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On the regularity of extension to strictly pseudoconvex domains of functions holomorphic in a submanifold in general position

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Dedicated to the memory of Jacek Szarski

Abstract. In this note we consider the regularity at the boundary of the extensions to strictly pseudoconvex domain D of functions holomorphic in certain subvarieties of D. Using the construction of Henkin [6] we show that if ∂D is sufficiently smooth, M' is a subvariety of some neighbourhood D' of $\bar{D}$ such that M' intersects ∂D transversally and has no singular points on ∂D, and f is in one of the spaces $A^k(M)$, $H^{\infty,k}(M)$ or $\Lambda_t(M)$, where $M = M' \cap D$, then there exists $F \in A^k(D)$, $H^{\infty,k}(D)$ or $\Lambda_t(D)$ respectively such that $F|_M \equiv f$. We show that for some class of subvarieties M', F can be given by an explicit integral formula.

We give also a theorem on the approximation of functions in $A^k(M)$, $H^{\infty,k}(M)$ and $\Lambda_t(M)$ by functions holomorphic in a neighbourhood of $\bar{M}$ in M'.

1. Notations. Let D be a bounded domain in C^n with $\mathscr{C}^1$ boundary. By $\mathcal{O}(D)$ we denote the space of all functions holomorphic in D.

For every $k = 0, 1, 2, \ldots$, set $A^k(D) = \mathcal{O}(D) \cap \mathscr{C}^k(\bar{D})$, where $\mathscr{C}^k(\bar{D})$ denotes, as usually, the space of all functions f of class $\mathscr{C}^k$ in D such that their derivatives of order $\leqslant k$ extend continuously to $\bar{D}$, and let $H^{\infty,k}(D)$ be the algebra of all functions holomorphic in D such that their derivatives of order $\leqslant k-1$ extend continuously to $\bar{D}$, and the derivatives of order k are bounded in D. $A^k(D)$ and $H^{\infty,k}(D)$ are Banach algebras with the norm

$$\|f\|_{D,k} = \sum_{|a| \leqslant k} \sup_D |D^a f|,$$

where $a = (a_1, \ldots, a_n) \in Z_+^n$ is a multiindex, $|a| = a_1 + \ldots + a_n$ and

$$D^a = \frac{\partial^{|a|}}{\partial z_1^{a_1} \ldots \partial z_n^{a_n}}.$$

We write $A(D) = A^0(D)$ and $H^\infty(D) = H^{\infty,0}(D)$.

Similarly, we set $A^\infty(D) = \mathcal{O}(D) \cap \mathscr{C}^\infty(\bar{D})$.

If t is a positive real number, which is not an integer, and k is a nonnegative integer such that $k<t<k+1$, let $\Lambda_t(D)$ denote the space of all functions $f\in A^k(D)$ such that for every $a\in Z_+^n$ with $|a| = k$, there exists a constant $c_a>0$ such that

$$|D^a f(z)-D^a f(z')|\leqslant c_a|z-z'|^{t-k}, \qquad z, z'\in D.$$

$\Lambda_t(D)$ is a Banach space with the norm

$$\|f\|_{D,t} = \|f\|_{D,k}+\sum_{|a|=k}\sup_{z,z'\in D}|D^a f(z)-D^a f(z')|/|z-z'|^{t-k}.$$

Let M' be a closed analytic subvariety of a domain D in C^n and let M be a relatively compact subdomain of M', such that ∂M is a manifold of class $\mathscr{C}^1$ and contains no singular points of M'. Let $A^k(M), k = 0,1,2,\ldots$ be the subspace of $\mathcal{O}(M)$, defined in the following way: Let $\{(U_i,\varphi_i)\}_{i=1}^l$ be any finite collection of the local coordinate maps on M', such that for each $i = 1,\ldots,l$, $\overline{U}_i$ is contained in the set of regular points of M', $\varphi_i\colon U_i\to V_i$ is a biholomorphic mapping of U_i onto an open neighbourhood V_i of zero in some C^{n_i}, $\partial(M\cap U_i)$ is of class $\mathscr{C}^1$, and such that $\partial M\subset\bigcup_{i=1}^{l}U_i$. We say that $f\in A^k(M)$ if $f\circ\varphi_i^{-1}|_{\varphi_i(U_i\cap M)}\in A^k\big(\varphi_i(U_i\cap M)\big)$ for every $i = 1,\ldots,l$. This definition is independent of the choice of the family $\{(U_i,\varphi_i)\}_{i=1}^l$. We may introduce the norm in $A^k(M)$, setting

$$\|f\|_{M,k} = \sum_{i=1}^{l}\|f\circ\varphi_i^{-1}\|_{\varphi_i(U_i\cap M),k}.$$

With this norm $A^k(M)$ is a Banach space. It is easy to check that any other choice of the maps $(U_1',\varphi_1'),\ldots,(U_{l'}',\varphi_{l'}')$ gives an equivalent norm.

