On the Relationship Between Capacity and Distance in an Underwater Acoustic Communication Channel *

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Path loss of an underwater acoustic communication channel depends not only on the transmission distance, but also on the signal frequency. As a result, the useful bandwidth depends on the transmission distance, a feature that distinguishes an underwater acoustic system from a terrestrial radio one. This fact influences the design of an acoustic network: a greater information throughput is available if messages are relayed over multiple short hops instead of being transmitted directly over one long hop. We asses the bandwidth dependency on the distance using an analytical method that takes into account physical models of acoustic propagation loss and ambient noise. A simple, single-path time-invariant model is considered as a first step. To assess the fundamental bandwidth limitation, we take an information-theoretic approach and define the bandwidth corresponding to optimal signal energy allocation – one that maximizes the channel capacity subject to the constraint that the transmission power is finite. Numerical evaluation quantifies the bandwidth and the channel capacity, as well as the transmission power needed to achieve a pre-specified SNR threshold, as functions of distance. These results lead to closed-form approximations, which may become useful tools in the design and analysis of acoustic networks.

I. Introduction

With the availability of high speed acoustic communication techniques, the maturing of underwater vehicles, and the advances in sensor technology, integration of point-to-point communication links into autonomous underwater networks has been steadily gaining interest over the past years, both from the research viewpoint [1], and that of the design and deployment of first experimental networks [2]. It is envisioned that some of the immediate applications of acoustic networking technology will include collaborative missions of multiple autonomous vehicles, and the deployment of ad hoc underwater sensor networks. The design of such systems is the subject of on-going research.

One of the questions that arise naturally at this time is what are the fundamental capabilities of underwater networks in supporting multiple users that wish to communicate to (or through) each other over an acoustic channel. While research has been extremely active on assessing the capacity of wireless radio networks (e.g., [3]) no similar analyses have been reported for underwater acoustic networks. The few available analyses focus on the acoustic channel capacity. For example, [4] uses a time-invariant channel model with additive Gaussian noise that may or may not be white, while [5] uses a Rayleigh fading model, with additive white Gaussian noise (AWGN). Neither of these analyses addresses the capacity dependence on distance.

Underwater acoustic communication channels are characterized by a path loss that depends not only on the distance between the transmitter and receiver, as it is the case in many other wireless channels, but also on the signal frequency. The signal frequency determines the absorption loss which occurs because of the transfer of acoustic energy into heat. This fact implies the dependence of acoustic bandwidth on the communication distance. The resulting bandwidth limitation is a fundamental one, as it is determined by the physics of acoustic propagation, and not by the con-

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straints of transducers.

The absorption loss increases with frequency as well as with distance, eventually imposing a limit on the available bandwidth within the practical constraints of finite transmission power. Consequently, a shorter communication link offers more bandwidth than a longer one in an underwater acoustic system. For example, transmission over 100 km can be performed in one hop, using a bandwidth of 1 kHz, or by relaying the information over 10 hops, each of which is 10 km long, but offers a bandwidth on the order of 10 kHz. Hence, in exchange for a more complicated system of relays, significant increase in information throughput can be obtained. At the same time, total energy consumption will be lower, but this is so for the radio channel as well.

Before one can answer the questions of network capacity, a functional dependence of the acoustic communication bandwidth with distance must be obtained. This is the subject of the present paper, which is organized as follows.

In Sec.II we summarize the basics of acoustic propagation, to formulate a model of the path loss and the ambient noise that will be used to assess the bandwidth. In Sec.III we propose two definitions of the acoustic bandwidth, one a heuristic definition based on the 3 dB loss in the band-edge SNR and a uniform energy allocation, and the other an information-theoretic definition based on optimal energy allocation for a fixed transmission power. In both cases, the total transmission power is determined as that needed to achieve a pre-specified SNR within the given bandwidth. Sec.IV illustrates the results numerically, providing a quantitative measures of the bandwidth in Hz and capacity in bps, as well as the transmission power in dB re μ Pa, as functions of distance. Numerical results lead to closed-form approximations which provide functional dependence of the system capacity on the transmission distance. Conclusions are summarized in Sec.V.

