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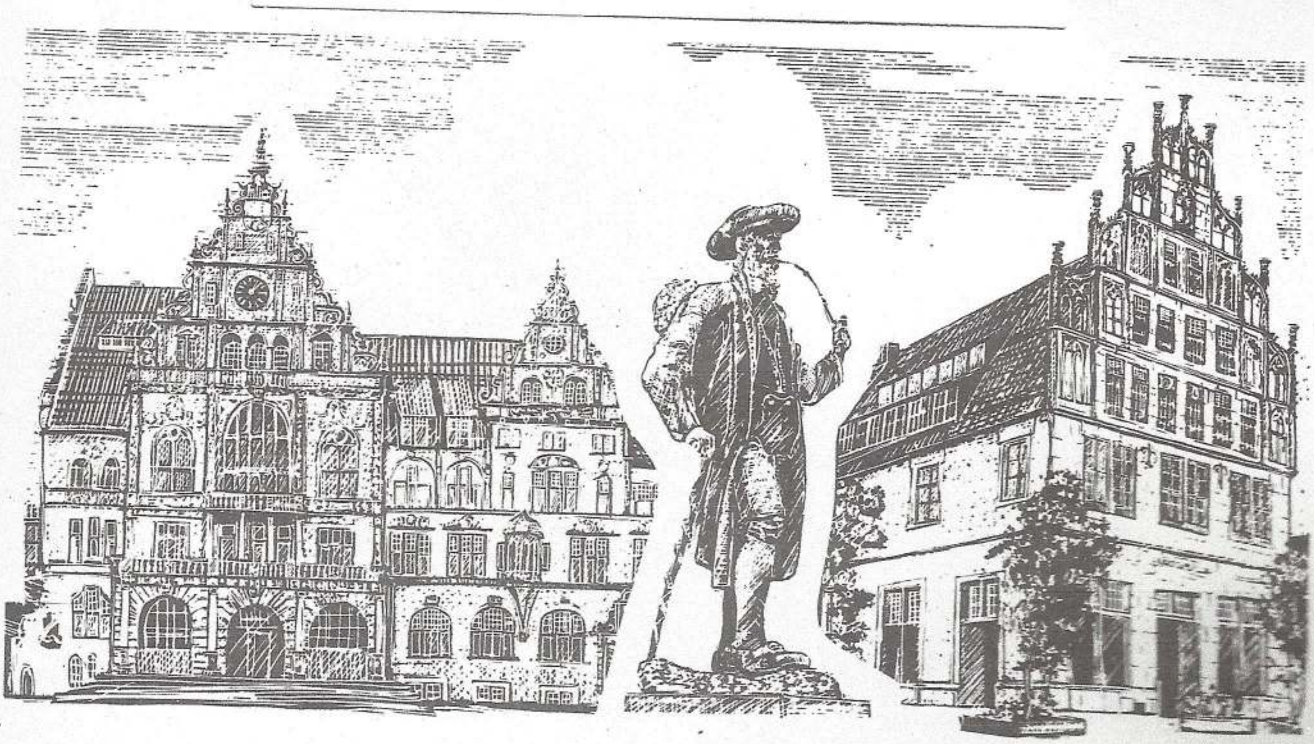

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On the Prominence Structure of the Decimal System

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Wulf Albers and Gisela Albers

Summary: This paper gives a first approach to a theory of the prominence of numbers of the decimal system.

Basic components of the theory are the limited rational principles of rule construction by iterated addition or subtraction of a given amount, the refinement of a scale by adding the means of any two neighbours, and its coarsening by omitting all uneven elements.

Applying these principles to the powers of 10 (i.e. the most prominent numbers of the decimal system) various scales with different degrees of exactness can be constructed. Empirical observations suggest that among different scales with about the same exactness generally that one is preferred which can be constructed in the easier way, where the construction principles are those from above and the coarsening procedure is assumed to have the same complexity as the two-fold refinement. This reduces the relevant scales to $(z \cdot a \cdot 10^n, z \in \mathbb{Z})$ with $n \in \mathbb{Z}$ and $a \in \{1,.5,.25 \text{ or } .2\}$.

The prominence of a number or set of numbers is defined as the length of the steps of the coarsest scale containing the number or the set. (For empirical applications it is suggested to permit 25 percent exceptions.)