We define the spaces $H^{\infty,k}(M)$ and $\Lambda_t(M)$ in the similar way.

A bounded domain $D\subset C^n$ is called a *domain* with $\mathscr{C}^k$ boundary if there exists a neighbourhood U of ∂D and a real-valued function $\varrho\in\mathscr{C}^k(U)$ such that

(i) $D = (D\setminus U)\cup\{z\in U\colon\ \varrho(z)<0\}$,

(ii) $\operatorname{grad}\varrho(z)\neq 0$ for $z\in\partial D$.

The function ϱ is called a *defining function* for D. D is called *strictly pseudoconvex* (with $\mathscr{C}^k$ boundary, $k\geqslant 2$) if the function ϱ can be chosen in such a way that it is strictly plurisubharmonic in a neighbourhood of ∂D. Moreover, if the defining function ϱ can be chosen so that $\varrho\in\mathscr{C}^k(C^n)$ satisfies the condition $\lim_{z\to\infty}\varrho(z) = +\infty$ and has a real hessian positive definite at every point of C^n, and $D = \{z\in C^n\colon\ \varrho(z)<0\}$, then D is called a *strictly convex domain* with $\mathscr{C}^k$ boundary ([5], p. 529).

2. The extension operators. Let D be a strictly pseudoconvex domain in C^n (with $\mathscr{C}^2$ boundary), and let M' be a complex subvariety of some neighbourhood D' of $\bar{D}$ such that M' intersects ∂D transversally and has no singular points on ∂D. G. M. Henkin proved in [7] that if M' is a complex manifold and $M = M' \cap D$, then there exists a continuous and linear extension operator

$$L\colon H^\infty(M) \to H^\infty(D)$$

such that $Lf \in A(D)$ if $f \in A(M)$. K. Adachi [1] and M. Elgueta [4] proved that $Lf \in A^\infty(D)$ provided $f \in A^\infty(M)$ and ∂D is of class $\mathscr{C}^\infty$. It is noted in [5] that the proof given in [7] extends to the case when M' has singular points, if none of these are on ∂D.

In this note we show that L has the same properties with respect to the spaces $A^k(M)$, $H^{\infty,k}(M)$ and $\Lambda_t(M)$ under the assumption that D has sufficiently smooth boundary:

THEOREM 1. *Let D, M' and M be as above.*

(a) *Suppose that ∂D is of class $\mathscr{C}^{k+5}$, $k = 1, 2, \ldots$ and $M' \cap \partial D$ contains no singular points of M'. Then $L\colon H^{\infty,k}(M) \to H^{\infty,k}(D)$ is a continuous extension operator. Moreover, $L(A^k(M)) \subset A^k(D)$.*

(b) *Suppose that $t > 0$ is not an integer and k is a non-negative integer such that $k < t < k+1$. Let ∂D be of class $\mathscr{C}^{k+6}$. Then $L\colon \Lambda_t(M) \to \Lambda_t(D)$ is a continuous extension operator.*

In the proof of the above theorem we use the same line of argument, as presented in [7], and the regularity properties of Fornaess' extension operator (see [5]), which were investigated in [8].

As a corollary, we obtain the following approximation theorem:

THEOREM 2. *Let M be a relatively compact subset of a reduced Stein space X, such that ∂M consists of regular points of X and M is strictly pseudoconvex with a $\mathscr{C}^k$ boundary, where k satisfies assumptions* (a) *or* (b) *of Theorem* 1. *Then there exists a neighbourhood M' of $\bar{M}$ in X, such that:*

(a) *Every function $f \in A^k(M)$ can be approximated in the topology of $A^k(M)$ by functions which are holomorphic in M'.*

(b) *For every function $f \in H^{\infty,k}(M)$ (resp. $f \in \Lambda_t(M)$) there exists a sequence $\{f_n\} \subset \mathcal{O}(M')$ such that $f_n(x) \to f(x)$ for every $x \in M$, and for every $n = 1, 2, \ldots$, $\|f_n\|_{M,k} \leqslant \|f\|_{M,k}$ (resp. $\|f_n\|_{M,t} \leqslant \|f\|_{M,t}$).*

(For the definition of strict pseudoconvexity on Stein spaces, see [5], p. 546.)

