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On the proper motion of the Magellanic Clouds and the halo mass of our Galaxy

D. N. C. Lin and D. Lynden-Bell
Lick Observatory, University of California, Santa Cruz and Institute of Astronomy, The Observatories, Cambridge CB3 0HA

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Summary. It is shown that the SMC lies close to the spin plane of the LMC.

There is a sufficient angle in the sky between the Magellanic Clouds that even if the LMC were moving purely transversely to the line of sight with the SMC moving with it, about one quarter of that transverse motion would be seen as a radial velocity of the SMC.

Supplementing this idea with other arguments we deduce that the Magellanic stream trails 'behind' the Magellanic Clouds.

Allowing for a possible halo to the Galaxy we compute models that last for 10^{10} yr and fit the observed velocities of the Magellanic stream. These give a large orbit for the Magellanic Clouds running from 50–200 kpc from the Galaxy. The stream was detached not during the current close passage, but at the last one. Differences of period have drawn out the stream during approach to the current passage which may have just severed the binary motion of the Magellanic Clouds. The 'circular' velocity required to produce a halo sufficiently massive to cause the Magellanic stream is $V_c = 244 \pm 20\,\mathrm{km\,s^{-1}}$ which must be maintained to distances of over 70 kpc from the Galactic Centre.

1 Introduction

Although we were able to simulate the Magellanic stream in good agreement with the observations there are several awkward implications of our previous work (Lin & Lynden-Bell 1977).

1. Since the orbital period was only 10^9 yr the Magellanic Clouds must have had about 12 close encounters with the Galaxy. How could the Clouds have survived so many close passages only to be violently torn into the Magellanic stream on the 13th time?

2. The SMC is 21° in the sky from the LMC. If they were ever much closer to the Galaxy it is unlikely that the SMC would not have been torn off the LMC along with the Magellanic stream (Toomre 1972), yet our earlier work had a perigalactic distance of only about 11 kpc.

3. We were forced to assign a high circular velocity for the Galaxy as well as a massive halo within 50 kpc. This involved an obvious inconsistency since we treated the Galaxy as a point mass while the halo must extend far beyond the small perigalactic distance.

In this paper we both account for the distributed mass of any possible heavy halo and we critically examine observations related to the orbit of the SMC about the LMC. We are led to quite different conclusions which avoid the difficulties with our former orbit mentioned above. The conclusions are closer to those of Fujimoto & Sofue (1969) and of Kunkel (1979) and are close to those first discovered by Murai & Fujimoto (1980) who have conducted a thorough campaign of calculation by backward integration.

Feitzinger, Isserstedt & Schmidt-Kaler (1977) were the first to determine observationally the sense in which the Magellanic Clouds move across the sky. They showed that the radial velocities of several classes of objects in the LMC determine a line of nodes some $20° \pm 5°$ different from the 'positional' line of nodes determined from the apparent axial ratio of the distribution of the objects. They assume these objects are really distributed in an axially symmetrical manner. They interpret the offset of the velocity line of nodes as due to the fact that the LMC itself subtends a large angle in the sky so that a purely transverse motion of its centre of mass would give radial velocity contributions at other points. The argument is ingenious and correct, but the method cannot be made very precise, both because it is hard to measure the offset accurately, and because any intrinsic ellipticity in the distribution would shift the apparent positional line of nodes from the true one. A barred spiral with its 'velocity centre' offset from its optical centre is not a promising candidate for assumptions of intrinsic circular symmetry (de Vaucouleurs & Freeman 1972). Our work will provide support for the conclusions of Feitzinger *et al.* (1977) by a method which is less sensitive to the precise position of nodal lines.

2 The transverse velocity of the Magellanic Clouds

Let the position vector from the sun to the LMC be $\mathbf{L} = |\mathbf{L}|\,\hat{\mathbf{L}}$. In galactic coordinates

$$\mathbf{L} = |\mathbf{L}|\,(\cos l \cos b, \sin l \cos b, \sin b) = 52\,(0.153, -0.826, -0.540)\,\text{kpc}.$$

similarly for the SMC

$$\mathbf{S} = |\mathbf{S}|\,\hat{\mathbf{S}}$$

$$S = 63\,(0.388, -0.602, -0.698)\,\text{kpc}.$$

Taking the LMC to be four times the mass of the SMC, the mass centre of the Magellanic Clouds is at

$$\mathbf{C} = |\mathbf{C}|\,\hat{\mathbf{C}} = 54.2\,(0.202, -0.790, -0.579)\,\text{kpc}.$$

Following the line of the Magellanic stream around the sky we find the direction along the stream towards **C**. At **C** this stream-line as seen in the plane of the sky is $\hat{\mathbf{s}} = (0.029, -0.586, 0.810)$. Now, after correction for the solar motion and the circular velocity of the Local Standard of Rest around the Galaxy, the velocity of **C** will consist of a transverse part v_{T} along $\hat{\mathbf{s}}$ and a radial motion v_{r} along **C**. Each Magellanic Cloud will have not only this motion of **C**, but also a velocity arising from their binary motion about $\hat{\mathbf{C}}$. If $\Delta\mathbf{v}$ is the velocity of the SMC with respect to the LMC, then the components of the binary motion about **C** seen in the Large and Small Clouds will be

$$\Delta v_{\mathrm{L}} = -\frac{1}{5}\Delta\mathbf{v}\cdot\hat{\mathbf{L}},$$

$$\Delta v_{\mathrm{S}} = +\frac{4}{5}\Delta\mathbf{v}\cdot\hat{\mathbf{S}}.$$

Now let v_L and v_S be the observed radial velocities of the mass centres of the Clouds after correction for the motions of the Sun around the Galaxy. Then, after further correction for the internal motions of the binary, these are components of the motion of **C**; thus

$$v'_L = v_L - \Delta v_L = (v_r \hat{\mathbf{C}} + v_T \hat{\mathbf{s}}) \cdot \hat{\mathbf{L}},$$

$$v'_S = v_S - \Delta v_S = (v_r \hat{\mathbf{C}} + v_T \hat{\mathbf{s}}) \cdot \hat{\mathbf{S}}.$$

We solve for v_T and v_r to obtain

$$v_T = 4.01\,(v'_L - v'_S) - 0.06\,(v'_L + v'_S) = 4.01\,(v_L - v_S) - 0.06\,(v_L + v_S) + 4.00\,\Delta\mathbf{v} \cdot \hat{\mathbf{D}}$$

$$v_r = 0.804\,v'_L + 0.206\,v'_S = 0.804\,v_L + 0.206\,v_S + 0.06\,\Delta\mathbf{v} \cdot \hat{\mathbf{D}}_1.$$

Here $\hat{\mathbf{D}}$ is the direction (0.346, −0.653, −0.674) which is not far from $\hat{\mathbf{S}}$ and $\hat{\mathbf{D}}_1$ is the direction (−0.667, −0.573, 0.476). Since the circular velocity at the Sun (Knapp, Tremaine & Gunn 1978) is not yet definitively determined we write it $V_c = 230\,\text{km s}^{-1} + \Delta V_c$.