Empirical results indicate that the prominence of the set of numbers resulting from a specific decision situation is in many cases about one tenth of the numbers in question.

More precisely the prominence (or exactness of scale) of a specific decision problem seems to be selected according to the rule that it is maximal subject to the condition that the range of reasonable alternatives contains at least three prominent numbers.

Results of n-person bargaining experiments (with real valued characteristic function games) suggest that the evaluation of outcomes (more exactly, of payoff differences) is done via the underlying prominence, such that the distance of two outcomes is evaluated by the number of prominence steps between them. By measureing incentives in this way a general theory of the probability that a "social" coalition (mainly an equal share coalition of all players) is formed could be developed for games of apex type, by which the empirical observation that the probabilities can be quite different if the same game is played on scales with different prominence can be explained. This result suggests that the prominence of the (spontaneously selected) scale can have basic importance for the analysis of individual or group decisions on numerical values.

1 Prominent Points of the Decimal System

The basic principles of constructing scales seem to be

(a) the iterated addition (and subtraction) of a given amount x, so that all integer multiples of x are obtained
(b) the refinement of a scale by adding the means of any two neighbours
(c) the coarsening of a scale by omitting all uneven elements.

In the decimal system the main reference points are the powers 10^n ($n \in \mathbb{Z}$), and according to principle (a) each of these powers produces a scale ($z \in 10^n$, $z \in \mathbb{Z}$) which can be refined or coarsened.

The first refinement of such a scale according to (b) gives the numbers ($z \cdot .5 \cdot 10^n$, $z \in \mathbb{Z}$), the second produces ($z \cdot .25 \cdot 10^n$, $z \in \mathbb{Z}$). These types of refinement seem to be empirically relevant. - A third application of (b) gives ($z \cdot .125 \cdot 10^n$, $z \in \mathbb{Z}$), but we did not observe sets of empirical data (with the character of decimals, not multiples of 1/8) in which the multiples of .125 were essentially more frequent than other multiples of .025. It seems that the system $(z\cdot .1\cdot 10^n, z\in\mathbb{Z}) = (z\cdot 10^{n-1}, z\in\mathbb{Z})$ is clearly preferred.

The coarsening principle (c) gives the systems $(z\cdot 2\cdot 10^n, z\in\mathbb{Z}) = (z\cdot .2\cdot 10^{n+1}, z\in\mathbb{Z})$ which can be chosen as an alternative of $(z\cdot .25\cdot 10^{n+1}, z\in\mathbb{Z})$, since both systems have about the same degree of exactness. - It seems that in a fixed situation each individual selects one of these two scales. That both of them have some empirical evidence can be seen from the fact that some currencies have pieces of value .02, .20, 2, and 20 (as Switzerland), while others introduced .25, 2.5, and 25 instead (as the Netherlands). - The two-fold application of principle (c) gives $(z\cdot 4\cdot 10^n, z\in\mathbb{Z})$ which, however, seems never to be used. The scale $(z\cdot 5\cdot 10^n, z\in\mathbb{Z})$ with a comparable degree of exactness seems to be clearly preferred.

The result of these considerations are the scales

$(z\cdot a\cdot 10^n, z\in\mathbb{Z})$
where $n\in\mathbb{Z}$ and $a\in\{1, .5, .25 \text{ or } .2\}$.

The length of the step $\Delta = 10^n$ of such a scale is called its *prominence*. We call a real number x *prominent with* Δ if it is contained in such a scale with prominence $\Delta' \geq \Delta$. A set M of real numbers is *prominent with* Δ if all (in empirical applications 75 % of all) elements of M are prominent with Δ and Δ is maximal with this property.

The idea is that every decision of an individual for a number is made on one of these scales considering only numbers with a given prominence where the prominence seems to be selected sufficiently high such that the set of alternatives which have to be checked as possible solutions of the decision problem remains sufficiently low.

We mention the following deviations from the principle that only scales of the types above are selected.

(1) In situations with more than one decision maker it is frequently considered to split the joint outcome equally. This leads to additional prominent numbers which, however, have not been used to generate scales according to principle

(a) in the cases we observed.

(2) If the decision task is to divide a given amount x among different people, it can happen that prominent divisors of x are selected to generate scales using principle (a) . (Some subjects selected a prominence of 30 , when x was 150 . - We explain this deviation from the theory by the assumption that the reference number x caused the subjects to leave the decimal system.)