Theorem 2 generalizes some approximation theorems, obtained previously in the case, when D is a strictly pseudoconvex domain in C^n ([6], p. 631; [9], Theorem 1).

E. L. Stout proved in [11] the existence of an integral formula for functions which are holomorphic on certain hypersurfaces in strictly pseudoconvex domains in C^n. More exactly, he showed that the following holds:

Suppose that F is a function defined and holomorphic in a neighbourhood $\tilde{D}$ of the closure of a strictly pseudoconvex domain D in C^{n+1} with $\mathscr{C}^\infty$ boundary, which is not identically zero, and is such that $\Delta = Z(F) \cap D$ is non-empty and connected, $dF(y) \neq 0$ for all $y \in Z(F) \cap \partial D$ and $Z(F)$ intersects ∂D transversally. (Here $Z(F) = \{y \in \tilde{D}: F(y) = 0\}$.) Let $\Phi(\zeta, y)$ be any function satisfying the properties of [6], Lemma 2.4, with respect to the domain D. Then there exists a differential form $L(\zeta, y)$ of degree $(n, n-1)$ with respect to ζ and $(0, 0)$ with respect to y, defined in a neighbourhood V of $\partial D \times \bar{D}$, such that the coefficients of $L(\zeta, y)$ are smooth in V and holomorphic in y for each ζ near ∂D, and that for every $h \in \mathcal{O}(\bar{\Delta})$ and for all $y \in \Delta$,

$$h(y) = \int_{\partial\Delta} h(\zeta) \frac{L(\zeta, y)}{\Phi(\zeta, y)^n}.$$

This formula is valid also for $h \in H^\infty(\Delta)$; in this case, $h(\zeta)$ denotes the boundary values of h on $\partial\Delta$ which exist a.e. with respect to the volume measure on $\partial\Delta$, in virtue of [10], Theorem 9, p. 37, and the definition of $H^\infty(\Delta)$.

Therefore the formula

$$Ph(y) = \int_{\partial\Delta} h(\zeta) \frac{L(\zeta, y)}{\Phi(\zeta, y)^n}, \qquad y \in D,$$

defines an extension operator

$$P: H^\infty(\Delta) \to \mathcal{O}(D).$$

Using the results of [7], [5] and [8], we shall show that under the special choice of $\Phi(\zeta, y)$, P has the following properties:

THEOREM 3. *P is a linear and continuous extension operator from $H^{\infty,k}(\Delta)$, $A^k(\Delta)$, $k = 0, 1, \ldots$, and $\Lambda_t(\Delta)$, $t > 0$, $t \neq 0, 1, \ldots$, into the corresponding function spaces in the domain D.*

This theorem says that if a variety M has a special form, as described above, then the extension operator from Theorem 1 can be given by an explicit integral formula.

3. Proofs of the Theorems. We recall first one construction given by Fornaess in [5]. Let D be a strictly pseudoconvex domain in C^n with $\mathscr{C}^k$ boundary, and let $\psi: \tilde{D} \to C^m$, $m \geq n$, be a biholomorphic mapping of some neighbourhood $\tilde{D}$ of $\bar{D}$ onto a closed complex submanifold $\psi(\tilde{D})$ of C^m, such that there exists a strictly convex domain $C \subset C^m$ with $\mathscr{C}^k$

boundary, such that $\psi(D) \subset C$, $\psi(\tilde{D}\setminus\bar{D}) \subset C^m\setminus\bar{C}$, and $\psi(\tilde{D})$ intersects ∂C transversally. Let $\varrho \in \mathscr{C}^k(C^m)$ be a defining function for C. Set

$$(1)\qquad f(\xi, z) = \sum_{i=1}^{m} \varrho_i(\xi)(\xi_i - z_i) \qquad \left(\varrho_i(\xi) \overset{\text{df}}{=} \frac{\partial\varrho}{\partial\xi_i}(\xi)\right),$$

and let

$$(2)\qquad \Phi(\zeta, y) = f\big(\psi(\zeta), \psi(y)\big), \qquad \zeta, y \in \tilde{D}.$$

