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ON THE QUANTUM MECHANICS OF NEUTRINO OSCILLATION*

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ABSTRACT

We try to understand and to develop some of the basic quantum mechanics of neutrino oscillation. First, we observe that measurements which identify the physical neutrino (mass eigenstate) involved in each event of an experiment destroy any oscillation pattern. We explain how these measurements do that. Then, we construct a wave packet treatment of neutrino oscillation. We find that it gives the same results as the standard treatment. Next, we estimate the distance a beam must travel before its different physical neutrinos, which have different speeds, will stop interfering with each other. Finally, we consider the possibility of observing the difference in the arrival times of the various physical neutrinos at a given point.

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I. INTRODUCTION

In trying to understand neutrino oscillation, even at a simple level, one is soon confronted by some basic questions of principle about the quantum mechanics of this phenomenon. One such question concerns the role of physical neutrinos with definite masses in oscillation experiments. The neutrinos of definite flavor, such as ν_e and ν_μ, each of which is emitted in weak decays in association with a particular charged lepton, are supposedly linear superpositions of these physical neutrinos. However, one can make measurements, at least in principle, which determine the mass of the neutrino in each event of an experiment, thereby determining which of the physical neutrinos was involved. As we shall explain, when one knows which physical neutrino participates in each event, there cannot be any oscillation pattern. But then, how do the measurements which fix the neutrino mass destroy the oscillation?

In this paper we try to give a clear statement of this puzzle, and then we solve it. The nature of the solution calls attention to the fact that a correct general treatment of neutrino oscillations may require the use of wave packets. Therefore, we construct a wave packet treatment, and show that it leads to the standard results.[1] Finally, we reexplore the prediction[2] that, many oscillation lengths from their source, the different physical neutrinos in a beam will become incoherent and will arrive at a given detector at different times.

II. NEUTRINO OSCILLATION

To set the stage for what follows, we briefly present a slightly modified version of the usual description of neutrino oscillation.

Suppose that there are N physical neutrinos (mass eigenstates) ν_m, with nondegenerate masses M_m. Neutrino oscillation will then occur if the neutrinos ν_f of definite flavor (ν_e, ν_μ, etc.) are not the mass eigenstates, but linear combinations of them:

$$\nu_f = \sum_m U_{fm}\, \nu_m \quad . \tag{2.1}$$

Here U is an orthonormal mixing matrix.

In the standard treatment,[1] it is supposed that we have a beam of neutrinos all having a common, fixed momentum p_ν. It is also usually assumed that all the masses M_m are much smaller than p_ν. In this beam, the mass eigenstate ν_m will have energy $E_m(p_\nu) = \sqrt{p_\nu^2 + M_m^2} \cong p_\nu + (M_m^2/2p_\nu)$. If a neutrino in this beam is born with definite flavor f at time t = 0, then at that time its wave function will be

$$\psi(x,t{=}0) = \sum_m U_{fm}\, \nu_m\, e^{ip_\nu x} \quad . \tag{2.2}$$

After a time t this will evolve into

$$\psi(x,t) = \sum_m U_{fm}\, \nu_m\, e^{ip_\nu x}\, e^{-iE_m t} \quad . \tag{2.3}$$

This state is a superposition $\sum_{f'} \alpha_{ff'}\, \nu_{f'}$ of all the flavors. Since our neutrino is highly relativistic, if it was born at $x = 0$, then at time t it will be at $x \cong t$. At that point the wave function is

$$\psi(t,t) \cong \sum_m U_{fm}\, \nu_m\, e^{-i\left(M_m^2/2p_\nu\right)t} \quad . \tag{2.4}$$

From this expression, it is easy to show that the probability $\left|\alpha_{ff'}\left(p_\nu,x\right)\right|^2$ of finding the neutrino to have flavor f' at a distance x from its source, if originally it had flavor f, is[1]

$$\left|\alpha_{ff'}(p_\nu,x)\right|^2 = \sum_m U^2_{f'm}U^2_{fm} + \sum_{m'\neq m} U_{f'm}U_{fm}U_{f'm'}U_{fm'}\cos 2\pi \frac{x}{\ell_{mm'}} \quad . \quad (2.5)$$