(3) If the selected number is - as in price setting - such that others are supposed to react on them, then it can make sense to replace the numbers ($z \cdot a \cdot 10^n$, $z \in \mathbb{Z}$) by alternatives with a higher efficiency anticipating the reactions of the others. (Frequently 2.98, 2.78, 2.48, 2.28 are selected instead of 3.00, 2.75, 2.50, 2.25 , such that the prices are kept just below the prominence steps of the buyers. But there are different rules of this type, also depending on the degree of reflection related with the act of buying, and we do not want to go into these details here. In the following we shall only consider examples with an obvious transformation rule.)

2 The Relative Size of the Prominence

Empirical results give the impression that sets of data (as prices of comparable goods in the same shop, proposals in similar bargaining situations, etc.) quite frequently have a prominence of about 10 % of the numbers and the range is between 5 % and 25 % .

To give an impression of a set of possible prices resulting from such a behaviour, we consider as an example the case that each price is calculated with a relative prominence not below 5 % . (We suppose that each price is calculated separately and selected from a separate system ($z\,\Delta$, $z \in \mathbb{Z}$) with a prominence $\Delta = a\,10^n$ which is as low as possible, but not below 5 % of the finally selected price.) Then the seller can chose among the following numbers in the range between 1 and 10 : [1])

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[1]) We assume that a = .25 is prefered to a = .2 . - Possible prices between 10^n and 10^{n+1} are obtained by multiplication with 10^n .

1.00, 1.10, 1.20, 1.25, 1.30, 1.40, 1.50, 1.60, 1.70, 1.75, 1.80, 1.90 (A1)
2.00, 2.25, 2.50, 2.75, 3.00, 3.25, 3.50, 3.75, 4.00, 4.25, 4.50, 4.75 (A2)
5.00, 5.50, 6.00, 6.50, 7.00, 7.50, 8.00, 8.50, 9.00, 9.50 (A3)

In this system of possible prices there are structural breaks at 1.00 (where the prominence changes from .05 to .10, i.e. from 5 % to 10 %), at 2.00 (where the prominence changes from .10 to .25, i.e. from 5 % to 12.5 %), and at 5.00 (where the prominence changes from .25 to .50, i.e. from 5 % to 10 %). So the rule that no article is calculated with a higher relative prominence than 5 % causes relative prominences between 5 % and 12.5 %.

(Generally, a minimal relative prominence of α % causes relative prominences between α % and at most $2.5 \cdot \alpha$ % in the corresponding system of possible alternatives.)

Another example which is also comparable with observed price systems is the set of possible alternatives between 1 and 10, where each number is calculated with a minimal relative prominence of 10 %:

1.00, 1.25, 1.50, 1.75, 2.00, 2.25 (B1)
2.50, 3.00, 3.50, 4.00, 4.50 (B2)
5.00, 6.00, 7.00, 8.00, 9.00 (B3)

there the structural breaks are at 1.00, 2.50, and 5.00, where the relative prominences change from .10 to .25 (i.e. from 10 % to 25 %), from .25 to .50 (i.e. from 10 % to 20 %), and from .50 to 1.00 (i.e. from 10 % to 20 %).

These theoretical examples illustrate how prices with a relative prominence between 5 % and 25 % can occur. To underline the empirical evidence of these considerations we give some examples of observed price systems. Each of the examples gives all observed prices of a selected department of one shop or all prices of an advertisement or leaflet. (We remark that examples have been selected such that the prominence structure can easily be recognized.)[1]

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1) All prices have been observed in shops at Bielefeld in the first months of 1983.

Example 1: slabs of chocolate (department store)[1]

ideal	.85	.90	1.00	1.10	1.20	1.25	1.30	1.40	1.50	1.60	1.70	1.75
real	.85	.89	.99 .98	1.08	1.18	1.23	1.28	1.38	1.48	1.58 1.59 1.60	-	-
frequ.	1	1	3	2	3	1	5	2	2	3	-	-

ideal	1.80	(1.85)	1.90	2.00	...	2.25(?)	...	2.75(?)
real	1.78 1.80	1.85	1.90	1.98 2.00		2.30		2.70
frequ.	2	1	1	5		1		1

This price system fits quite well with the predictions of row (A1). Note that the structural break at 2.00 is also according to (A1).