Then

$$\Phi(\zeta, y) = \sum_{j=1}^{n} (\zeta_j - y_j) P_j(\zeta, y),$$

where

$$P_j(\zeta, y) = \sum_{i=1}^{m} \varrho_i\big(\psi(\zeta)\big) F_{ij}(\zeta, y)$$

with convenient functions $F_{ij} \in \mathcal{O}(\tilde{D}\times\tilde{D})$. Let $\tilde{F}_{ij}$, $\tilde{\zeta}_j$ be the functions holomorphic in $C^m \times C^m$ and C^m respectively, such that $\tilde{F}_{ij}\big(\psi(\zeta), \psi(y)\big) = F_{ij}(\zeta, y)$ for $\zeta, y \in \tilde{D}$ and $\tilde{\zeta}_j\big(\psi(\zeta)\big) = \zeta_j$ for $\zeta \in \tilde{D}$, $i = 1, \ldots, m$, $j = 1, \ldots, n$. For ξ, $z \in C^m$, set

$$\tilde{g}_j(\xi, z) = \sum_{i=1}^{m} \varrho_i(\xi)\tilde{F}_{ij}(\xi, z), \qquad j = 1, \ldots, n.$$

Let

$$K(\xi, z) = \frac{(n-1)!}{(2\pi i)^n} \frac{\sum_{j=1}^{n} (-1)^{j-1}\tilde{g}_j(\xi, z) \wedge \bigwedge_{k\neq j} \bar{\partial}_\xi \tilde{g}_k(\xi, z) \wedge d\tilde{\zeta}_1(\xi) \wedge \ldots \wedge d\tilde{\zeta}_n(\xi)}{\big(f(\xi, z)\big)^n},$$

$(\xi, z) \notin f^{-1}(0)$. It is proved in [5] that $\Phi(\zeta, y)$ satisfies the properties of [6], Lemma 2.4, and that for every $h \in H^\infty(D)$ and for all $y \in D$,

$$(3)\qquad h(y) = \int_{\partial D} h(\zeta) K\big(\psi(\zeta), \psi(y)\big),$$

where $h(\zeta)$ denotes the boundary values of h on ∂D. Therefore, for every $h \in H^\infty\big(\psi(D)\big)$, the formula

$$Rh(z) = \int_{\psi(\partial D)} h(\xi) K(\xi, z), \qquad z \in C,$$

defines an extension operator

$$R\colon H^\infty\big(\psi(D)\big) \to \mathcal{O}(C).$$

Moreover, R is the linear and continuous extension operator from $H^\infty\big(\psi(D)\big)$ to $H^\infty(C)$ such that $R\Big(A\big(\psi(D)\big)\Big) \subset A(C)$, [5], Theorem 4, p. 563. Fornaess proved also, that for every strictly pseudoconvex domain $D \subset C^n$ with $\mathscr{C}^k$ boundary, there exist a neighbourhood $\tilde{D}$ of $\bar{D}$, a number $m \geq n$, a mapping $\psi\colon \tilde{D} \to C^m$, and a strictly convex domain $C \subset C^m$, with the properties described above.

It was shown in [8], Theorem 1, that, under the assumption on the regularity of ∂D as in Theorem 1, R maps $H^{\infty,k}(\psi(D))$ into $H^{\infty,k}(C)$, $A^k(\psi(D))$ into $A^k(C)$ and $\Lambda_l(\psi(D))$ into $\Lambda_l(C)$ linearly and continuously.

Proof of Theorem 1. Following Henkin [7] we consider first the case when D is a strictly convex domain in C^n and $M' = \{z_{k+1} = \ldots \ldots = z_n = 0\} \cap D'$, where D' is some neighbourhood of $\bar{D}$. Let $M = M' \cap D$, and let ϱ be a defining function for D. Set

$$f(\xi, z) = \sum_{i=1}^{n}{}' \varrho_i(\xi)(\xi_i - z_i)$$

and let

$$C(\xi, z) = \frac{(k-1)!}{(2\pi i)^k} \frac{\sum_{i=1}^{k} (-1)^{i-1} \varrho_i(\xi) \wedge \bigwedge_{l \neq i} d\varrho_l(\xi) \wedge d\xi}{\big(f(\xi, z)\big)^k},$$

where $d\xi = d\xi_1 \wedge \ldots \wedge d\xi_k$. Then, for $h \in H^\infty(M)$, the formula

$$Lh(z) = \int_{\partial M} h(\xi) C(\xi, z), \quad z \in D, \tag{4}$$

defines ([7], Theorems 1 and 2), the linear and continuous extension operator $L\colon H^\infty(M) \to H^\infty(D)$ such that $L(A(M)) \subset A(D)$. Note that Henkin's extension operator (4) coincides with that in (3) if we identify M with the domain in C^k and let $\psi(z_1, \ldots, z_k) = (z_1, \ldots, z_k, 0, \ldots, 0) \in C^n$. Therefore the operator L satisfies the conclusion of Theorem 1 if D is strictly convex and M is a plane section of D.