The observed values of the radial velocities of the rotation centres of the Clouds are, after correction for the motions of the Sun in the Galaxy (see Table 1)

$$v_L = 72 \pm 3\,\text{km s}^{-1} - 0.826\,\Delta V_c,$$

$$v_S = 11 \pm 2 - 0.602\,\Delta V_c.$$

Thus the transverse and radial velocities of their mass centre are

$$v_T = 240 \pm 20\,\text{km s}^{-1} - 0.812\,\Delta V_c + 4\,\Delta\mathbf{v} \cdot \hat{\mathbf{D}},$$

$$v_r = 60 \pm 4\,\text{km s}^{-1} - 0.788\,\Delta V_c + 0.06\,\Delta\mathbf{v} \cdot \hat{\mathbf{D}}_1. \quad (1)$$

Since it is probable that $|\Delta V_c| < 30\,\text{km s}^{-1}$ corresponding to the range $200 < V_c < 260\,\text{km s}^{-1}$ those terms are unlikely to make a qualitative difference to either v_T or v_r. Likewise, the small coefficient of 0.06 means that v_r will be positive and much smaller than the 200 km s^{-1} observed as a radial velocity of approach at the tip of the Magellanic stream. This small positive radial velocity implies that the Magellanic Clouds have either just passed their closest approach to the Galaxy or that they are approaching their apogalacticon.

Now the Magellanic stream, if formed tidally, must be either a bridge pulled downwards towards the Galaxy and running ahead of the Magellanic Clouds due to its angular momentum, or a tail flung outwards and trailing. If the stream were a bridge then the Magellanic Clouds must lag behind the stream in its motion across the sky. This would imply that the Magellanic Clouds are approaching apogalacticon and that they were formerly still closer to the Galaxy, disastrously close for their binary motion, in fact, as we shall see. This 'leading stream' case has $v_T < 0$. If, however, the stream were a tail then its greater distance implies

Table 1. Radial velocities with respect to the Sun in km s^{-1}.

Authors	LMC			SMC	
	Optical centre	Rotation centre	Mean hydrogen velocity	Rotation centre	Mean velocity
Feast, Thackeray & Wesselink (1961)	260 ± 2	274 ± 2			166 ± 3
Kerr, Hindman & Robinson (1954)			280		160
Kerr & de Vaucouleurs (1955)		276	280 ± 1	160	161 ± 1
Average	260 ± 2	275 ± 2	280	160	162 ± 2
Adopted			278 ± 3		161 ± 2
Corrected for solar motion and galactic rotation	$v_L = 72 \pm 3 - 0.826\,\Delta V_c$			$v_S = 11 \pm 2 - 0.602\,\Delta V_c$	

more material and that a magnificent tidal tail has appeared just as the Magellanic Clouds are moving past perigalacticon. While this may sound natural to those who have not watched tidal tearing it is most exceptional. The big tides near perigalacticon are accelerations which produce velocities and these require time to produce the large displacements and spread out the tail. Thus tails appear well after perigalacticon. Furthermore, the observed tail has a zero in radial velocity only 30° from the Magellanic Clouds, so that the further 80° of observed tail has not yet reached its perigalacticon and must have been torn off still earlier. The way out of this conundrum is to have the Magellanic stream torn off not during the present close passage, but during the last one some 2×10^9 yr ago.

We now argue that a leading stream ($v_T < 0$) is most unlikely. To start with the maximum deprojected rotational velocity of the LMC is 70 km s^{-1}. The expected circular velocity of 60 km s^{-1} at the present separation of the Clouds is in agreement with de Vaucouleurs's estimate of $2 \times 10^{10} M_\odot$ for their combined mass. However, if the SMC were rotating with the circular velocity around the LMC in its plane, then the geometry is such that $|\Delta \mathbf{v} \cdot \hat{\mathbf{D}}| < 10 \text{ km s}^{-1}$, see Section 3. Even if the full 60 km s^{-1} is in the $-\hat{\mathbf{D}}$ direction that would only be sufficient to reduce v_T to zero, see equation (1). However, that cannot lead to a consistent orbital solution because with apocentres near 60 kpc orbits of very low angular momentum penetrate the Galaxy. The Magellanic Clouds can not have remained as a binary for 10^{10} yr in such an orbit. To make the binary safe for so long a period requires a large perigalacticon not far less than the present distance of the Clouds. For example, even with a tight SMC–LMC binary we have been unable to find orbits that preserve the binary and which have perigalactica anywhere near as low as 20 kpc. To keep the perigalacticon > 20 kpc requires a high angular momentum which in turn leads to $|v_T| > 110 \text{ km s}^{-1}$. Detailed modelling makes more serious demands here and it is probable that $|v_T| > 150 \text{ km s}^{-1}$ is a more realistic lower limit. To get v_T negative with a sensible orbit for the Clouds we need from equation (1)

$$\Delta \mathbf{v} \cdot \hat{\mathbf{D}} < -90 \text{ km s}^{-1}.$$

This implies that the SMC–LMC pair is no longer bound.

Such a conclusion is not necessarily unacceptable and we began our computations with orbits in this sense which is the one we previously advocated. We at once encountered a serious difficulty – one needs a rather eccentric orbit to give the relatively high ratio of the radial velocity of approach observed at the tip of the stream to the circular velocity. With a leading stream the Magellanic Clouds are now close to apogalacticon so an eccentric orbit requires a small perigalactic distance. However, a small perigalactic distance is very disruptive to the binary motion of the Clouds. We found it impossible to maintain the binary orbit for more than two or three passages when the perigalactic distance was as low as 20 kpc. On the other hand, eccentricities as low as 1/3 were already too low to produce high enough approach velocities at the stream tip (we previously advocated $e = 0.6$ or 0.7). The situation looked sufficiently hopeless that after several trials (200 particles per trial) with high inclination orbits and perigalactica between 15 and 30 kpc, we abandoned this sense of orbital motion in favour of orbits with a trailing stream. For these the Magellanic Clouds are now close to perigalacticon, so greater eccentricity causes no embarrassment, indeed it helps to preserve the binary because the greater period of the more eccentric orbits produces fewer close passages in 10^{10} yr. In the halo potential we got longitude–velocity profiles similar to those of the Magellanic stream, much more easily with it trailing than with it leading.