It is interesting to note that price systems of different articles with different selling prices can have the same relative prominence. For example in two of the department stores we observed the prominence of television sets and portables was also comparable with (A1). They were prominent with 100 in a range between 800 and 2000, where exclusively prices of the type $z \cdot 100 + x$ with $z \in \mathbb{Z}$ and $x = 99$, 98 or 48 were observed, and in one store $x = 48$ occurred only in 5 out of 64 cases.

Example 2: unpacked pralines (department store)

ideal + real	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
frequency	1	1	4	3	1	3	3	5	-	8

ideal + real	4.75	5.00
frequency	7	1

This set is comparable with (A2). It should be noted that prices ending with .25 have been avoided and are essentially less frequent than those ending with .75 while in other cases the .25 values are preferred:

[1]) Articles offered with different tastes have been counted only once.

Example 3: leather blousons (department store)

ideal	200	225	250	275	300	325	350	375	400	425
real	195	-	250	-	295	325	350	-	395	425

Another candy has a considerably higher degree of prominence, comparable with (B3):

Example 4: chocolate eggs (department store)

ideal + real	40	(45)	50	(55)	60	(65)	70	75	80 ...	100 ...	120
frequency	10	1	4	2	14	1	7	11	3	1	1

That there are also price systems where the prominence 2 is preferred to 2.5 is illustrated by

Example 5: dress material (reduced prices of seasonal sales) (dept. store)

ideal + real	2	3	4	5	6	8	10	12	15	18	20	25	1)
frequency	28	24	7	91	17	76	19	18	11	6	4	1	

The relative prominence of these prices is comparable with (B) but it is exceptionally high at 2 and 3. This may be a consequence of the quick calculation in the sales situation or result from the principle to avoid broken numbers.

Another example of a price system according to (B) is

Example 6: shoes (discounter)

ideal	10.00	12.50	15.00	20.00	25.00	30.00	35.00	40.00	50.00	60.00
real	9.99	12.98	15.98	19.98	25.98	29.98	35.95	39.95	49.95	59.95
					25.00		35.00		49.50	59.50
					24.98					

ideal	70.00	75.00	80.00	90.00	100.00	110.00
real	69.95	75.95	79.95	89.50	-	109.00

The last example gives a price system which is somewhat more exact than (B). It permits insight into some additional conditions under which numbers with a lower prominence are selected:

Example 7: all articles of a leaflet with do-it-yourself materials and equipment (discounter) [1]

ideal	1.25	2.00	2.50	3.00	3.50	4.00	5.00	5.50	6.00	(6.25)
real	1.29	1.99	2.49	2.99	3.49	3.99	4.99	5.49	5.99	6.29
frequ.	1	1	1	3	1	1	5	1	3	1

ideal	6.50	7.00	7.50	8.00	9.00	10.00	12.00	12.50(?)	15.00
real	6.49	6.99	7.49	7.99	8.99	9.99	11.98	12.98	14.98
frequ.	1	3	3	2	7	5	1	3	2

ideal	17.00	17.50(?)	19.00	20.00	22.00	25.00	27.00	30.00	35.00
real	16.98	17.98	18.98	19.98	21.98	24.98	26.98	29.98	34.95
frequ.	1	2	2	7	2	4	1	3	3

ideal	40.00	45.00	50.00	55.00	60.00	65.00	70.00	75.00	80.00
real	39.95	44.95	49.95	55.00	59.95 [2]	65.00	69.95	75.00	79.95
frequ.	3	4	4	1	5	1	5	1	2

ideal	85.00	90.00	100.00	110.00	120.00	130.00	140.00	150.00	160.00
real	85.00	89.50	99.50	-	119.00	129.00	139.00	149.00	-
frequ.	1	1	4	-	2	1	3	4	-

ideal	170.00	180.00	190.00	200.00	... 240.00	... 260.00
real	169.00	179.00	189.00	198.00 [3]	239.00	259.00
frequ.	3	1	1	2	1	1

It should be remarked that the prices with a comparably high degree of exactness are set for articles to which similar or comparable articles existed with

[1] 15 articles from the food section which were in a separate board have been omitted.

[2] 1 article had the price 59.50.