Here the oscillation lengths $\ell_{mm'}$ are given by

$$\ell_{mm'} = 2\pi \frac{2p_\nu}{\left|M^2_m - M^2_{m'}\right|} \quad . \quad (2.6)$$

Note that the oscillating term in $\left|\alpha_{ff'}\right|^2$ comes from interference between the different mass eigenstates in the neutrino wave function.

III. WHEN THE NEUTRINO MASS IS MEASURED

Imagine performing the typical neutrino oscillation experiment sketched in Fig. 1. Here the neutrino source is a region in which pions in flight decay via the mode $\pi^+ \to \mu^+\nu_\mu$. Downstream from the source is a target-detector that looks for neutrino interactions which produce an electron. This experiment searches, in other words, for $\nu_\mu \leftrightarrow \nu_e$ oscillations. Suppose that these are found and that, in particular, when the source-to-detector distance x is varied, the oscillatory x-dependence predicted by Eq. (2.5) is observed.

Now imagine adding apparatus which measures the momenta of the pion and muon in each event. Suppose that these momenta are measured so precisely that they determine the mass-squared of the neutrino, M^2_ν, with an error $\Delta(M^2_\nu)$ less than all $\left|M^2_m - M^2_{m'}\right|$! Then one will know, for each event, which physical neutrino ν_m was involved.

In this situation, event rates at the detector clearly no longer oscillate with x. Now only a single physical neutrino, as ordinary a beam particle as a proton, contributes to any given event. One does not have the coherent contributions from several ν_m whose interference with one another is the origin of oscillatory x-dependence.

The events involving a particular ν_m will have a rate proportional to the probability for a π to decay to a μ together with this specific ν_m, $U^2_{\mu m}$, times the probability for this ν_m to produce an e in the target, U^2_{em}. While this rate does not vary with x, it does reflect the fact that the neutrinos of definite flavor are linear combinations of the mass eigenstates, since it would vanish otherwise.

The disappearance of oscillation with x as a result of momentum measurements which determine M^2_ν raises an interesting question. Namely, what do these measurements do to destroy the oscillation pattern? The answer to this question is given by the uncertainty principle. To determine M^2_ν one must measure the momenta of both the pion and the muon. Now, the more accurately the pion momentum is measured, the more uncertain the pion position will be. Consequently, the more uncertain the point where the neutrino is born will be. One might guess that just when the pion momentum is measured accurately enough for $\Delta(M^2_\nu)$ to be less than all $\left|M^2_m - M^2_{m'}\right|$, the uncertainty in the neutrino source point will exceed all the oscillation lengths $\ell_{mm'}$. Obviously, it will then be impossible to observe any oscillation pattern.

This guess is precisely correct. Indeed, we can easily show that, independent of the details of the neutrino source and detector, if measurements made at either place become sufficiently accurate to reveal which ν_m is involved in each event, then the consequent uncertainty in the neutrino source or detection point (depending on where the measurements are made) grows larger than all the oscillation lengths.

In a given event, the neutrino mass M_ν is related to the neutrino energy E_ν and momentum p_ν by

$$M_\nu^2 = E_\nu^2 - p_\nu^2 \quad . \tag{3.1}$$

If measurements from which E_ν and p_ν can be deduced are made with uncorrelated errors ΔE_ν and Δp_ν, then the error $\Delta(M_\nu^2)$ in the resultant value of M_ν^2 will be

$$\Delta\left(M_\nu^2\right) = \left[\left(2E_\nu\right)^2 \left(\Delta E_\nu\right)^2 + \left(2p_\nu\right)^2 \left(\Delta p_\nu\right)^2\right]^{\frac{1}{2}} \quad . \tag{3.2}$$