In the trailing case, the Magellanic Clouds have just passed perigalacticon. The SMC is now further from the Galaxy than the LMC and there has not been time since perigalactic passage to reverse this. Thus, any radial tidal kick during the close passage will have produced a positive contribution to $\Delta \mathbf{v} \cdot \hat{\mathbf{D}}$. Thus, v_T is probably greater than 240 km s^{-1} and positive.

One final point in favour of the stream being behind the Magellanic Clouds in their orbit, ($v_T > 0$), is that the large perigalactic distance automatically leads to a Magellanic stream which makes a near great circle in the sky. The offset of the Sun from the Galactic Centre nicely explains the small deviation whereas if parts of the stream were much closer, some extra cancelling effect would have to be invoked.

3 Magellanic geometry

Although the SMC and the LMC have their centres 21° apart in the sky, the existence of a common hydrogen envelope with a continuum velocity structure joining the Clouds shows that they must have been orbiting together for a long time. Their common envelope (Kerr, Hindman & Robinson 1954) is so spread out in the sky that only a weak tidal force from the Galaxy is needed to tear it off and so to cause the Magellanic Stream while leaving the Magellanic Clouds in binary motion.

We now estimate the plane of the orbit of the two clouds by first considering the rotation plane of the LMC. Table 2 lists determinations of the inclinations and position angles of the Large and Small Clouds. We have omitted determinations based on velocity measurements because these will be offset by the very transverse motion we wish to measure. The determination of the inclination by Gascoigne & Shobbrook (1978) is the only one that actually measures an inclination. They detect the difference in distance between the two sides of the LMC which is confirmed by Martin, Warren & Feast (1979). All the others rely on the guess that some structure is in reality round and that its observed ellipticity is due to its inclination. With irregular bodies such assumptions can lead to nonsense. However, de Vaucouleurs's

Table 2. Inclinations and position angles for LMC and SMC.

		i (°)	p_0 (°)
de Vaucouleurs (1955)	Red isophotes	25 ± 5	160 ± 20
de Vaucouleurs (1957)	Red isophotes	27 ± 3	170 ± 5
Hindmann, Kerr & McGee (1963)		35	
Westerlund (1964)	Outlying clusters	45	187 ± 10
McGee & Mitton (1966)	Clusters	29 ± 9	171
Feitzinger *et al.* (1977)	Red and blue isophotes	33 ± 3	168 ± 4
Gascoigne & Shobbrook (1979)	Cepheid magnitudes	27 ± 10	(170)
	Adopted	27 ± 7	170 ± 10
		i	p_0
de Vaucouleurs (1957)★	Counts and isophotes	60 ± 3	45 ± 3
Lynden-Bell★	Published photographs and isophotes by de Vaucouleurs and Elsasser (1958)	55 ± 10	45 ± 6
	Adopted	60 ± 15	45 ± 5

★ de Vaucouleurs mentions that the wing of the SMC causes difficulties in the accurate estimation of i but in his detailed study he manages to determine a result with a very small mean error. Unable to find alternative values with which to compare his result, we measured axial ratios off photographs and off published isophotes and counts. The brightest parts of the SMC are more elongated than the fainter parts where the wing causes difficulties. Allowing that the bright central region may be intrinsically elongated in roughly the same direction as the apparent major axis we were unable to reject 45° as a possible inclination. Our best value of 55° comes from a mean of measurements off de Vaucouleurs's photograph (Fig. 13 of de Vaucouleurs & Freeman 1972) and off the '50' contour of Elsasser's (1958) counts. de Vaucouleurs's value seems excellent but we do not agree in his assessment of the accuracy of this method of finding i.