In order to pass to the general case when D is a strictly pseudoconvex domain in C^n and M' a complex subvariety of some neighbourhood D' of $\bar{D}$, without singular points on ∂D, and which intersects ∂D transversally, we proceed as in [7]. First we verify that the following theorem on the separation of singularities holds:

THEOREM 4. *Suppose that ∂D is of class $\mathscr{C}^k$, where k satisfies the hypothesis of Theorem* 1 (a) *or* (b), *and let $M = M' \cap D$. For every $\varepsilon > 0$ there exists a covering $\{\tilde{M}_i\}$ of ∂M by domains $\tilde{M}_i \supset M$, $i = 1, \ldots, N = N(\varepsilon)$, and continuous linear operators $L_i\colon \tilde{A}(M) \to \tilde{A}(\tilde{M}_i)$, $i = 1, \ldots, N$, where $\tilde{A}$ denotes A^k, $H^{\infty,k}$ or Λ_l respectively, so that*

(a) $f(z) = \sum_{i=1}^{N} (L_i f)(z)$ *for* $f \in \tilde{A}(M)$ *and* $z \in M$,

(b) *for every* $i = 1, \ldots, N$, $\operatorname{diam}(\bar{M} \setminus \tilde{M}_i) < \varepsilon$.

The construction of the operators L_i is the same as in [7], Theorem 3 and Lemma 12; therefore, we give only the necessary modifications:

1° If ∂D is of class $\mathscr{C}^k$, then the same is true for ∂D_ν, $\partial \tilde{D}_\nu$, ∂M_ν and $\partial \tilde{M}_\nu$ (for definitions, see [7], p. 562–563). Moreover, the domains M_ν and $\tilde{M}_\nu$ in M' can be chosen in such a way that $\bar{M}_\nu \setminus M$ and ∂M_ν do not contain singular points of M', and the same is true for $\tilde{M}_\nu$.

2° Combining [3], Theorems 1 and 3, and the proof of [6], Theorem 1.2, and assuming that the constants σ and δ from [7], Lemma 11, are chosen sufficiently small, we conclude, that the operators R_ν^0, R_ν^1, $\nu = 1, \ldots$ $\ldots, N$, defined in [7], p. 563, are continuous and linear as the operators between the following spaces:

$$R_\nu^0\colon \tilde{A}(M_{\nu-1}) \to \tilde{A}(M_\nu \cap S_{\zeta_0, 3\delta/4}),$$
$$R_\nu^1\colon \tilde{A}(M_{\nu-1}) \to \tilde{A}(\tilde{M}_\nu \cap S_{\zeta_0, 3\delta/4}),$$
$$R_\nu^1\colon \tilde{A}(M_{\nu-1}) \to \tilde{A}(M_0'' \cap M_1'')$$

(for further description, see [7]). This implies that the operators L_ν^0: $\tilde{A}(M_{\nu-1}) \to \tilde{A}(M_\nu)$ and L_ν^1: $\tilde{A}(M_{\nu-1}) \to \tilde{A}(\tilde{M}_\nu)$ (see [7], Lemma 12) are continuous. (We use here the fact that if $G, G' \subset C^n$ are open and $G' \subset \subset G$ the convergence of the sequence $\{f_n\} \subset \mathcal{O}(G)$ uniformly on compact subsets of G implies the convergence of $\{f_n|_{G'}\}$ in any of the spaces $\tilde{A}(G')$.)

The end of the proof of Theorem 1 goes also the same line as the final part of the proof of the Main Theorem in [7]: the only modification that we need is that the operator L defined in [7], p. 566, is now linear and continuous operator L: $\tilde{A}(\tilde{M} \cap G_{\zeta^*}^*) \to \tilde{A}(G_{\zeta^*}^*)$ in virtue of the result just established for the case of strictly convex domain D and the plane section M of D.

