledge as 'justified true belief'. Such a view has, as is well known, been subjected to searching criticisms.[84] The difficulties with the 'possible world' approach become even more acute when extended to the logic of quantification—for here we encounter the problems of 'essentialism' and 'trans-world identity'.[85] To be sure, one should not put off the study of such logics because of the feeling that such 'possible worlds' are bizarre. As Hintikka reassures us:

> One danger here is to think of 'possible worlds' as being something weird and conse-
> quently philosophically suspect. Yet nothing is more commonplace in human life and
> in the life of science than to find someone considering several possibilities as to how
> some sequence of events might turn out (e.g. considering several possible outcomes of
> an experiment). Whoever does so, is dealing with as many 'possible worlds' in the
> general sense proposed here.[86]

In another passage which also seems to bring such 'possible worlds' more down to earth he says: "It would be more natural to speak of different possibilities concerning our 'actual' world than to speak of several possible worlds."[87] In spite of such reassurances, we have found that Hintikka's system is best understood as a normative ideal—holding in general for knowers that are 'not of this world'. After examining the system one might come to agree with Ackermann that the prospects for completing such an ideal analysis do not seem bright.[88] Perhaps it will lead us to reconsider the merit of the 'philosophical' sense of 'know'. Nevertheless, credit should be given to Hintikka for formalizing the 'logic' of such a notion. One may doubt whether it is the case—as Føllesdal claims—that Hintikka's book ". . . has contributed more to the clarification of the notion of knowledge

than has any other work",[89] but one would be hard pressed to disagree with Ackermann's claim that: "Hintikka's system is currently the model against which other proposals need to be tested".[90]

NOTES

1. See E. J. Lemmon, "Is there only one correct system of modal logic?," *Proceedings of the Aristotelian Society*, Supl. vol. XXXIII (1959), pp. 23-40; G. H. Von Wright, *An Essay in Modal Logic*, North Holland Publishing Co., Amsterdam (1951); and Jaakko Hintikka, *Knowledge and Belief*, Cornell University Press (1962).

2. Max Hocutt, "Is epistemic logic possible?," *Notre Dame Journal of Formal Logic*, vol. XIII (1972), pp. 433-453.

3. Jaakko Hintikka, "Epistemic logic and the methods of philosophical analysis," reprinted in his *Models for Modalities*, D. Reidel Publishing Co., Dordrecht, Holland (1969), p. 4.

4. Jaakko Hintikka, "Different constructions in terms of the basic epistemological terms: a survey of some problems and proposals," reprinted in *Contemporary Scandinavian Philosophy*, p. 107. See also *Knowledge and Belief*, pp. 17 ff.

5. See Hintikka, *Knowledge and Belief*, p. 48. See also pp. 22-23.

6. Hocutt, *op. cit.*, p. 434.

7. See *Knowledge and Belief*, pp. 17 ff. and p. 43.

8. *Ibid.*, pp. 45-47.

9. *Ibid.* See especially chapter 5.

10. We will later see that Hintikka came to believe that Ayer's condition of 'having a right to be sure' was not strong enough.

11. See *Knowledge and Belief*, pp. 42-43.

12. Some epistemic logicians, e.g., Føllesdal, do not accept this interdefinability. See his "Knowledge, identity, and existence," *Theoria*, vol. 33 (1967), pp. 1-27.

13. Hintikka, *Knowledge and Belief*, pp. 29-30.

14. *Ibid.*, pp. 40 ff.

15. *Ibid.*, pp. 29 ff.

16. Hintikka, *Knowledge and Belief*, p. 30.

17. See, e.g., Hector-Neri Castañeda's review of Hintikka's *Knowledge and Belief, The Journal of Symbolic Logic*, vol. 29 (1964), pp. 132-134.

18. See, e.g., Roderick Chisholm, "The logic of knowing," *Journal of Philosophy*, vol. LX (1963), pp. 773-795. See especially p. 781.

19. Hocutt, *op. cit.*, p. 436. Hocutt is referring here to the views of Lemmon but his remarks are also applied to Hintikka's position.

20. *Ibid.*

21. See " 'On knowing oneself' and other problems in epistemic logic," *Theoria*, vol. 32 (1966), pp. 1-13.

22. Hintikka, *Knowledge and Belief*, p. 36.

23. Hintikka, " 'Knowing oneself' and other problems in epistemic logic," p. 2.

24. See previous note.

25. *Knowledge and Belief*, pp. 34-35.