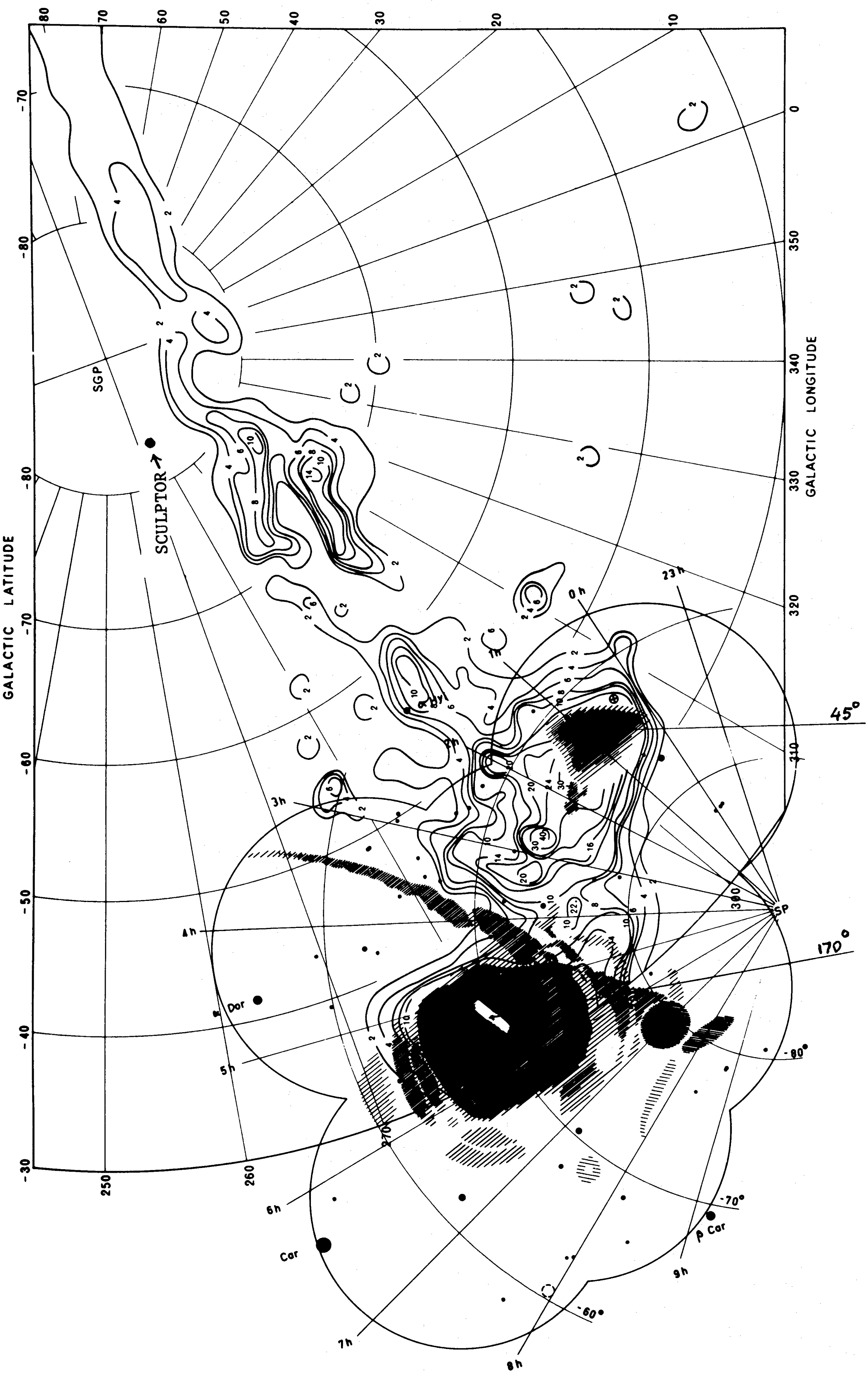

Figure 1. de Vaucouleurs's features of the deep photography of the Magellanic Clouds in celestial coordinates projected on to the galactic coordinated map of the Magellanic stream of Mathewson *et al.* (1974). The major axes of the Magellanic Clouds are indicated by their position angles (170°, 45°).

perspicacity appears to be well vindicated in both the value of the tilt and in the determination that the east side of the LMC is nearest to us. Thus the LMC as seen in the sky rotates clockwise.

Let the unit vector to the N celestial pole be $\hat{\mathbf{N}} = (-0.483, 0.745, 0.460)$. Then the unit vector pointing east at the LMC is

$$\hat{\mathbf{E}} = \hat{\mathbf{N}} \times \hat{\mathbf{L}} / |\hat{\mathbf{N}} \times \hat{\mathbf{L}}| = (-0.071, -0.557, 0.827) \tag{2}$$

Let the unit vector along the spin axis of the LMC be $\hat{\boldsymbol{\omega}}_L$ then using our knowledge of which side is nearest and the observed spin sense we have

$$\hat{\boldsymbol{\omega}}_L = \sin i \sin p_0 \hat{\mathbf{L}} \times \hat{\mathbf{E}} - \sin i \cos p_0 \hat{\mathbf{E}} + \cos i \hat{\mathbf{L}} = (0.027, -0.992, -0.125). \tag{3}$$

We want to know the angle between this and the line LS joining the LMC to the SMC

$$\vec{\mathrm{LS}} = \mathbf{S} - \mathbf{L} = \mathbf{a} = |\mathbf{a}|\hat{\mathbf{a}} = 23.9\,(0.706, 0.216, -0.675)\ \mathrm{kpc}. \tag{4}$$

The unit vector $\hat{\mathbf{a}}$ depends solely on the difference of the distance moduli of the Magellanic Clouds. This ought to be much easier to determine than the absolute distances but, nevertheless, there is a shocking spread in the estimates as Table 3 demonstrates. Accurate determination of the difference is most important for an improved distance scale for all extragalactic astronomy and it must be possible to clear up the discrepancies. The angle between $\hat{\boldsymbol{\omega}}_L$ and $\hat{\mathbf{a}}$ is given by

$$\hat{\boldsymbol{\omega}}_L \cdot \hat{\mathbf{a}} = -0.111 = \cos(96^\circ\!.3)$$

so the SMC lies only $6^\circ\!.3$ off the plane of the LMC. Repeating the calculation with Van den Bergh's distances yields $\hat{\boldsymbol{\omega}}_L \cdot \hat{\mathbf{a}} = +0.026 = \cos(88^\circ\!.4)$ only $1^\circ\!.6$ off the LMC's plane and on the opposite side of it.

Allowing for uncertainties in i we deduce that the SMC lies $6^\circ \pm 10^\circ$ off the plane of the LMC. It could certainly lie in the plane and we shall now explore the hypothesis that the SMC has always been close to this plane. Under this hypothesis we deduce that the orbital and spin planes are close and the nodal line of the plane of the SMC's orbit about the LMC must lie close to the nodal line of the LMC. Taking them to be the same, $p_0 = 170^\circ$, we find

$$\hat{\mathbf{a}} \times (\cos p_0 \hat{\mathbf{L}} \times \hat{\mathbf{E}} + \sin p_0 \hat{\mathbf{E}}) \propto \hat{\boldsymbol{\omega}}_M = (0.062, -0.968, -0.244), \tag{5}$$

where the last step is obtained by normalization to $|\hat{\boldsymbol{\omega}}_M| = 1$. The orbital spin, $\hat{\boldsymbol{\omega}}_M$, is 7° off the LMC's spin axis adopted (equation 3).

On this basis we can now estimate how the internal motion of the LMC–SMC binary affects their radial velocities as seen from the Sun. However, we first complete discussion of

Table 3. $\Delta\mu$ the difference of the distance moduli of the SMC and LMC.

	$(\Delta\mu)_{AB}$	$\Delta E(B-V)$	$\Delta\mu$
Sandage (1972)	0.44	0.06 ± 0.02	0.68 ± 0.13
Gascoigne (1969)	0.52	0.03	0.63
van den Bergh (1975)	0.40	0.04	0.56 ± 0.2
de Vaucouleurs (1978)	0.17 ± 0.20	0.04	0.33 ± 0.21
Graham (1977)	0.40 ± 0.10	0.03	0.52 ± 0.10
Martin (private communication)			0.36 0.11
Used here	(52 and 63 kpc Allen 1973)		0.42

the internal motions by considering the SMC's spin, $\hat{\omega}_s$, which we determine in a similar fashion to that of the LMC. This gives

$$\hat{\omega}_s = (-0.081, -0.992, 0.094) \tag{6}$$

which yield

$$\hat{\omega}_s \cdot \hat{\omega}_L = \cos 14^\circ.0, \qquad \hat{\omega}_s \cdot \hat{\omega}_M = \cos 21^\circ.2$$

with angular errors of about $\pm 15^\circ$. We speculate that both these angles might be zero so that $\hat{\omega}_s$, $\hat{\omega}_M$ and $\hat{\omega}_L$ are all parallel and equal to $\hat{\omega} \simeq (0.019, -0.996, -0.091)$ which gives $i'_{LMC} = 29^\circ$ but the rather discrepant values $i'_{SMC} = 48^\circ$ in place of 60° and $\Delta\mu = 0.58$. The nodal line of the SMC at $p_0 = 45^\circ$ is within 2° of the direction expected from this hypothesis. The difference in this value from p_0 for the LMC is due to their proximity to the South Celestial Pole. Fig. 1 demonstrates that their nodal lines lie almost 'parallel' in the sky. We shall not adopt the above speculation here.