[3] 1 article had the price 199.00.

prices with a higher prominence: curtain rails (with different lengths) had the prices 18.98 (120 cm), 19.98 (140 cm), 21.98 (160 cm) and 24.98 (180 cm), where the 140 cm and the 180 cm versions give the reference points 20.00 and 25.00. The prices of spotlights were

	pine	oak
single	19.98	21.98
two on a beam	39.95	39.95
three on a beam	55.00	59.95
three on a rondell	75.00	85.00

It can be easily realized that the price of the single pine version sets the reference point 20.00. The single oak version is calculated one step (10 % = 2.00) higher. The prices of the double and threefold versions are set cheaper than the corresponding multiples of the single versions, however, the selected degree of prominence did not permit a reduction of the double pine version. The price of the pine rondell version is again a number with a high prominence (75), whereas the oak rondell got a price which is one step (10) higher than 75.

These two examples explain most of the prices with a comparably low prominence, namely 18.98, 21.98 (2x), 55.00 and 85.00. Also the price of 6.29 (per **square** meter of rock wool) results from a price of 5.00 per running meter for the normal width of .80 meters.

3 The prominence selection rule

The observation that goods with the same sales price but different margins differ in the exactness of calculation suggests another approach to the prediction of the prominence of price or value systems. The main criterion on this approach is the range of reasonable alternatives of the decision problem.

> prominence selection rule: For a given range [x, y] of reasonable alternatives the prominence is selected in such a way that the range includes at least three numbers with his prominence.

Although this rule seems quite tough it points better predictions than the

10 % - rule of the preceding paragraph. To explain the behavioral ideas behind this rule we consider an example of price setting :

A seller reflects upon the adequate sales price of an article. From his experience he has some idea of possible prices on the market. He can describe the corresponding range of prices roughly by an upper and a lower bound. Generally, these bounds have a comparably high prominence. Now our assumption is that the seller selects the prominence of the scale in such a way that besides the extremes at least one alternative between them becomes prominent.

Figure 1 gives range and prominence for a selection of articles from the food market. The corresponding data were collected in 68 shops at Bielefeld in May 1982. We selected all branded articles of this survey which had a selling price above 2.00 and for which the prominence structure was not perturbed by a price recommendation. The range of reasonable alternatives is estimated by the range of observed prices omitting the 10 % tails of both sides.

In figure 1 the closed points refer to branded articles (special brands of coffee, honey, brandy, champagne, instant food and pre-cooked meal) while the circles belong to other goods (apples ("golden delicious"), tomatoes, cheap liquor and sliced bread of a certain quality).

The results show that the range has a size between $2\,\Delta$ and $5\,\Delta$ where Δ is the prominence of the set of actually selected prices. This supports the prominence selection rule permitting a maximal extension of the range of reasonable alternatives between $2\,\Delta$ and $5\,\Delta$, if the boundaries of the range are assumed to have at least prominence Δ. To illustrate this: if the range of reasonable alternatives is $\{x,\ 2 \leq x \leq 4\}$ then $\Delta = 1$ and the range has an extension of $2\,\Delta$, if the reasonable alternatives are $\{x,\ 1.5 \leq x \leq 3.5\}$ then $\Delta = .5$ and the range has an extension of $5\,\Delta$.

It is interesting that the nonbounded articles have a larger relative range ($> 5\,\Delta$, the difference to the branded articles is significant on the 0.02 level). This can be explained by the fact that these articles do not have a comparable homogenity of calculation. The only exception (cheap coffee) which is unbranded but has a relative range of $4.3\,\Delta$ seems to have a quite homogeneous calculation.

