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On the pressure dependence of the rate factor in Glen's flow law

1. INTRODUCTION

Using the terminology and notation of Greve and Blatter (2009), Glen's flow law, which describes secondary creep of polycrystalline glacier ice, reads

$$D = A(T, p) \, f(\sigma_e) \, t^D, \tag{1}$$

where $D = \operatorname{sym} \operatorname{grad} \mathbf{v}$ is the strain-rate or stretching tensor, $\mathbf{v}$ is the three-dimensional velocity vector, t^D is the deviatoric part of the Cauchy stress tensor, T is the absolute temperature, p is the pressure, $\sigma_e = \left[\frac{1}{2}\operatorname{tr}\left(t^D\right)^2\right]^{1/2} = \left(\frac{1}{2} t_{ij}^D t_{ij}^D\right)^{1/2}$ is the effective stress, $A(T, p)$ is the rate factor (softness) and $f(\sigma_e)$ is the creep function. The latter is commonly expressed as a power law,

$$f(\sigma_e) = \sigma_e^{n-1}, \tag{2}$$

and for the stress exponent n most frequently the value $n = 3$ is used (e.g. Hutter,1983; Cuffey and Paterson, 2010; Van der Veen, 2013, and references therein).

As for the rate factor, it is usually expressed in the form of an Arrhenius law,

$$A(T, p) = A_0 \exp\left(-\frac{Q + pV}{RT}\right), \tag{3}$$

where A_0 is the pre-exponential constant, Q is the activation energy, V is the activation volume and $R = 8.314 \, \mathrm{J \, mol^{-1} \, K^{-1}}$ is the universal gas constant.

It has often been stated (e.g. Rigsby, 1958; Paterson, 1994; Hooke, 2005; Greve and Blatter, 2009; Cuffey and Paterson, 2010) that the pressure dependence in Eqn (3) can be properly accounted for by dropping the activation volume V, while replacing the absolute temperature T by the temperature relative to pressure melting T',

$$T' = T - T_m + T_0. \tag{4}$$

In this expression, $T_0 = 273.15 \, \mathrm{K}$, and T_m is the pressure-dependent melting temperature of ice,

$$T_m = T_0 - \beta p, \tag{5}$$

where $\beta = 7.42 \times 10^{-8} \, \mathrm{K \, Pa^{-1}}$ is the Clausius–Clapeyron constant for pure ice (Hooke, 2005). Combining Eqns (4) and (5) yields

$$T' = T + \beta p, \tag{6}$$

and the simplified rate factor with the dependence on T' takes the form

$$A(T') = A_0 \exp\left(-\frac{Q}{RT'}\right). \tag{7}$$

2. CONSISTENCY OF COMPUTED AND MEASURED ACTIVATION VOLUMES

We now investigate whether replacing the rate factor in the form of Eqn (3) by that in the simplified form of Eqn (7) is consistent with measured values of the activation volume V. We start by demanding that Eqns (3) and (7) are equal, so that their exponential arguments are equal,

$$\frac{Q + pV}{RT} \overset{!}{=} \frac{Q}{RT'}. \tag{8}$$

Using Eqn (6), this is equivalent to

$$\frac{Q + pV}{RT} = \frac{Q}{R(T + \beta p)} = \frac{Q}{RT} \frac{1}{1 + \frac{\beta p}{T}}. \tag{9}$$

A short scaling analysis with $\beta = 7.42 \times 10^{-8} \, \mathrm{K \, Pa^{-1}}$, $p = 400 \, \mathrm{bar} = 4 \times 10^7 \, \mathrm{Pa}$ (appropriate for the thickest ice in recent ice sheets) and $T \approx 250 \, \mathrm{K}$ gives

$$\frac{\beta p}{T} \approx 1.2 \times 10^{-2} \ll 1, \tag{10}$$

so that, to a very good approximation (relative error $\sim (\beta p / T)^2 = O(10^{-4})$), Eqn (9) simplifies to