We now return to the radial velocities expected from the internal binary motion. If v_i is the component of the binary orbital motion tangential to the line joining the Clouds then, taking the LMC to be four times the mass of the SMC, we expect to see a radial velocity in the SMC due to such motion of $4/5 v_i \hat{\omega}_M \times \hat{\mathbf{a}} \cdot \hat{\mathbf{S}} = -0.108 v_i$ and a velocity of the LMC due to binary motion of $-1/5 v_i \hat{\omega}_M \times \hat{\mathbf{a}} \cdot \hat{\mathbf{L}} = 0.032 v_i$. Taking $v_i = 60\,\mathrm{km\,s^{-1}}$ we deduce that the circular component of the binary motion gives contributions to the observed radial velocity of the LMC and SMC of only

$$\Delta v_L = +2\,\mathrm{km\,s^{-1}},$$

$$\Delta v_S = -6\,\mathrm{km\,s^{-1}}.$$

These give $\Delta \mathbf{v} \cdot \mathbf{D} = -8\,\mathrm{km\,s^{-1}}$.

Crude estimates of contributions from the eccentricity of the binary motion give slightly smaller contributions to $\Delta \mathbf{v} \cdot \hat{\mathbf{D}}$.

The above illustrative example shows that it is quite possible that $\Delta \mathbf{v} \cdot \mathbf{D}$ is small, but it has not taken into account the tidal disturbance to the orbit which is one of the main subjects of our more detailed simulations. The example also leads us to explore simulations in which the SMC was born as a condensation in the principal plane of the LMC. It is important to determine the inclination of the LMC with respect to the plane of its orbit about the Galaxy. Taking the Galactic Centre to be at (9, 0, 0) kpc we find the vector from that Centre to the barycentre of the Magellanic Clouds is $\mathbf{C}_0 = \mathbf{C} - (9, 0, 0) = 53.1\,(0.037, -0.806, -0.591)$. The direction of the angular momentum of the orbit around the Galaxy is given by

$$\hat{\mathbf{h}} \propto \hat{\mathbf{C}}_0 \times (v_r \hat{\mathbf{C}} + v_T \hat{\mathbf{s}}) = \hat{\mathbf{C}}_0 \times \mathbf{v},$$

although v_r and v_T are not yet determined accurately, nevertheless v_r must be considerably less than v_T and $\hat{\mathbf{C}}_0 \times \hat{\mathbf{C}}$ is small since these vectors are nearly parallel. Hence the direction of $\mathbf{h}$ is well determined and close to $\hat{\mathbf{C}}_0 \times \hat{\mathbf{s}}_i$ our best estimate is $\hat{\mathbf{h}} = (-0.995, -0.083, 0.051)$ which is good to 2°.

It follows that

$$\hat{\omega}_M \cdot \hat{\mathbf{h}} = \cos(89^\circ.6) \text{ and } \hat{\mathbf{C}}_0 \cdot \hat{\omega}_M = \cos 22^\circ,$$

$$\hat{\omega}_L \cdot \hat{\mathbf{h}} = \cos(87^\circ) \text{ and } \hat{\mathbf{C}}_0 \cdot \hat{\omega}_L = \cos 29^\circ.$$

Thus high inclination orbits follow from the hypothesis that the SMC was born in the outer parts of the LMC. In the next section we shall find orbits with such inclinations and orientations which keep the binary for 10^{10} yr.

4 Modelling the Magellanic orbit

A more detailed description of our procedure was given in our earlier work. A single massive sphere represents the LMC and is clothed with 200 test particles, one of which is later chosen to be the representative of the SMC. We take the view that it was the common hydrogen envelope around both that was torn to make the Magellanic stream. In our new computations we use the halo potential $\psi = -V_c^2 \log r$ for the Galaxy, so the galactic force law is $-V_c^2 \mathbf{r}/r^2$ with V_c the constant circular velocity.

Once again we compute in dimensionless units with $V_c = 1$ and the initial perigalactic distance equal to one. After a good fit was found in the dimensionless profile of radial velocity with angle, we looked for orbits that yielded similar inclinations and distributions after two, three or more passages. As found earlier, the effective size of the distribution of particles increased as the number of close passages increased. Toomre points out that tearing is enhanced when at pericentre the Galaxy is close to the spin plane of the Magellanic Clouds. It is those effects rather than orbital shrinkage due to dynamical friction that cause the eventual tearing. Nevertheless in the final computations we have incorporated dynamical friction in the equations of motion following the formulation given by Tremaine (1976).

The primary differences caused by the different gravitational potential were:

1. The smoother potential for the Galaxy makes its tides less jerky near pericentre. As a result the debris from the tidal tearing follows the original orbit much more closely than previously.

2. The orbits no longer close, and the rising and falling sectors of the v_r against angle ϕ are no longer symmetrical. The falling branch that brings the orbit through apogalacticon spreads over a smaller angle in the sky than the rising branch. As a result of 1 and 2 combined, it is now hard to get the 110° of almost straight line change in radial velocity seen along the Magellanic stream if its zero is assumed to be at apogalacticon. This provides another incentive for taking v_T positive, for then the observations fall on the longer rising branch and the slightly longer straight portion makes fitting easier.

3. As a result of 1, the most recent pericentre must now lie close to the point where the stream has zero velocity. This is some 30° from the current barycentre of the Magellanic Clouds and quite close to the South Galactic Pole. The stream extends some 70° further around the sky, thus the tidal tearing must have occurred a very long way prior to this pericentric passage. The only way we can get this to happen is to have the Magellanic Clouds' envelope torn near the previous pericentric passage. Both the Magellanic Clouds and the debris then move out to apocentre and get separated into a long thin stream during the subsequent reapproach to the Galaxy and the current pericentric passage (Figs 2 and 3). We found it relatively easy to get quite good fits to the velocities along the stream with this model (Fig. 4). Davies's 'age' for the stream (Davies, Buhl & Jafolla 1976) does not hold because:

(a) the large velocities now seen were generated fairly recently whereas the tearing is more ancient;

(b) any two particles of the stream will move in planar orbits about the galaxy with angular momentum directions that cannot differ greatly. The planes will cross, so the stream will appear to narrow in such regions.

4. We tried several inclinations but, even before we deduced the likelihood of $i = 90°$ from observations (Section 3), we found that 90° gave much better results than 135° or even 120°.

5. Our introduction of dynamical friction had the effect of decreasing the apocentric distance by about 5 per cent per passage.