Now, to know which ν_m is involved in each event, we require that

$$\Delta\left(M_\nu^2\right) < \left|M_m^2 - M_{m'}^2\right| \tag{3.3}$$

for all m,m'. If this requirement is to be met, we see from Eq. (3.2) that the error in p_ν must satisfy

$$2p_\nu \, \Delta p_\nu < \left|M_m^2 - M_{m'}^2\right| \quad . \tag{3.4}$$

Then, depending on where the p_ν and E_ν measurements are made, either the neutrino source point or its detection point will have an uncertainty Δx obeying

$$\Delta x > \frac{2p_\nu}{\left|M_m^2 - M_{m'}^2\right|} \tag{3.5}$$

for all m,m'. From Eq. (2.6) we see that, apart from a factor of 2π, the right-hand side of this relation is precisely the oscillation length $\ell_{mm'}$.[3]

IV. WAVE PACKET TREATMENT OF OSCILLATIONS

As we have seen, the usual treatment of neutrino oscillation assumes that one has neutrinos of a single, fixed momentum p_ν. However, our discussion of momentum measurements and the uncertainty principle calls attention to the obvious fact that neutrino oscillations cannot be observed unless the neutrino source is localized within a region much smaller than the oscillation lengths $\ell_{mm'}$. But then, by the uncertainty principle, the neutrino momentum must have a spread of at least $\sim 1/h$, where h is the length of the source along the beam direction. Now, one can imagine determining the value of p_ν in each event in terms of a known target momentum and measurements of the momenta of the final state particles in the detector. However, one wants the error in the neutrino interaction point to be much less than an oscillation length, or much less than ℓ_{osc}, the shortest of the oscillation lengths, when there are several. Thus, any determination of p_ν had better involve an uncertainty Δp_ν obeying $\Delta p_\nu >> 1/\ell_{osc}$. This uncertainty may be smaller than 1/h, if h is tiny compared to ℓ_{osc}, but it must not be smaller than, say, λ/ℓ_{osc}, with $10 < \lambda < 100$. The amplitudes corresponding to different momenta within this minimal interval must be considered coherently. This means that a proper treatment of neutrino oscillation should use a wave packet of this minimal width in momentum space. Will such a treatment yield the usual results?

The fractional momentum-space width of this wave packet,

$$\frac{\Delta p_\nu}{p_\nu} = \frac{\lambda}{\ell_{osc}} \frac{1}{p_\nu} \sim \lambda \frac{|M_m^2 - M_{m'}^2|}{p_\nu^2} \quad , \qquad (4.1)$$

is infinitesimal for neutrino masses in the electron-volt range and typical neutrino momenta. Thus, the wave packet treatment will surely reproduce the results obtained assuming fixed p_ν, unless some factor in the packet has unusually rapid p_ν-dependence. To see whether anything unusual does happen, we construct the wave packet treatment. We do this also to help lay to rest obvious questions raised by the fixed-p_ν discussion. Namely, why should one assume that the differnt mass eigenstates ν_m in a beam have a common momentum but different energies? Why not assume they have a common energy but different momenta? Or different momenta and different energies? And what oscillation pattern is predicted if one does make one of these alternate assumptions? The wave packet treatment eliminates the need to make some idealizing assumption by taking both momentum and energy variations properly into account.

While the correct way to construct the wave packet description is not totally obvious, we believe one should proceed as follows. Consider neutrinos born in association with a definite charged lepton ℓ_f of flavor f in, say, π decay. If we neglect momenta transverse to the beam direction, then a given neutrino momentum p_ν corresponds, for any particular mass eigenstate ν_m, to a unique pion momentum $p_\pi^m(p_\nu)$. We then take the amplitude for creation of a neutrino of type ν_m and momentum p_ν to be $U_{fm}\,a[p_\pi^m(p_\nu)]$. Here U_{fm}, defined by Eq. (2.1), is the amplitude for production of ν_m in association with ℓ_f. The factor $a[p_\pi^m(p_\nu)]$ is the amplitude for the parent π to have the momentum $p_\pi^m(p_\nu)$ which leads to neutrino momentum p_ν in the decay $\pi \to \ell_f \nu_m$. One may think of the π as being confined in a box of dimension $h << \ell_{osc}$ along the beam direction so that the oscillation pattern to which its daughter neutrino contributes can be studied. The amplitude a for the π to have a given momentum then simply reflects its confinement within the box.

