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On the Possibility of the Creation of Particles by a Classical Gravitational Field

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In a recent article, Hawking [1] pointed out that "if T^{ab} satisfies a physically reasonable condition ... a space time ... which is empty at one time must be empty at all times". Knowing about annihilation of pairs of particles into gravitous and vice versa, he traces the discrepancy "to the difficulty of defining a local energy-momentum operator for the matter fields in a curved space-time" and "to the fact that we have quantised the matter fields but not the metric" (citations from [1] p. 301, 302 and 305). More precisely, he asserts that if at an initial moment of time the energy density T_0^0 is zero in some region, (as well as all other T_k^i) then it will remain zero there henceforth. The proof is based on the inequatity

$$T_0^0 \geqq |T_\alpha^\beta| \tag{1}$$

for the components of the energy-momentum tensor.

Our purpose in this note is to point out that the inequality (1), while valid for the part of T_k^i corresponding to particles already in existence, is in general violated by the part of T_k^i corresponding to the polarization which the classical gravitational field causes in the quantized particle vacuum (e.g. the electron-positron vacuum).

It is precisely the latter part of T_k^i which describes the creation of particles.

As a simple example, we discuss the creation of particles by a weak gravitational field in a linear approximation. Let

$$g_{ik} = g_{ik}^{(0)} + h_{ik} . \tag{2}$$

The weak perturbation h_{ik} gives rise, as a result of polarization of the vacuum, to a contribution to the energy-momentum tensor

$$T_{ik}^{(1)} \sim h_{ik}$$

this contribution can be calculated by the methods of quantum field theory. In an approximation linear in h_{ik}, it is described by the diagramm: We consider two boson creation, so that inverse propagation in time of the solid line is not necessary.

Considering the right hand side of equations we do not need a second wavy line (graviton propagator) to the left of the diagramm.

$T_{ik}^{(1)} =$  (3)

Where the wavy line stands for the field h_{ik}, the smooth lines are the propagators for the created particles, and the dots are the vertices for the interaction of these particles with the field. We shall not write out here the corresponding analytical expressions which are different for different sorts of particles. We emphasis only the fact that they are non-zero.

Inasmuch as it is question of principle we work with small perturbations of Minkowskian metric, $g_{00} = 1$, $g_{11} = g_{22} = g_{33} = -1$.

For a weak field, we may expand h_{ik} in plane waves and consider each Fourier component separately; thus we set

$$h_{ik} \sim e^{-ikx} \tag{4}$$

Then the relationship between $T_{ik}^{(1)}$ and h_{ik}, described by the graph (3)

$$T_{ik}^{(1)} = a_{ikem} h^{em} . \tag{5}$$

(The field components which can create particles are, of course, these with $k^2 = \omega^2 - \vec{k}^2 > 0$. For instance we must have $k^2 > 4m^2$ for the creation of an electron-position pair.)

In m the approximation linear in the field, $T_{ik}^{(1)}$ must satisfy an equation of continuity in the simple form

$$\frac{\partial T_i^{k(1)}}{\partial x^k} = 0 . \tag{6}$$

In this connection we note some symmetry properties of a_{ikem}. First of all we must have

$$a_{ikem} = a_{ikme} = a_{kime} = a_{meki} .$$

The first two equations are obvious from the definition; the third corresponding to the Onsager symmetry principle, follows directly from the diagramm (3). The Eq. (6) gives

$$k^i a_{ikem} = 0 \tag{7a}$$

and then also

$$k^e a_{ikem} = 0 . \tag{7b}$$

The last equations expresses the peculiar gauge invariance of the theory. In fact for an infinitesimal coordinate transformation (ξ_i is an arbitrary infinitesimal vector)

$$h_{ik} \to h_{ik} + ik_i \xi_k + ik_k \xi_i$$

the tensor $T_{ik}^{(1)}$, in an approximation linear in h and ξ, should be invariant; and this in ensured by (7b). Consequently a_{ikem} can be expressed in terms of two scalar functions

$$\begin{aligned} a_{ikem} = {} & a_1(k^2)\,[(g_{ik}^{(0)}k^2 - k_i k_k)\,(g_{em}^{(0)}k^2 - k_e k_m)] \\ & + a_2(k^2)\,[(g_{ie}^{(0)}k^2 - k_i k_e)\,(g_{km}^{(0)}k^2 - k_k k_m) \\ & + (g_{ke}^{(0)}k^2 - k_k k_e)\,(g_{im}^{(0)}k^2 - k_i k_m)]\,. \end{aligned} \tag{8}$$

For a plane wave with $k^2 > 0$ it always possible to choose a system of coordinates in which the field is homogeneous ($\vec{k} = 0, k^2 = \omega^2$). Then it follows from (6) that

$$-i\omega T_0^{0\,(1)} = 0; \qquad T_0^0 = 0\,.$$

This in any case contradicts (1) and resolves Hawking's paradox. In the next order of approximation in powers of h we have

$$\frac{\partial \overline{T_0^{0\,(1)}}}{\partial x^0} = \overline{\frac{1}{2}\frac{\partial h_{\alpha\beta}}{\partial x^0}\,T^{\alpha\beta\,(1)}} \tag{9}$$

where the bar over T signifies an average

In complex form

$$\frac{\partial T_0^0}{\partial x^0} = \frac{1}{4}\,\omega h_{\alpha\beta} h^{\gamma\delta} Im(a_{\gamma\delta}^{\alpha\beta})\,. \tag{10}$$

where Im means imaginary part.

The right-hand side of (10) is positive, and as one can easily show, is just the energy of the particles created by the field in unit time. We note also that the direct calculation of $Im \cdot a$ from the diagramm (3) leads to a convergent expression, and needs no renormalization.

From (10) it can be seen that in the chosen system of coordinates $T_0^0 \sim h^2$, while $T_\alpha^\beta \sim h$. This is fully analogous to the usual quantum-mechanical situation, when the change of conserved quantity say the energy of an atom is of order E^2 in an electric field E, while for example the dipole moment of the atom contains terms which are linear in the field. We note also the obvious analogy between formula (10) and the familiar expression for the dissipation of the energy of an electromagnetic field in a medium whose dielectric constant has an imaginary part

In conclusion we emphasize that in a free gravitational field for plane waves, of course, $k^2 = 0$ so that our discussion is concerned with the creation of particles by a gravitational field due to matter. $T_{ik}^{(1)}$ is then only a part of the full energy-momentum tensor – that part which is due to the field. This is, however, connected only with the linear approximation and we believe does not affect our main assertion. Already in the second order matter can be created even by a free gravitational field, e.g. by two plane transverse gravitational waves, with different spatial directions. A phenomenological description of this process can be developed in a similar fashion. The components which arise once again will not satisfy the inequality (1). It should here be emphasised that there can hardly be any doubt about the possibility of the creation of particles by a classical gravitational field. The problem of defining the local energy momentum operator in a slightly curved space has the same kind of difficulty as in electrodynamics; as well known, these difficulties are overcome by renormalisation technique, leaving finite expressions with all needed properties. Indeed, the quantum process of conversion of gravitons into particles is a possibility – as mentioned already in [1]. But bosons in large number constitute a classical field, and the conversion process may of course take place in the presence of large numbers of gravitons.

The creation of particles in a non-stationary metric may be important cosmologically; see Ref. [2] and [3]. In this connection our discussion of the possibility of calculating the probability of creation in the approximation of an unquantized gravitational field (i.e. – of an unquantized metric) has a central significance. A general discussion of pair-creation in external gravitational and electric fields can be found in the writings of one of the authors [4]. The importance of anisotropic metric evolution is stressed in [5].

References

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