26. See " 'Knowing oneself' and other problems in epistemic logic," p. 3.

27. *Ibid*.

28. See, e.g., Bas van Frassen, *Formal Semantics and Logic*, The Macmillan Co., New York (1971), pp. 2-5.

29. Ackermann is discussing the notion of 'belief' but the point is the same.

30. Robert J. Ackermann, *Belief and Knowledge*, Doubleday and Co., New York (1972), p. 8.

31. Hocutt, *op. cit.*, p. 437.

32. See, e.g., *Knowledge and Belief*, p. 10.

33. Hintikka, "Epistemic logic and the methods of philosophical analysis," p. 5.

34. Jaakko Hintikka, " 'Knowing that one knows' reviewed," *Synthese*, vol. 21 (1970), p. 141.

35. *Ibid*., pp. 142 ff.

36. See Hocutt, *op. cit.*, p. 437.

37. *Ibid*.

38. Hocutt, *op. cit.*, p. 438.

39. Hintikka, *Knowledge and Belief*, pp. 38-39.

40. Hocutt, *op. cit.*, p. 438.

41. Hocutt concedes that "Hintikka enlarges this circle a little, but not much." See p. 437.

42. *Ibid*., pp. 438-439.

43. See, e.g., *Knowledge and Belief*, p. 158.

44. *Ibid*., p. 156.

45. Hocutt, *op. cit.*, p. 440.

46. *Ibid*., p. 441.

47. Hocutt, *op. cit.*, pp. 442 ff.

48. *Ibid*., p. 442.

49. Hintikka, *Knowledge and Belief*, pp. 58-59.

50. Hocutt, *op. cit.*, pp. 442-443.

51. *Ibid*., p. 445.

52. Hintikka, *Knowledge and Belief*, p. 105.

53. Similar comments also apply to the application of the semantic tableau technique to modal logic—which Hocutt also finds unintelligible. See Hocutt, *op. cit.*, p. 438, n. 5.

54. Hocutt, *op. cit.*, p. 443.

55. *Ibid.*, p. 445.

56. *Ibid.*, p. 446.

57. See Hocutt, *op. cit.*, p. 449.

58. Hintikka, *Knowledge and Belief*, p. 6.

59. *Ibid.*, pp. 60 ff.

60. *Ibid.*, pp. 47 ff.

61. *Ibid.*, pp. 49 ff.

62. Hintikka, *Knowledge and Belief*, p. 50.

63. *Ibid.*, p. 67.

64. *Ibid.*, p. 72.

65. *Ibid.*, p. 77.

66. *Ibid.*, p. 74.

67. *Ibid.*, p. 6.

68. Hintikka, *Knowledge and Belief*, p. 8.

69. *Ibid.*

70. *Ibid.*, p. 111.

71. Hintikka, *Knowledge and Belief*, p. 20.

72. *Ibid.*, p. 5.

73. Hintikka, " 'Knowing that one knows' reviewed," p. 148.

74. *Ibid.*, p. 150.

75. Hintikka, " 'Knowing that one knows' reviewed," p. 153.

76. *Ibid.*, p. 154.

77. *Ibid.*, p. 158.

78. *Ibid.*, p. 154.

79. Hocutt says another interpretation could be given of the 'performatory' character of knowledge claims—one on which the assertion of '$K_a p$' is equivalent simply to the assertion of 'p'. On this view, $K_a p$ is said to be truth-functionally equivalent to p and epistemic logic either reduces to propositional logic or is committed to the absurd principle $p \supset K_a p$. See Hocutt, *op. cit.*, pp. 449-450.

80. Hocutt, *op. cit.*, p. 450.

81. Hocutt, *op. cit.*, pp. 450-451.

82. *Ibid.*, p. 451.

83. *Ibid.*

84. We have in mind of course the Gettier counter-examples and the discussion arising out of these. See, e.g., Ackermann, *op. cit.*, chapter 5.

85. See, e.g., the discussion in Hintikka's "Semantics for propositional attitudes," reprinted in his *Models for Modalities*.

86. Jaakko Hintikka, "The semantics of modal notions and the indeterminacy of ontology,"
     Synthese, vol. 21 (1970), pp. 408-424. See p. 422.

87. Hintikka, "Semantics for propositional attitudes," p. 90.

88. Ackermann, *op. cit.*, p. 97.

89. Føllesdal, *op. cit.*, p. 27.

90. Ackermann, *op. cit.*, p. 135.

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