Suppose that an experiment studies neutrinos with momenta in the narrowest band allowed, with width $\Delta p_\nu = \lambda/\ell_{osc}$ around p_ν. (Recall that $10 < \lambda < 100$ and $\ell_{osc} \equiv \underset{m\neq m'}{\text{Min}} \left\{4\pi p_\nu / |M_m^2 - M_{m'}^2|\right\}$.) At the time of neutrino emission, $t = 0$, the amplitude for finding a neutrino ν_m in this band at a distance x from the neutrino source is

$$b_m(x,t=0) = \int_{p_\nu - \frac{1}{2}\frac{\lambda}{\ell_{osc}}}^{p_\nu + \frac{1}{2}\frac{\lambda}{\ell_{osc}}} dp_\nu' \; U_{fm} \; a\left(p_\pi^m(p_\nu')\right) e^{ip_\nu' x} \quad . \tag{4.2}$$

This corresponds to a neutrino state $\psi(x,t=0) = \sum_m b_m(x,t=0)\, \nu_m$, which will evolve after time t into

$$\psi(x,t) = \int_{p_\nu - \frac{1}{2}\frac{\lambda}{\ell_{osc}}}^{p_\nu + \frac{1}{2}\frac{\lambda}{\ell_{osc}}} dp_\nu' \sum_m U_{fm}\, a\left(p_\pi^m(p_\nu')\right) \nu_m\, e^{ip_\nu' x}\, e^{-iE_m(p_\nu')t} \tag{4.3}$$

$$\cong \int_{p_\nu - \frac{1}{2}\frac{\lambda}{\ell_{osc}}}^{p_\nu + \frac{1}{2}\frac{\lambda}{\ell_{osc}}} dp_\nu' \sum_m U_{fm}\, a\left(p_\pi^m(p_\nu')\right) \nu_m\, e^{ip_\nu'(x-t)}\, e^{-i\left(M_m^2/2p_\nu'\right)t} \quad . \tag{4.4}$$

Now, over the range of integration, the phase of the factor $e^{ip_\nu'(x-t)}$ varies by $(\lambda/\ell_{osc})(x-t)$. Thus, $\psi(x,t)$ will be appreciable only in the region $(x-t) \ll \ell_{osc}$, where this phase variation is not so large as to lead to cancellations. Therefore, let us specialize to the point $x=t$ in this region:

$$\psi(t,t) = \sum_m U_{fm}\, \nu_m \int_{p_\nu - \frac{1}{2}\frac{\lambda}{\ell_{osc}}}^{p_\nu + \frac{1}{2}\frac{\lambda}{\ell_{osc}}} dp'_\nu\, a\left(p_\pi^m\left(p'_\nu\right)\right) e^{-i\left(M_m^2/2p'_\nu\right)t} \quad . \tag{4.5}$$

Over the range of integration, the phase of the factor $e^{-i\left(M_m^2/2p'_\nu\right)t}$ varies by

$$\Delta\phi \simeq \frac{M_m^2}{2p_\nu^2}\, t\, \frac{\lambda}{\ell_{osc}} \quad . \tag{4.6}$$

For t of the order of the oscillation lengths or less, $\Delta\phi \ll 1$, and this factor may be removed from the integral. Then

$$\psi(t,t) \cong \sum_m U_{fm}\, \nu_m\, e^{-i\left(M_m^2/2p_\nu\right)t}\, g_m \quad , \tag{4.7}$$

where

$$g_m \equiv \int_{p_\nu - \frac{1}{2}\frac{\lambda}{\ell_{osc}}}^{p_\nu + \frac{1}{2}\frac{\lambda}{\ell_{osc}}} dp'_\nu\, a\left(p_\pi^m\left(p'_\nu\right)\right) \quad . \tag{4.8}$$