II. Acoustic propagation: path loss and noise

II.A. Attenuation

Attenuation, or path loss that occurs in an underwater acoustic channel over a distance l for a signal of frequency f is given by

$$A(l,f) = A_0 l^k a(f)^l \quad (1)$$

where A_0 is a unit-normalizing constant, k is the spreading factor, and $a(f)$ is the absorption coefficient. Expressed in dB, the acoustic path loss is given by

$$10\log A(l,f)/A_0 = k \cdot 10\log l + l \cdot 10\log a(f) \quad (2)$$

The first term in the above summation represents the spreading loss, and the second term represents the absorption loss. The spreading factor k describes the geometry of propagation, and its commonly used values are $k = 2$ for spherical spreading, $k = 1$ for cylindrical spreading, and $k = 1.5$ for the so-called practical spreading. (The counterpart of k in a radio channel is the path loss exponent whose value is usually between 2 and 4, the former representing free-space line-of-sight propagation, and the latter representing two-ray ground-reflection model.) The absorption coefficient can be expressed empirically, using the Thorp's formula which gives $a(f)$ in dB/km for f in kHz as [6]:

$$10\log a(f) = 0.11\frac{f^2}{1+f^2} + 44\frac{f^2}{4100+f^2} + 2.75\cdot 10^{-4} f^2 + 0.003 \quad (3)$$

This formula is generally valid for frequencies above a few hundred Hz. For lower frequencies, the following formula may be used:

$$10\log a(f) = 0.002 + 0.11\frac{f^2}{1+f^2} + 0.011 f^2 \quad (4)$$

The absorption coefficient is shown in Fig.1. It increases rapidly with frequency, thus imposing a limit on the maximal usable frequency for an acoustic link of a given distance.

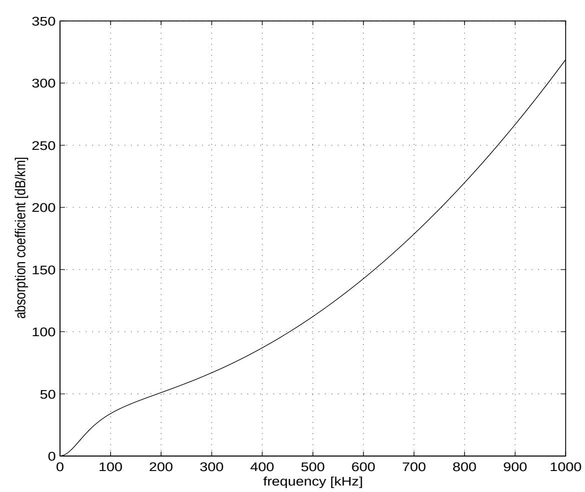

Figure 1: Absorption coefficient, $a(f)$ [dB/km].

The path loss describes the attenuation on a single, unobstructed propagation path. If a tone of frequency f and power P is transmitted over this path, the received signal power will be $P/A(l,f)$. If there are multiple propagation paths, each of length $l_p, p = 0, \ldots P-1$, then the channel transfer function can be described by

$$H(l,f) = \sum_{p=0}^{P-1} \Gamma_p / \sqrt{A(l_p,f)} e^{-j2\pi f \tau_p} \quad (5)$$

where $l = l_0$ is the distance between the transmitter and receiver, Γ_p models additional losses incurred on the pth path (e.g. reflection loss), and $\tau_p = l_p/c$ is the delay (c=1500 m/s is the nominal speed of sound underwater). If the transmission is not directional, such that propagation paths other than the direct one contribute to the received signal, then the received power will be $P|H(l,f)|^2$. In our treatment, we shall focus on a propagation model that takes into account only the basic path loss. Extensions to the multipath propagation case are straightforward, if the attenuation $A(l,f)$ is replaced by $1/|H(l,f)|^2$, evaluated for the particular channel geometry or determined experimentally.

II.B. Noise

The ambient noise in the ocean can be modeled using four sources: turbulence, shipping, waves, and thermal noise. Most of the ambient noise sources can be described by Gaussian statistics and a continuous power spectral density (p.s.d.). The following empirical formulae give the p.s.d. of the four noise components in dB re μ Pa per Hz as a function of frequency in kHz [7]:

$$\begin{aligned} 10\log N_t(f) &= 17 - 30\log f \\ 10\log N_s(f) &= 40 + 20(s-0.5) + 26\log f - \\ &\quad 60\log(f+0.03) \\ 10\log N_w(f) &= 50 + 7.5w^{1/2} + 20\log f - \\ &\quad 40\log(f+0.4) \\ 10\log N_{th}(f) &= -15 + 20\log f \end{aligned} \quad (6)$$

Turbulence noise influences only the very low frequency region, $f < 10$ Hz. Noise caused by distant shipping is dominant in the frequency region 10 Hz -100 Hz, and it is modeled through the shipping activity factor s, whose value ranges between 0 and 1 for low and high activity, respectively. Surface motion, caused by wind-driven waves (w is the wind speed in m/s) is the major factor contributing to the noise in the frequency region 100 Hz - 100 kHz (which is the operating region used by the majority of acoustic systems). Finally, thermal noise becomes dominant for $f > 100$ kHz.