Proof of Theorem 2. By [5], Theorems 9 and 10, there exist an open Stein neighbourhood M' of $\bar{M}$, a non-negative integer m, a holomorphic mapping ψ: $M' \to C^m$ which maps D' biholomorphically onto a closed subvariety of C^m, and a strictly convex domain $C \subset C^m$ such that ∂C is of the same regularity as ∂M, $\psi(M) \subset C$, $\psi(M' \setminus \bar{M}) \subset C^m \setminus \bar{C}$, and $\psi(M')$ intersects ∂C transversally. We can assume that $0 \in C$. Let $F \in A^k(C)$. If $F_r(z) = F(rz)$, $0 < r < 1$, and $\{r_n\}$ is a sequence such that $r_n \nearrow 1$, then $F_{r_n} \to F$ in the $\| \|_{C,k}$-norm. Since $\bar{C}$ is convex, it is a simple consequence of the Oka–Weil approximation theorem, that every function F_{r_n} can be approximated in $\| \|_{C,k}$-norm by polynomials. The conclusion now follows from Theorem 1.

The proof for the spaces $H^{\infty,k}(M)$ and $\Lambda_t(M)$ is similar.

Proof of Theorem 3. Let $\tilde{D}$, m, ψ and C for the domain D be constructed as above, and choose $\Phi(\zeta, y)$ for D to be defined as in (2). It follows from the results of [5] and the proof of [11], Theorem II.1, that there exists a differential form $\tilde{L}(\xi, z)$ of degree $(n, n-1)$ with respect to ξ and $(0, 0)$ with respect to z, defined and smooth in a neighbourhood of $\psi(\partial\Delta)\times \times\bar{C}$, with coefficients holomorphic in z, such that for every $y \in D$ and every $h \in H^\infty(\Delta)$,

$$Ph(y) = \int\limits_{\psi(\partial\Delta)} h\circ\varphi(\xi)\frac{\tilde{L}(\xi, \psi(y))}{f(\xi, \psi(y))^n},$$

where $\varphi = (\psi|_{\bar{\Delta}})^{-1}$ and f is defined by (1). Therefore, in order to prove the theorem, it is sufficient to show that if

$$\tilde{P}h(z) = \int\limits_{\psi(\partial\Delta)} h(\xi)\frac{\tilde{L}(\xi, z)}{f(\xi, z)^n},$$

$h \in H^\infty(\psi(\Delta))$, $z \in C$, then $\tilde{P}$ is a continuous extension operator from $H^{\infty,k}(\psi(\Delta))$, $A^k(\psi(\Delta))$ or $\Lambda_t(\psi(\Delta))$ to the corresponding function spaces in the domain C.

It follows from the properties of $f(\xi, z)$ [5], that for every $h \in H^\infty(\psi(\Delta))$, $\tilde{P}h$ extends holomorphically across $\partial C\setminus\psi(\partial\Delta)$. Therefore it is sufficient to verify the required smoothness of $\tilde{P}h$ at the boundary of C near an arbitrary point $\xi_0 \in \psi(\partial\Delta)$. Fix such a ξ_0. Set $\tilde{\Delta} = Z(F)\cap\tilde{D}$. Choose a neighbourhood V of ξ_0 in C^m such that no singular points of $\psi(\Delta)$ are in V, and that the set $\tilde{\Delta}\cap\psi^{-1}(V)$ is contained in some coordinate neighbourhood U of $\psi^{-1}(\xi_0)$ in $\tilde{\Delta}$, which is mapped biholomorphically onto a neighbourhood of zero in C^n by some function φ. There exists a strictly convex domain $\tilde{C} \subset C^m$ with smooth boundary, such that $\tilde{C} \subset C$, $\tilde{C} \subset\subset V$ and $\partial\tilde{C}\cap\partial C$ is a neighbourhood of ξ_0 in ∂C. We can assume also that if ϱ and $\tilde{\varrho}$ denote the defining functions of the domains C and $\tilde{C}$ respectively, then $\varrho \equiv \tilde{\varrho}$ in a neighbourhood of ξ_0 in C^m. Therefore the function