Note that if the integral g_m does not depend significantly on m, then Eq. (4.7) agrees with Eq. (2.4), apart from an overall factor of no consequence. Now, since the amplitude a reflects the localization of the pions within a region of size h, it will not vary appreciably until its argument changes by an amount of order 1/h. Thus, g_m will have negligible m-dependence if

$$\left| p_\pi^m(p'_\nu) - p_\pi^{m'}(p'_\nu) \right| << \frac{1}{h} \quad . \tag{4.9}$$

We shall demonstrate that this condition does indeed hold for nonrelativistic parent pions.

Recall that $p_\pi^m(p_\nu)$ is the π momentum which leads to neutrino momentum p_ν in the decay $\pi \to \ell_f \nu_m$ when momenta transverse to the neutrino beam direction are neglected. If the π is nonrelativistic,

$$p_\pi^m = \frac{p_\nu - p_m^*}{E_m^*} M_\pi \quad . \tag{4.10}$$

Here p_m^* and E_m^* are, respectively, the momentum and energy of ν_m in the pion rest frame, and M_π is the pion mass. If M_{ℓ_f} is the mass of ℓ_f, we have

$$E_m^* = \frac{M_\pi^2 + M_m^2 - M_{\ell_f}^2}{2M_\pi} \quad . \tag{4.11}$$

Then for fixed p_ν and M_{ℓ_f},

$$\frac{dp_\pi^m}{dM_m^2} = \frac{1}{2p_m^*}\left(\frac{M_\pi}{E_m^*} - 1\right) - \frac{1}{2E_m^*}\beta_\pi \quad . \tag{4.12}$$

Here β_π is the speed of the nonrelativistic pion, so the second term on the right-hand side is negligible compared to the first. Also, for small β_π, $p_m^* \cong p_\nu$, so Eq. (4.12) indicates that the values of p_π^m which correspond to two different neutrinos ν_m and $\nu_{m'}$ differ by

$$\left|p_\pi^m(p_\nu) - p_\pi^{m'}(p_\nu)\right| \cong \frac{1}{2p_\nu}\left(\frac{M_\pi}{E_m^*} - 1\right)\left|M_m^2 - M_{m'}^2\right| \quad . \tag{4.13}$$

Since $[(M_\pi/E_m^*)-1]$ is of order unity, $|p_\pi^m - p_\pi^{m'}|$ is of order $1/\ell_{mm'}$ [cf. Eq. (2.6)]. This implies that $|p_\pi^m - p_\pi^{m'}| << 1/h$, since $h << \ell_{mm'}$. Hence, we have shown in this example that g_m is indeed essentially m-independent. Then, apart from an insignificant overall factor, the neutrino state of Eq. (4.7) does agree with that of Eq. (2.4). Thus, the wave packet and fixed-p_ν treatments lead to the same prediction, Eq. (2.5), for neutrino oscillations.

In any practical experiment, the neutrino momentum spectrum will be considerably broader than λ/ℓ_{osc}. Amplitudes for momenta in non-overlapping bands of width λ/ℓ_{osc} need not be considered coherently with each other, of course, since in principle one can make final state measurements at the detector which determine p_ν to an accuracy of λ/ℓ_{osc} without disturbing the oscillation pattern. However, from Eqs. (2.5) and (2.6) we see that if there is a broad momentum spectrum of width Δp_ν, the oscillation pattern will be washed out for[4,5]

$$x > \frac{p_\nu}{\Delta p_\nu} \ell_{mm'} \quad . \tag{4.14}$$

This means that in any realistic experiment, oscillations will certainly be gone beyond, say, one hundred oscillation lengths. Now, Eq. (4.6) shows that when

$$x = t > \frac{2p_\nu^2}{M_m^2} \frac{\ell_{osc}}{\lambda} \quad , \tag{4.15}$$

the factor $e^{-i\left(M_m^2/2p_\nu'\right)t}$ can no longer be removed from the integral in Eq. (4.5), and the analysis which led to the wave packet result of Eq. (4.7) no longer goes through. However, this value of x is an enormous number of oscillation lengths; by the time this point is reached, oscillations have long since disappeared, and the wave packet treatment is no longer relevant.