The overall p.s.d. of the ambient noise, $N(f) = N_t(f) + N_s(f) + N_w(f) + N_{th}(f)$, is illustrated in Fig.2, for the cases of no wind (solid) and wind at a moderate 10 m/s (dotted), with varying degrees of shipping activity in each case. The noise decays with frequency, thus limiting the useful acoustic bandwidth from below. It may be useful to note that in a certain frequency region the noise p.s.d. decays linearly on the logarithmic scale. The following approximation may then be useful:

$$10\log N(f) \approx N_1 - \eta \log f \quad (7)$$

This approximation is shown in the figure (dash-dot) with $N_1 = 50$ dB re μ Pa and η=18 dB/decade.

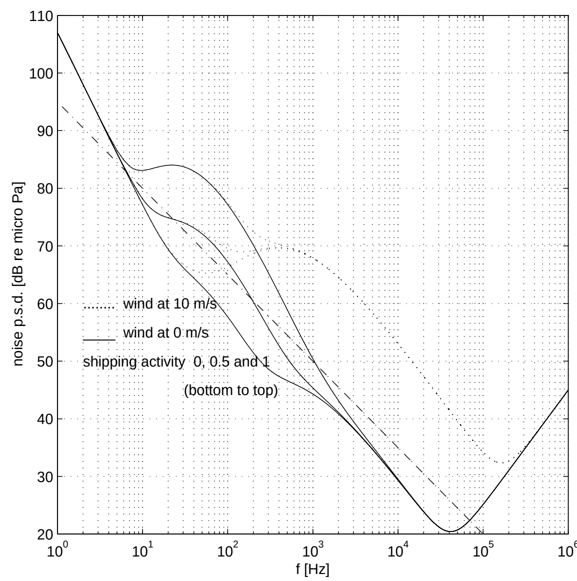

Figure 2: Power spectral density of the ambient noise, $N(f)$ [dB re μ Pa]. The dash-dot line shows an approximation $10\log N(f) = 50 - 18 log f$.

II.C. The AN Product and the SNR

Using the attenuation $A(l,f)$ and the noise p.s.d. $N(f)$ one can evaluate the signal-to-noise ratio (SNR) observed over a distance l when the transmitted signal is a tone of frequency f and power P. Not counting the directivity gains and losses other than the path loss, the narrow-band SNR is given by

$$SNR(l,f) = \frac{P/A(l,f)}{N(f)\Delta f} \quad (8)$$

where Δf is the receiver noise bandwidth (a narrow band around the frequency f). The AN product, $A(l,f)N(f)$, determines the frequency-dependent part of the SNR. The factor $1/A(l,f)N(f)$ is illustrated in Fig.3. For each transmission distance l, there clearly exists an optimal frequency $f_o(l)$ for which the maximal narrow-band SNR is obtained. The optimal frequency is plotted in Fig.4 as a function of transmission distance. In practice, one may choose some transmission bandwidth around $f_o(l)$, and adjust the transmission power so as to achieve the desired SNR level. We comment more on such choices in the following section.

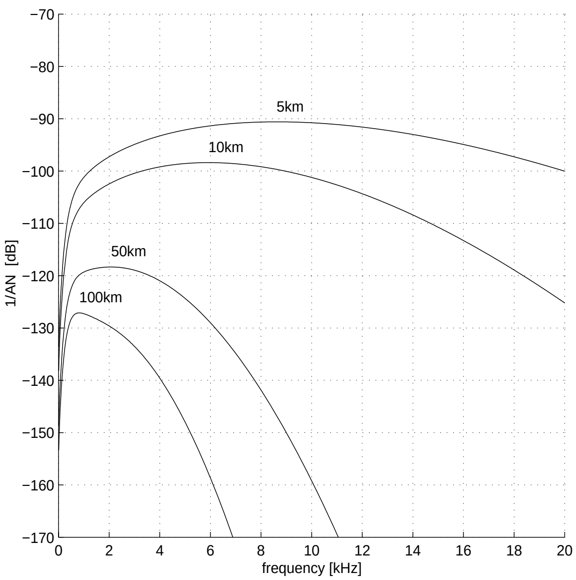

Figure 3: Frequency-dependent part of narrow-band SNR, $1/A(l,f)N(f)$. Practical spreading ($k=1.5$) is used for the path loss $A(l,f)$. Moderate shipping activity ($s=0.5$) and no wind ($w=0$) are used for the noise p.s.d. $N(f)$.