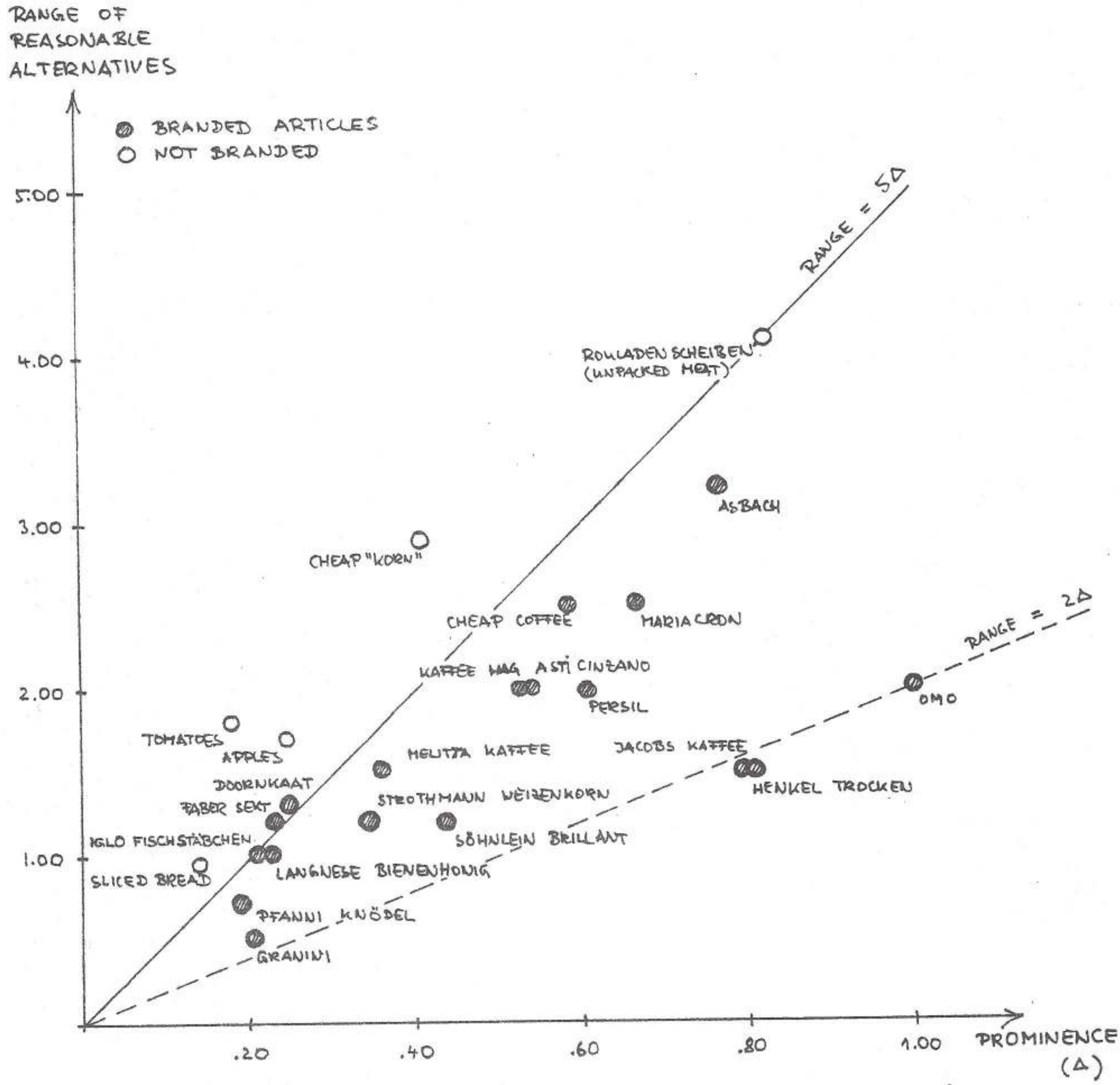

Figure 1: Range of reasonable alternatives versus prominence for some branded and unbranded articles.

4 Evaluation of payoff differences by means of the prominence

Results from game theoretical experiments indicate that the prominence structure of a decision problem does not only serve to select possible numerical alternatives for a decision but is also used to evaluate differences between possible outcomes. The following results which shall be reported in short seem to admit this interpretation.

We refer to a series of n-person characteristic function games with free communication and transferable payoffs. Each of these n-person games was played by n^2 players. Each player played the game n times in n subsequent rounds meeting no other player twice.

The games can be described by the set of players $N = \{1, 2, ..., n\}$ (here $n = 4$ or 5) and the payoffs of the coalitions $S \subseteq N$ which can be distributed among the players of the coalition. Here these payoffs were of the following type:

$$v(s) = \begin{cases} A & \text{if } A \in S \text{ and } s \setminus \{1\} \neq \emptyset \\ B & \text{if } s = N \setminus \{1\} \\ o & \text{otherwise} \end{cases}$$

where $a \gtrapprox B \geq A/n$ are positive real numbers.