V. SEPARATION OF MASS EIGENSTATES OF DIFFERENT SPEEDS

For highly relativistic neutrinos, normal spreading of the wave packet can easily be shown to be negligible. However, at a given momentum, the different mass eigenstates ν_m in the packet travel at different speeds. Nussinov[2] has made the very interesting point that, as a result, the packet will split into nonoverlapping pieces when the various ν_m components become more widely separated than the length d of the original packet. Once they no longer overlap, the different ν_m cannot interfere to produce neutrino oscillations. Nussinov estimated that $d \sim c\tau$ (with c the speed of light) when neutrinos are emitted by particles of lifetime τ, and collisions among the parent particles may be neglected.

Having discussed several aspects of the quantum mechanics of neutrino oscillations, we would now like to reexamine this question of separation of the different ν_m. First, we wish to argue that d will typically be less than $c\tau$. Consider, for example, neutrinos produced in $\pi \to \mu\nu$. In principle, one can record the arrival time of each μ at a detector a known distance from the pion decay region (assumed small), and from that arrival time infer the instant of decay, and hence the distance travelled down the beam line by the neutrino since the decay.[6] If the arrival time is measured with an accuracy of, say, 10^{-9} seconds, the neutrino position can be inferred to an accuracy of less than 5% of $c\tau_\pi \simeq 8$ m. A more striking example is provided by the neutrinos from the β decay of a nucleus with $\tau_{Nuc} \sim 1$ second. Here $c\tau_{Nuc} \sim 10^5$ km! Clearly, by observing the β particle, one can pin down the location of its associated neutrino to a region whose length is many orders of magnitude less than 10^5 km.

What, then, is the length of the wave packet? It is h, the length of the region within which the parent of the neutrino is effectively localized, either through preparation of the state of the parent, or by measurements of the decay fragments which accompany the neutrino, or both. If we are interested in a neutrino emitted at time $t = 0$, but we can learn only that the emitter was somewhere in a region of length h, then the amplitudes for the emission to have occured at $t = 0$ at the various points in this region must be added coherently. Thus, the neutrino wave packet will have a length $d \sim h$. In Section IV, we have in Eq. (4.3) an explicit wave packet corresponding to a pion in a box of length h. If we specialize to the simple case of a nonrelativistic pion, and extend the momentum integration to include the full range where $a[p_\pi^m(p'_\nu)]$ is appreciable, we easily find that the length of the resulting packet is $\mathcal{O}(h)$. Finally, we may consider a gedanken experiment such as that sketched in Fig. 2. Here a slow pion is confined to a "box" of length h, and we neglect motion transverse to the beam direction. A muon detector a known distance ℓ upstream from the end of the box records, with infinite precision, the time of arrival t_A of the μ from the π decay. As remarked earlier, such a set-up can reduce the uncertainty in the neutrino position at time t_A and thereafter to well below $c\tau_\pi$. However, since the decay occurs at an unknown point in the box, there will obviously still be an uncertainty of order h in the neutrino position, so once again we conclude that the neutrino wave packet has length $d \sim h$.