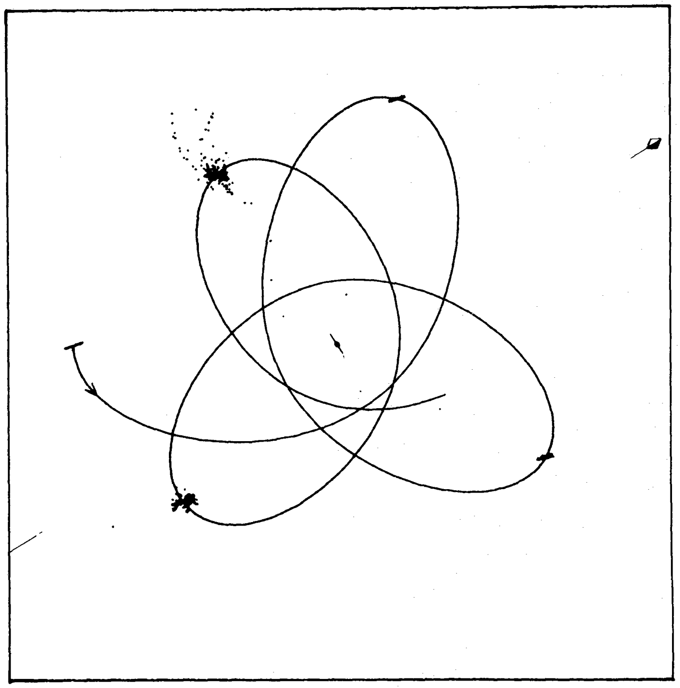

Figure 2. The orbit of the Magellanic Clouds for the last 10^{10} yr. The plane is the plane of the Magellanic Cloud's orbit seen looking back from the Galactic Anticentre. The Galactic Poles are indicated. The Sun lies almost above the Galactic Centre, 9 kpc from the plane of the figure.

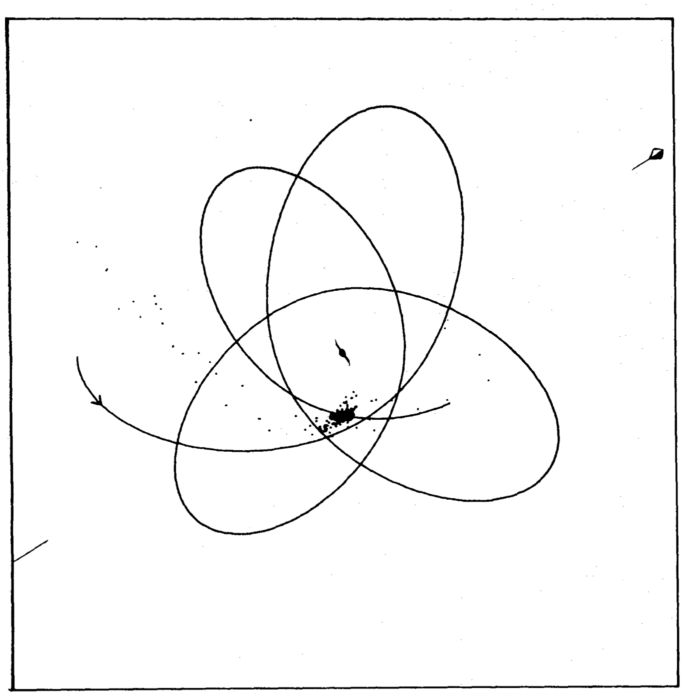

Figure 3. The current configuration of the Magellanic stream as calculated. S is the particle picked out to represent the SMC. The viewpoint is the same as Fig. 2.

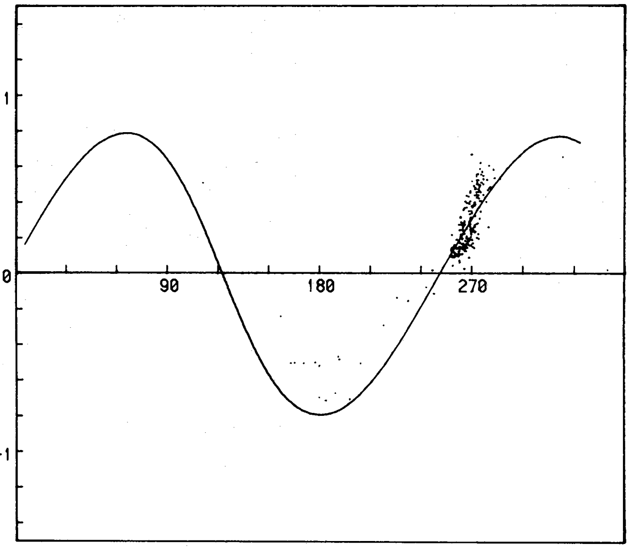

Figure 4. Current values of radial velocity in the simulation plotted against angle along the stream (in units of the circular velocity $\simeq 240\,\mathrm{km\,s^{-1}}$). Both velocity and angle are as seen from the Galactic Centre. Note the roughly linear variation of stream velocity with angle. The solid line is the velocity traced out by the LMC in its orbit.

6. The large apocentric distances make it somewhat unreasonable to take the circular velocity to be constant there. Thus, in our final results illustrated we have used the potential of a truncated halo

$$\psi = -\frac{1}{2} V_c^2 \log \left[\frac{(1 + r^2/r_h^2)^{1/2} - 1}{(1 + r^2/r_h^2)^{1/2} + 1}\right],$$

$$\nabla\psi = \frac{-\mathbf{r}\, V_c^2}{r^2\sqrt{1 + (r^2/r_h^2)}},$$

this corresponds to a spherical halo distribution of density

$$4\pi G\rho = \frac{V_c^2}{r^2(1 + r^2/r_h^2)^{3/2}},$$

where V_c is the circular velocity for $r \ll r_h$. The halo radius r_h was taken to be five times the pericentric distance. Experiments with r_h only three times that size were less successful. The total mass in this distribution is that which would be contained in a $1/r^2$ halo that terminated abruptly at r_h. Figs 2–4 give our best fitting orbit whose elements are described in Table 4.

We emphasize that the heavy halo is only necessary to explain the high velocities in the sparse material at the tip of the Magellanic stream and that we have assumed that gravity is the only force acting on it. Explanations with other forces may be possible (Mathewson, Schwarz & Murray 1977; Oort & Hulsbosch 1978; Hulsbosch 1978).