In these games player 1 has a relatively strong position since he needs only one other player as a partner while the other players can coalesce without player 1 only if all of them stick together. However, this symmetric weakness of players 2, 3, ...,n causes them to consider the coalition $\{2, 3, ..., n\}$ as their first coalition alternative which is strongly supported by social arguments (compare ALBERS [1978]). Player 1 can try to corrupt one of the other to form a coalition with him.

Within the "social" coalition $\{2, 3, ..., n\}$ the payoff was distributed equally. In many cases this coalition has been extendea to the grand coalition $\{1, 2, ..., n\}$ again according to the equal share principle. There are two arguments for such an extension: one is not to exclude player 1 for social reasons, and the other is that the coalition is safer if no player is outside (the grand coalition N has not been broken in any of our experiments).

A central problem of the game was whether a social coalition should be formed or one of the two-person coalitions. The incentives for this decision can be characterized ex post by the mean payoffs x_1, x_i of player 1 and his partner in the two-player coalition and the outcomes a_1, a_i ($i = 2, 3, ..., n$) in the social coalition. Here we set $a_1 = a_i = A/n$ since in nearly all cases where a social coalition became stable the grand coalition N was formed. (Only in one sequence of games the players preferred $\{ 2, 3, \ldots, n \}$ to the grand coalition. For this sequence we defined a_i by $B/n - 1$.)

It seems reasonable to assume that the incentive of a player j to leave the social coalition and form a two person coalition is proportional to the corresponding payoff difference $x_j - a_j$. Empirical results show that this incentive (which can be measured indirectly be the proportion of non-social results in the final outcomes of the games) remains more or less unchanged if the constants A and B defining the value structure of the game are simultaneously replaced by 10 A and 10 B. On the other hand, they could be essentially influenced by the degree of involvement of the players in the game (measured by the relative prominence of the set of numerical proposals).

Both results can be explained by the assumption that the evaluation of the payoff differences is influenced by the prominence in such a way that the incentives to leave the social coalition are

$$d_j = \frac{x_j - a_j}{\Delta}$$

where Δ is the prominence of the analysis (measured by the prominence of the set of proposals). Of course, Δ should be measured individually but we had not enough observations for such an analysis. We, therefore, assumed that all players analyse the game with more or less the same prominence and we estimated Δ for each type of the game by the prominence of the set of all proposals of all players.

By this formula the incentive is given by the number of prominence steps between x_j and a_j, for instance if $x_j = 500$, $a_j = 300$ and $\Delta = 50$, then the incentive is 4, since there are 4 prominence steps (350, 400, 450, 500) from 300 to 500.

If in the same situation with the same expected outcomes another player (or group of players) only plays with a prominence $\Delta = 100$, then the incentive to leave the social coalition is only half as high, since for this prominence there are only two steps (400 , 500) from 300 to 500.

It can be seen from table 3 that in two cases the same game has been played with different prominences. In both cases the corresponding motivational situations became completely different and very different degrees of cooperation resulted.

We assume that the probability p_j of a player j to decide for the two-person coalition is proportional to his incentive d_j and our data show that the proportionality factor has the size of about .1 (least square analysis gives an optimal value of .094). So we obtain

$$p_j = .1 \cdot d_j = .1 \cdot \frac{x_j - a_j}{\Delta}$$

(since p_j is a probability, it must be replaced by 1 if the formula gives a higher value.)

Assuming that the decisions of the players between a two-person coalition and a social coalition are made independently, we get the following formula for the probability that a nonsocial coalition is formed.

$$p = p_1 \left(1 - \prod_{i=2}^{n} (1 - p_i) \right)$$

where $\pi(1 - p_i)$ gives the probability that all weak players decide to form a social coalition, $1 - \pi(1 - p_i)$ is the probability that at least one weak player decides for a nonsocial coalition and the final expression selects those cases where player 1 also decides to play in a nonsocial way.