Now, suppose a neutrino is born at t and $x \sim 0$ with some definite flavor, and with reasonably sharp momentum p_ν. If two mass eigenstates ν_1 and ν_2 have speeds β_1 and β_2, respectively, at this momentum, then the ν_1 and ν_2 components of the neutrino wave packet of length h will no longer overlap when $t\beta_1 - t\beta_2 > h$, or when

$$x > \frac{h}{\beta_1 - \beta_2} \cong h \frac{2p_\nu^2}{|M_2^2 - M_1^2|} \sim (hp_\nu)\, \ell_{12} \quad . \tag{5.1}$$

Here $\ell_{12} = 4\pi p_\nu / |M_2^2 - M_1^2|$ is the oscillation length corresponding to the $M_2 - M_1$ mass difference. Now, current neutrino oscillation experiments involve values of p_ν between 1 MeV and 100 GeV. For p_ν in this range, $hp_\nu = 1$ for some h between 10^{-16} cm and 10^{-11} cm. Thus, even though ν_1 and ν_2 stop overlapping sooner than they would if d were $c\tau$ instead of h, Eq. (5.1) implies that for any macroscopic value of h, they will still continue to overlap for a tremendous number of oscillation lengths. Long before neutrino oscillations are eradicated by the separation of mass eigenstates of a given p_ν, they will have disappeared anyway due to the reasonably broad p_ν spectrum in any practical neutrino experiment.

On the other hand, it is interesting that after the mass eigenstates separate, they will arrive at any given detector at progressively more and more widely separated times as the beam continues to travel. Can this difference in arrival times be observed? For two mass eigenstates with masses M_1 and M_2 the difference is

$$\Delta t \cong x \; \frac{M_2^2 - M_1^2}{2p_\nu^2} \tag{5.2}$$

after the beam has gone a distance x. For $M_2^2 - M_1^2 = 1\ eV^2$ and $p_\nu = 10$ MeV, this becomes $\Delta t \sim 10^{-14}\ x$. If we assume that the shortest time interval that can be measured is 10^{-9} sec, then x must exceed 10^{10} km for $\nu_1 - \nu_2$ separation to be observable. Thus, unless $M_2^2 - M_1^2$ is much bigger than we supposed, this separation can be seen only among extraterrestrial neutrinos.

An intriguing suggestion to look for precisely this effect in the neutrino burst from a supernova has been made by LoSecco.[7] He points out that the distance from a supernova explosion in our galaxy to us can easily be of the order of 10 kiloparsecs, or 10^{17} km, since that is the galactic radius. For this value of x, $\Delta t \sim 10^{-2}$ sec if $M_2^2 - M_1^2 = 1\ eV^2$ and $p_\nu = 10$ MeV. Such a time interval is easy to observe. However, the proposed experiment is clearly difficult. There is, of course, the fact that supernova explosions are rather infrequent. Quite apart from that, we would like to raise two issues. First, if the neutrino has on average a mass M_ν, then the momentum spread Δp_ν of the neutrinos coming out of a supernova will by itself lead to a spread of arrival times

$$\Delta t\Big|_{\substack{\text{from}\\ \Delta p_\nu}} \cong x \frac{M_\nu^2}{p_\nu^2} \frac{\Delta p_\nu}{p_\nu} \quad . \tag{5.3}$$

For the experiment to work, this must be less than the difference of arrival times, Eq. (5.2), of neutrinos with different masses and the same momentum. We see that if M_ν^2 is of the order of $M_2^2 - M_1^2$, this will be the case if $\Delta p_\nu/p_\nu \ll 1$. Unfortunately, it appears that the neutrino spectrum emerging from a supernova is probably rather broad, with $\Delta p_\nu/p_\nu \sim 1$.[8] On the other hand, it may be that when the momentum-dependence of the cross section at the detector is taken into account, the effective spectrum is harmlessly narrow.[9] The second issue is the question of whether the neutrino pulse from the supernova is shorter than 10^{-2} sec, or at least has significant structure shorter than that, to begin with. It has been estimated that the great majority of the neutrinos in the burst stream out of the supernova during a period that lasts about 100 sec.[10] Thus, for the experiment to succeed, there must be some structure, or $M_2^2 - M_1^2$ must be somewhat bigger than we assumed. This experiment would indeed be a very interesting one; we hope it can be made to work.