Our final figure shows the two Clouds and the stream as they will appear 10^9 yr hence. It is evident that the particle that most nearly models the position and velocity of the SMC now, has detached itself from the LMC. Our choice of eccentricity is quite well tied down. Smaller eccentricities give too little radial velocity to the tip of the stream while larger eccentricities send the Clouds well beyond 200 kpc at apocentre and such a long period that there is an excessive time between the tearing off of the Magellanic stream and its reapproach.

Table 4. The Magellanic orbit about the Galaxy.

P_r	2.3×10^9 yr; radial period
P_ϕ	3.3×10^9 yr; mean azimuthal period
r_{max}	200 kpc; apocentric distance from Galactic Centre
R_{min}	50 kpc at $l = 300°$ $b = -70°$; pericentric distance from Galactic Centre
h	19 000 $(-0.995, -0.083, 0.051)$ km s^{-1} kpc^{-1}; orbital specific angular momentum
$\Delta\phi$	260°; angle swept between successive apocentres
$\nabla\psi$	$= -V_c^2 (r/r^2) [1 + r^2/r_h^2]^{-1/2}$
V_c	$= 244 \pm 20$ km s^{-1}
μ_{LMC}	$= 9.7$ km s^{-1} kpc^{-1} = 0.0020 arcsec yr^{-1} due east; proper motion of LMC

If the best estimate of the LMC distance, L, changes, the predicted proper motion should be multiplied by $(L/52\text{ kpc})^{-1}$
Galactocentric radial velocity of barycentre = 0.31 V_c
Galactocentric transverse velocity of barycentre = 1.52 V_c
These give $\Delta\mathbf{v} \cdot \mathbf{D} = 36$ km s^{-1} and $v_T = 373$ km s.

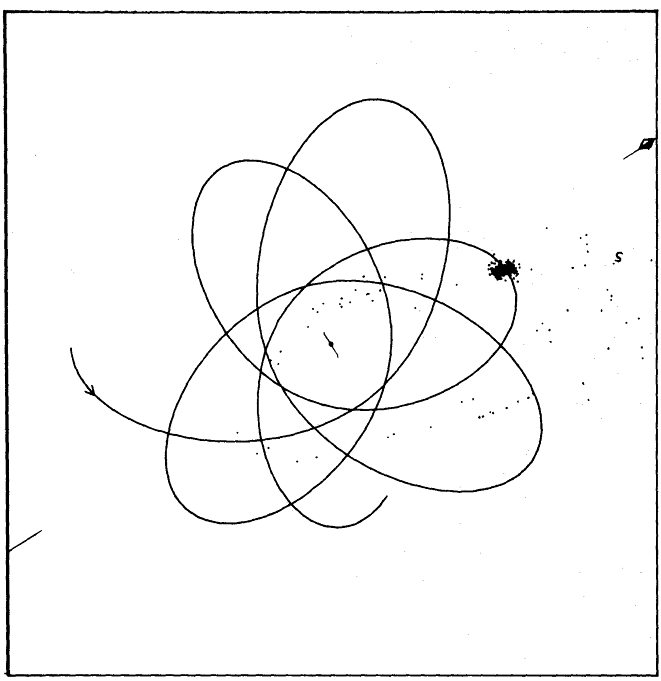

Figure 5. In 10^9 yr the LMC will reach its next apocentre. The SMC will lie behind it by some 70 kpc. The diagram shows the stream at that time.

This gives rise to unacceptably large gaps in the stream when it passes the Galaxy as we see it now. It is of interest to give the proper motion predicted for the LMC as this may be directly measurable by future astrometry

$$\mu_{LMC} = \frac{\mathbf{L} \times (\mathbf{v} - V_c\hat{\mathbf{C}} - \mathbf{v}_\odot)}{|\mathbf{L}|^3} \times \mathbf{L} = 9.7\text{ km s}^{-1}\text{ kpc}^{-1} = 0.0020\text{ arc sec yr}^{-1}\text{ due east.}$$

If we were to take no dark halo for the Galaxy then we cannot get a good fit to the stream without involving other forces, but it becomes consistent to take $\Delta\mathbf{v} \cdot \mathbf{D}$ small so that $V_T \simeq 240$ km s^{-1}. The corresponding proper motion of the LMC is $\mu_{LMC} = 0.0015$ arc sec yr^{-1} and is still within 3° of due east.

Our orbit is close to that of Murai & Fujimoto (1980) which differs only in its plane and gravitational potential from that advocated by Kunkel (1979). Our perigalactic distance is larger than that advocated earlier by Fujimoto & Sofue (1969) and the orbit is described in the opposite sense to both our former attempts and that of Davies & Wright (1977).

In Cartesian coordinates with origin at the Galactic Centre the heavy sphere that represents the LMC and the test particle chosen to represent the SMC have current positions and velocities given below. The distance unit is 50 kpc, the velocity unit is $V_c = 244\,\mathrm{km\,s^{-1}}$ and the time unit is their ratio

	x	y	z	v_x	v_y	v_z	t
LMC	-0.13	0.935	0	1.518	0.337	0	50.79
Initially	-4.0	0	0	0	4.107	0	0
SMC	-0.256	1.133	-0.095	1.527	0.509	0.150	50.79
Initially	-4.1128	4.1042×10^{-2}	-4.3676×10^{-2}	0.1044	0.3727	-0.3054	0

5 Determination of the circular velocity

As in our previous paper (Lin & Lynden-Bell 1977) we have computed in dimensionless units with our unit of velocity the circular velocity at the initial perigalactic passage which we took to be at unit radius. To fit these units to observations we use the current distance to the LMC and the observed radial velocity difference between the observed tip of the stream and the LMC. The latter is $636\,\mathrm{km\,s^{-1}} - 1.55\,V_c$. The computations make this 1.05 ± 0.1 units of velocity (see Fig. 4) which means $1.05\,V_c$ for the V_c = constant halo. Hence

$$(2.6 \pm 0.1)\ V_c = 636 \pm 20\,\mathrm{km\,s^{-1}},$$

$$V_c = 244 \pm 12\,\mathrm{km\,s^{-1}}.$$

The error is the fitting error of one dimensionless computation to the observed velocities; different dimensionless models give fits $10\,\mathrm{km\,s^{-1}}$ apart so the error should be doubled. This determination of V_c rests on the assumption that it is constant between the Sun and well beyond the Magellanic Clouds. A lowering of the velocity close to the Sun by $10\,\mathrm{km\,s^{-1}}$ would increase the circular velocity at and beyond 50 kpc by $14\,\mathrm{km\,s^{-1}}$. Other changes would be in proportion to those numbers.