$$\frac{Q + pV}{RT} = \frac{Q}{RT}\left(1 - \frac{\beta p}{T}\right). \tag{11}$$

Solving this for the activation volume V yields

$$V = -\frac{Q\beta}{T}. \tag{12}$$

We compute numerical values of the activation volume found in Eqn (12) for the temperature range 220–273 K that is relevant for ice in terrestrial ice sheets and glaciers. For the activation energy Q, we follow Cuffey and Paterson (2010) and use $Q = 6 \times 10^4 \, \mathrm{J \, mol^{-1}}$ for temperatures below 263 K, and $Q = 1.15 \times 10^5 \, \mathrm{J \, mol^{-1}}$ for temperatures above 263 K. This yields a range of activation volumes (units of $\mathrm{m^3 \, mol^{-1}}$)

$$-2.02 \times 10^{-5} < V < -1.69 \times 10^{-5} \quad \text{for } 220 \, \mathrm{K} < T < 263 \, \mathrm{K}, \tag{13}$$

and

$$-3.24 \times 10^{-5} < V < -3.13 \times 10^{-5} \quad \text{for } 263 \, \mathrm{K} < T < 273 \, \mathrm{K}. \tag{14}$$
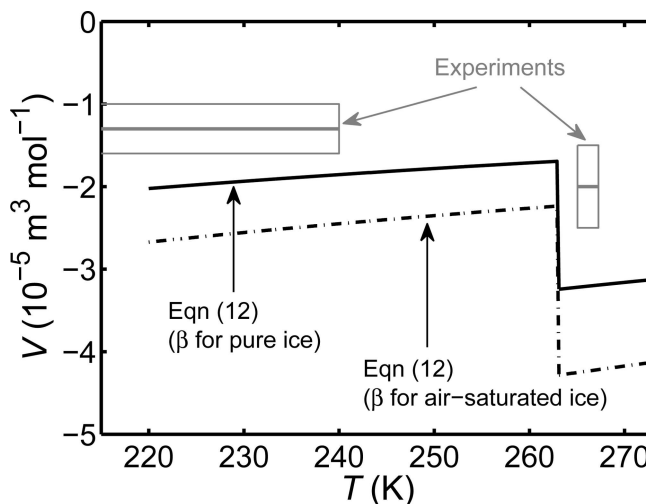
These results are illustrated in Figure 1.

Fig. 1. Computed (Eqn (12), with both values of the Clausius–Clapeyron constant β for pure and air-saturated ice) and experimentally determined (Weertman, 1973; Durham and others, 1997; error margins also shown) values of the activation volume V as a function of the temperature T.

Laboratory experiments on polycrystalline ice reported by Durham and others (1997) for the temperature range 195–240 K gave a value for the activation volume V of $(-1.3 \pm 0.3) \times 10^{-5} \, \text{m}^3 \, \text{mol}^{-1}$. Weertman (1973) analysed experiments for the temperature range –8.1°C to –5.9°C (265.05–267.25 K) and gave a value of $(-2 \pm 0.5) \times 10^{-5} \, \text{m}^3 \, \text{mol}^{-1}$. Therefore, the computed values of Eqns (13) and (14) are ~50% larger than the experimentally determined values. When the computation is done with the larger value $\beta = 9.8 \times 10^{-8} \, \text{K} \, \text{Pa}^{-1}$ of the Clausius–Clapeyron constant for air-saturated rather than pure ice (Hooke, 2005), this mismatch increases to about a factor 2 (see also Fig. 1). Nevertheless, taking into account the great difficulty in measuring this small effect, we consider this a reasonable agreement.

3. CONCLUSION

We have demonstrated that expressing the pressure dependence of the rate factor (Eqn (3)) of Glen's flow law only by a dependence on the temperature relative to pressure melting is consistent, within the rather large uncertainties, with measured values of the activation volume. Our findings include the possibility that this very common practice may over-predict the impact of pressure on ice deformability to some degree. However, until more comprehensive experimental data become available that will allow a more robust assessment, we conclude that it is a valid assumption to employ the rate factor in the simplified form of Eqn (7).

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