VI. SUMMARY

We have tried to understand some of the quantum mechanics of neutrino oscillations. First, we considered an oscillation experiment in which particle momenta and energies are measured, either at the neutrino source or at the detector, with enough precision to determine which of the physical neutrinos ν_m is involved in each event.

When this is done, the oscillation pattern is wiped out, and we saw that this can easily be understood in terms of the uncertainty principle. Namely, when the momentum measurements are accurate enough to identify the ν_m in each event, they make the neutrino source point, or its detection point, more uncertain than an oscillation length.

This underlines the fact that neutrino oscillations cannot be observed unless the neutrino source and detection points are both localized to well within an oscillation length, so that there is necessarily some uncertainty in the neutrino momentum. We proceeded to take account of this momentum spread by constructing a wave packet treatment of neutrino oscillations. We found that this treatment gives the same results as the standard one, at least for distances less than an extremely large multiple of an oscillation length. Beyond that, it is not clear what the wave packet treatment gives, but at such distances there are no longer any oscillations anyhow.

We then examined the eventual separation of the ν_m from each other as a neutrino beam travels to large distances from its source. We argued that loss of interference between the ν_m will occur sooner than previously estimated. However, so long as the dimensions of the region within which the neutrino's parent is effectively localized are macroscopic, this loss will not occur until oscillations have already been washed out by the broad momentum spread in any realistic neutrino experiment.

Once the ν_m have separated, the difference between their arrival times at a given detector may be observable. For neutrino masses in the electron-volt range, this would be the case only for extraterrestrial neutrinos. For them, however, it remains an intriguing possibility.

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6. We are indebted to A. Goldhaber for this argument.
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8. R. Sawyer and several astrophysicists at the Institute for Theoretical Physics at Santa Barbara have assured the author of this.

9. We thank J. LoSecco for making this point.

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FIGURE CAPTIONS

Fig. 1. A typical neutrino oscillation experiment.

Fig. 2. A gedanken experiment in which pion confinement and muon detection determine the position of a neutrino, but with a residual uncertainty.

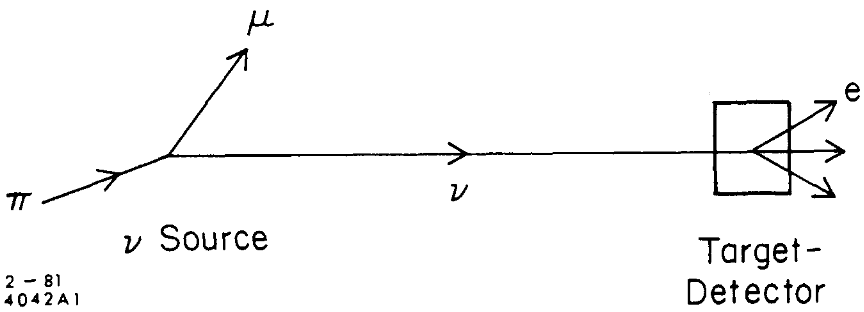

Fig. 1

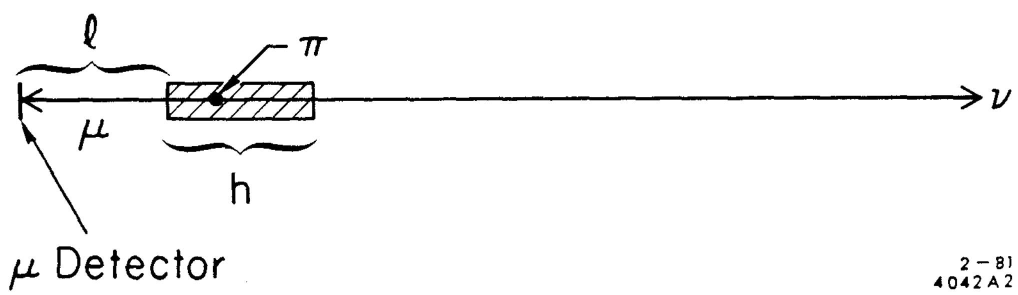

Fig. 2