Of these calculations, the most thorough numerical exploration is that of Murai & Fujimoto, which is in several respects a definite advance on anything we have attempted. They have three heavy bodies not just two and conduct a more complete search of orbits by backward integrations. However, in a field in which there has been as many different opinions as investigators, one indication of the truth is an emerging consensus between those who have held previously divergent views. We have come independently to a rather similar orbit and a similar picture for the origin of the stream. Fujimoto always advocated this sense of motion across the sky, whereas we did not and Murai & Fujimoto arrived at this sort of orbit first. There remain some differences in orbit:

1. Our orbit has larger apogalacticon and therefore longer period.

2. The way our search was conducted ensures that in the remote past the SMC was in an initially circular orbit around the LMC and we have found it possible to take that circle to lie in the plane of the LMC.

3. The particles that make up our Magellanic Stream are there for the whole history of the system and are not just introduced for the last two orbits. The latter procedure, used by Murai & Fujimoto, could lead to streams from material that would, in reality, have been removed on earlier encounters. Our procedure avoids that awkward possibility.

We hope our more observational remarks on Magellanic geometry and the lateral velocity produce some worthwhile further evidence as to the truth of the overall picture and we offer our prediction of the proper motion of the Magellanic Clouds as a more decisive test of it.

At one time we were wrongly led to believe that $\Delta \mathbf{v} \cdot \hat{\mathbf{D}}$ was so small that the transverse velocity v_T must be about $240\,\mathrm{km\,s^{-1}}$. This low speed lead us to try again point mass Newtonian orbits of similar eccentricity and inclination to the halo orbits figured above. It is interesting to record that although we expended a similar effort, we were not able to match the dimensionless parameters at all well in this case. The smoother tide of the halo potential is a real help. We believe this means that the halo must extend beyond about 70 kpc from the Galactic Centre.

6 Conclusions

As seen from the Galactic Centre, the orbit of the Magellanic Clouds runs over the Galactic Poles. The angular momentum vector of the orbital motion about the Galaxy lies close to the direction of the Galactic anticentre while the last pericentre of the orbit lies some 20° past the South Galactic Pole towards the Clouds. The spin of the LMC and the angular momentum vector of the SMC's orbit about the LMC lay almost in the direction opposite to the linear velocity of our circular motion about the Galaxy. The pericentric distance of the orbit of the Magellanic Clouds' barycentre is 50 kpc and the apocentric distance is some 200 kpc. The Galaxy must have a heavy halo out to at least 70 kpc if gravity alone is responsible for the velocities observed in the Magellanic stream. It is probable that the recent perigalactic passage has just freed the SMC from the LMC.

The circular velocity of our Galaxy is $244 \pm 20\,\mathrm{km\,s^{-1}}$ on the assumption that it is constant to great distances.

Prediction

The proper motion of the LMC will be 0.0020 arcsec $\mathrm{yr^{-1}}$ and due east. This is observable if many stars are used. It will be interesting to distinguish this from 0.0015 arcsec $\mathrm{yr^{-1}}$ which is the value expected if the Galaxy has no halo.

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References

Allen, C. W., 1973. *Astrophysical Quantities,* Athlone Press, London.
Davies, R. D., Buhl, D. & Jafolla, J., 1976. *Astr. Astrophys. Suppl.,* **23,** 181.
Davies, R. D. & Wright, A. E., 1977. *Mon. Not. R. astr. Soc.,* **180,** 71.
de Vaucouleurs, G., 1957. *Astr. J.,* **62,** 69.
de Vaucouleurs, G., 1978. *Astrophys. J.,* **223,** 730.
de Vaucouleurs, G. & Freeman, K. C., 1972. *Vistas Astr.,* **14,** 163.

Elsasser, H., 1958. *Z. Astrophys.*, **45**, 25.

Feast, M. W., Thackeray, A. D. & Wesselink, A. J., 1961. *Mon. Not. R. astr. Soc.*, **122**, 433.

Feitzinger, J. V., Isserstedt, J. & Schmidt-Kaler, Th., 1977. *Astr. Astrophys.*, **57**, 265.

Fujimoto, M. & Sofue, Y., 1969. *Astr. Astrophys.*, **61**, 199.

Gascoigne, S. C. B., 1969. *Mon. Not. R. astr. Soc.*, **146**, 1.

Gascoigne, S. C. B. & Shobbrook, R. R., 1978. *Proc. astr. Soc. Aust.*, **3**, 285.

Graham, J. A., 1977. *Publs astr. Soc. Pacif.*, **89**, 425.

Hindman, J. V., Kerr, F. J. & McGee, R. X., 1963. *Aust. J. Phys.*, **16**, 570.

Hulsbosch, A. N. M., 1978. *Astr. Astrophys.*, **66**, L5.

Kerr, F. J., Hindman, J. V. & Robinson, B. J., 1954. *Aust. J. Phys.*, **7**, 297.

Kerr, F. J. & de Vaucouleurs, G., 1955. *Aust. J. Phys.*, **8**, 508.

Knapp, G. R., Tremaine, S. D. & Gunn, G. R., 1978. *Astr. J.*, **83**, 1585.

Kunkel, W., 1979. *Astrophys. J.*, **228**, 718.

Lin, D. N. C. & Lynden-Bell, D., 1977. *Mon. Not. R. astr. Soc.*, **181**, 37.

Martin, W. L., Warren, P. W. & Feast, M. W., 1979. *Mon. Not. R. astr. Soc.*, **188**, 139.

Mathewson, D. S., Clearey, M. N. & Murray, J. D., 1974. *Astrophys. J.*, **190**, 291.

Mathewson, D. S., Schwarz, M. P. & Murray, J. D., 1977. *Astrophys. J.*, **217**, L5.

McGee, R. X. & Milton, J. A., 1966. *Aust. J. Phys.*, **19**, 343.

Murai, T. & Fujimoto, M., 1980. *Publs astr. Soc. Japan*, **32**, 581.

Oort, J. H. & Hulsbosch, A. N. M., 1978. *Astronomical Papers Dedicated to Bengt Stromgren*, ed. Reiz, A. & Andersen, T., Copenhagen University Observatory.

Sandage, A. R., 1972. *Q. Jl R. astr. Soc.*, **13**, 202.

Toomre, A., 1972. *Q. Jl R. astr. Soc.*, **13**, 266.

Tremaine, S. D., 1976. *Astrophys. J.*, **178**, 623.

van den Bergh, S., 1975. *Galaxies and the Universe*, p. 525, eds Sandage, A. R., Sandage, M. & Kristian, J., University of Chicago Press.

Westerlund, B. L., 1964. *IAU Symp. No. 20: The Galaxy and the Magellanic Clouds*, eds Kerr, F. J. & Rodgers, A. W., Australian Academy of Sciences.