Figure 2 gives the results of 8 such series of experiments. (They involved 155 subjects who played 155 games.) Δ has been measured as the prominence of the respective sets of proposals, x_j as the mean outcome in the two-person coalition of player 1 with another player, a_j is the equal share in the social coalition. Using these values our theory gives

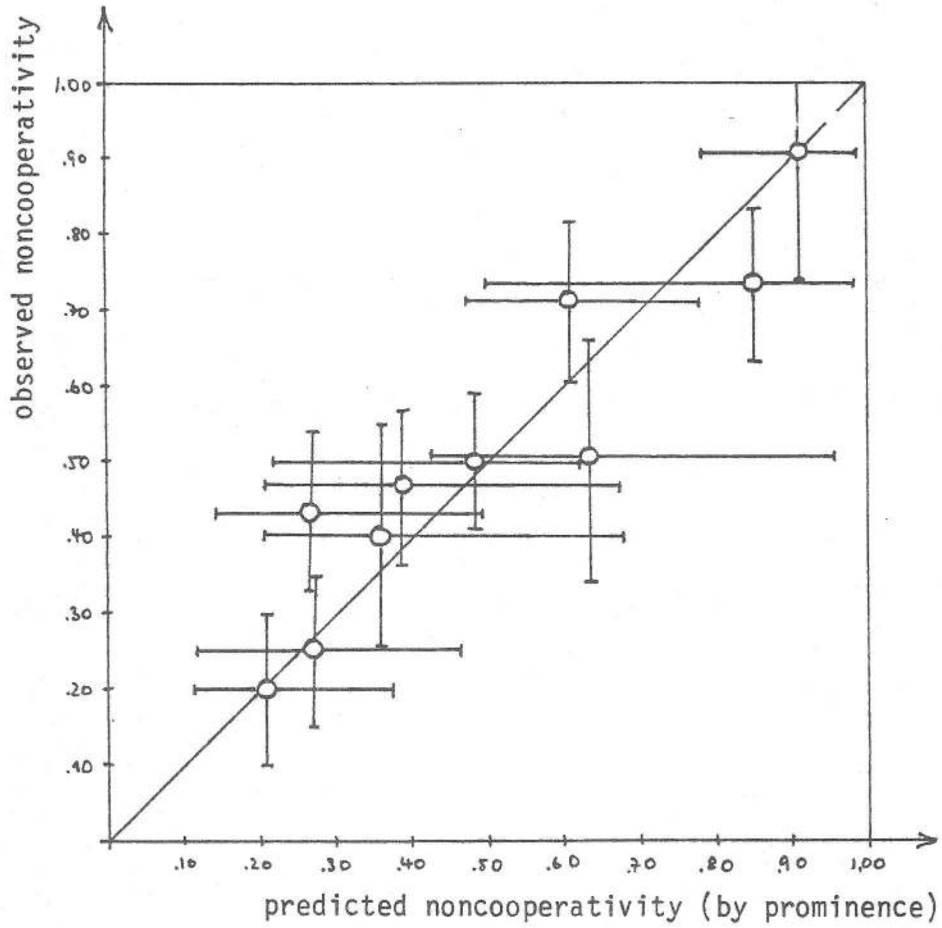

Figure 2: Predicted (by prominence) and observed noncooperativity in games of Apex type.

a prediction of the proportion p of noncooperative results. On the other hand, p can be measured by the percentage of nonsocial results if outcomes which do not involve the social coalition or the two-person coalition (these were less than 10 % of the results) are omitted.

The error of the percentage of nonsocial results is estimated by the assumption that 1.5 of the 15 - 25 results of a sequence of games could have a social instead of a nonsocial result or vice versa. The error of the prominence is estimated by the probability according to the geometric mean of the observed prominence and the next higher (respectively lower) promi-

				noncoop. results	
n	v(N)	v(234)	prominence	predicted	observed
5	100	100	5	.47	.40
5	1000	1000	20	.64	.71
5	1000	1000	50	.92	.91
5	1000	1000	50	.70	.50
5	1000	1000	100	.24	.20
4	100 [1]	75	10	.51	.50
4	160	165	5	.87	.73
4	160	165	10	.31	.43
4	320	315	20	.32	.25
4	1000	1000	50	.44	.47

Table 3: Predicted (via temptation) and observed degree of noncooperativity in games of Apex type.

nence. The error of the mean outcome x_j could be neglected because it was comparably small.

The data give a correlation of .83 of observed and predicted outcomes which is significant on the .01 level.

Maybe it cannot be finally decided whether the prominence really influences the evaluation of the payoff differences $x_j - a_j$ or if both, the evaluation of the differences and the prominence,depend on an underlying behavioral pattern. However, the observed results indicate that the prominence influences the evaluation since the error of p caused by the inexactness of Δ seems to be essentially lower than admitted by the "natural" error bounds given by the geometric means to the next higher or lower prominences.

[1] In this game we set $v(1) = 25$

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