$$\tilde{f}(\xi, z) = \sum_{i=1}^{m}\tilde{\varrho}_i(\xi)(\xi_i - z_i)$$

is equal to $f(\xi, z)$ for ξ in some neighbourhood W of ξ_0, with $W \subset\subset V$, and $z \in C^m$. Choose a $\mathscr{C}^\infty$ function χ, $0 \leqslant \chi \leqslant 1$, which is supported in W and which is equal to one in a neighbourhood of ξ_0. Clearly the function

$$\tilde{P}_1h(z) = \int\limits_{\psi(\partial\Delta)} h(\xi)[1-\chi(\xi)]\frac{L(\xi, z)}{f(\xi, z)^n}$$

is $\mathscr{C}^\infty$ in a neighbourhood of ξ_0, and it rests to prove that

$$\tilde{P}_2h(z) = \int\limits_{\psi(\partial\Delta)} h(\xi)\chi(\xi)\frac{L(\xi, z)}{f(\xi, z)^n} = \int\limits_{\psi(\partial\Delta)\cap W} h(\xi)\chi(\xi)\frac{L(\xi, z)}{\tilde{f}(\xi, z)^n}$$

has the desired smoothness property for z near ξ_0. Note that $E = \varphi\circ\psi^{-1}(\tilde{C})$ is a strictly pseudoconvex domain in C^n with smooth boundary and that

$\tilde{\psi} = \psi \circ \varphi^{-1}$ is a biholomorphic mapping of some neighbourhood $\tilde{E}$ of $\bar{E}$ onto a closed complex submanifold $\tilde{\psi}(\tilde{E})$ of some neighbourhood C_1 of $\bar{\tilde{C}}$, such that $\tilde{\psi}(E) \subset \tilde{C}$, $\tilde{\psi}(\tilde{E} \setminus \bar{E}) \subset C_1 \setminus \bar{\tilde{C}}$, and $\tilde{\psi}(\tilde{E})$ intersects $\partial\tilde{C}$ transversally. Therefore it is sufficient to prove the following lemma:

LEMMA 5. *Let E, $\tilde{E} \subset C^n$, ψ: $\tilde{E} \to C^m$, $\tilde{C}$, C_1 and $\tilde{f}$ be as above. Let $K(\xi, z)$ be a differential form of degree $(n, n-1)$ with respect to ξ and $(0, 0)$ with respect to z, defined in a neighbourhood V of $\psi(\partial E) \times \bar{\tilde{C}}$, such that its coefficients are smooth in V and holomorphic with respect to z. Let h be in $H^{\infty,k}(\psi(E))$, $A^k(\psi(E))$ or $\Lambda_t(\psi(E))$.*

Then the function

$$\tilde{h}(z) = \int_{\psi(\partial E)} h(\xi) \frac{K(\xi, z)}{\tilde{f}(\xi, z)^n}, \quad z \in \tilde{C},$$

belongs to $H^{\infty,k}(\tilde{C})$, $A^k(\tilde{C})$ or $\Lambda_t(\tilde{C})$ respectively.

This lemma can be proved by means of [2], Theorem A, [3], Theorems 1 and 3, and the results of [5] and [8]. The way of the proof is similar to that given in [8], and we do not yield the details here.

Note. It follows from [3] and [8] that Lemma 5, and Theorem 3 are valid also under less restrictive assumption on the differentiability of ∂D. However, the precise statement of the result would be somewhat complicated, therefore we consider here only the $\mathscr{C}^\infty$ case.

Note. It is shown in [4], Theorem 2, that if D is a strictly convex domain in C^n and $M = \{z \in D: z_{k+1} = \ldots = z_n = 0\}$ is a plane section of D, then every function $f \in A^\infty(D)$ such that $f|_M \equiv 0$, can be represented in the form

$$f(z) = z_{k+1} f_{k+1}(z) + \ldots + z_n f_n(z), \quad z \in D,$$

with convenient functions $f_i \in A^\infty(D)$, $i = k+1, \ldots, n$. This result is then used in [4] in the proof of the existence of the extension operator from $A^\infty(M)$ to $A^\infty(D)$, as announced in the introduction. We remark that the corresponding theorem for the spaces considered in this note is not true in general, as can be shown by the following simple example:

Let $B = \{(z_1, z_2) \in C^2: |z_1|^2 + |z_2|^2 < 1\}$, $M = \{(z_1, 0): |z_1| < 1\}$, and let h be any function which is in $H^\infty(U)$, but not in $A(U)$, where U denotes the unit disc in C, centered at 0; then set $f(z_1, z_2) = h(z_1) \cdot z_2$.

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