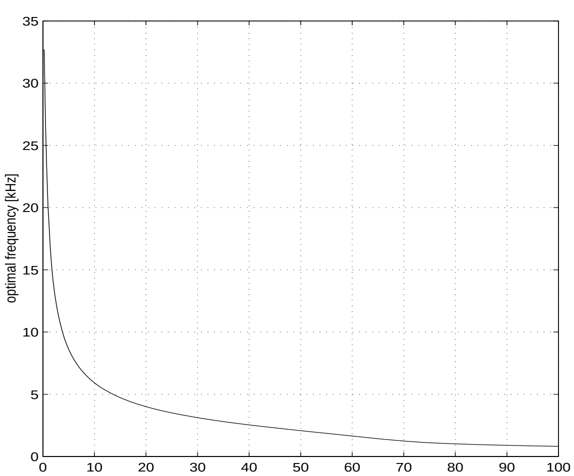

Figure 4: Optimal frequency $f_o(l)$ is the one at which $1/A(l,f)N(f)$ reaches its maximum.

III. Bandwidth and capacity

III.A. A heuristic bandwidth definition

A possible definition of the system bandwidth is that of a 3 db (or some other level) bandwidth. We define the 3 dB bandwidth $B_3(l)$ as that range of frequencies around $f_o(l)$ for which $SNR(l,f) > SNR(l,f_o(l))/2$, i.e., for which $A(l,f)N(f) < 2A(l,f_o(l))N(f_o(l)) = 2AN_{min}(l)$.

Once the transmission bandwidth is set to some $B(l) = [f_{min}(l),$
$f_{max}(l)]$ around $f_o(l)$, the transmission power $P(l)$ can be adjusted to achieve the desired narrow-band SNR level at $f_o(l)$. Alternatively, and perhaps more meaningfully, one may set the desired transmission power in accordance with the total SNR corresponding to the bandwidth $B(l)$. If we denote by $S_l(f)$ the p.s.d. of the transmitted signal chosen for the distance l, then the total transmitted power is

$$P(l) = \int_{B(l)} S_l(f)df \tag{9}$$

and the SNR is

$$SNR(l,B(l)) = \frac{\int_{B(l)} S_l(f)A^{-1}(l,f)df}{\int_{B(l)} N(f)df} \tag{10}$$

In this definition, the SNR depends on the transmitted signal p.s.d., and so does the total transmission power $P(l)$. In the simplest case, the transmitted signal p.s.d. is flat, $S(l,f) = S_l$ for $f \in B(l)$, and 0 elsewhere. The total transmission power is then $P(l) = S_l B(l)$. If it is required that the received SNR be greater than some pre-specified threshold SNR_0, then the minimal transmission power can be determined from SNR_0 and $B(l)$. When the 3 dB bandwidth is used, the corresponding transmission power is determined as

$$P_3(l) = SNR_0 B_3(l) \frac{\int_{B_3(l)} N(f)df}{\int_{B_3(l)} A^{-1}(l,f)df} \tag{11}$$

While this definition of the acoustic system bandwidth may be intuitively satisfying, there is nothing to guarantee its optimality. It may be possible to achieve a better utilization of resources through a different energy distribution across the system bandwidth. In other words, we may adjust the signal p.s.d. $S_l(f)$ in accordance with the given channel and noise characteristics $A(l,f)$ and $N(f)$ so as to optimize some performance metric. We do so in the following section.

III.B. Capacity-based bandwidth definition

A performance metric that naturally comes to mind is the channel capacity. Assuming that the noise is Gaussian, and that the channel is time-invariant for some interval of time, the capacity can be obtained by dividing the total bandwidth into many narrow sub-bands, and summing the individual capacities. The ith sub-band is centered around frequency $f_i, i = 1, 2, \ldots$ and it has width Δf, which is small enough that the channel transfer function appears frequency-nonselective, i.e. the only distortion comes from a constant attenuation factor $A(l, f_i)$. The noise in this narrow sub-band can be approximated as white, with the p.s.d. $N(f_i)$, and the resulting capacity is given by

$$C(l) = \sum_i \Delta f \log_2 \left[1 + \frac{S_l(f_i)A^{-1}(l, f_i)}{N(f_i)}\right] \quad (12)$$

Maximizing the capacity with respect to $S_l(f)$, subject to the constraint that the total transmitted power $P(l)$ is finite, yields the optimal energy distribution. The signal p.s.d. should satisfy the water-filling principle [8]:

$$S_l(f) + A(l, f)N(f) = K_l \quad (13)$$

where K_l is a constant whose value is to be determined from the power $P(l)$, and it is understood that $S_l(f) \geq 0$.

The power $P(l)$ can be chosen to provide a desired SNR, SNR_0, similarly as before. The SNR corresponding to the optimal energy distribution is given by

$$\begin{aligned} SNR(l, B(l)) &= \frac{\int_{B(l)} S_l(f)A^{-1}(l, f)df}{\int_{B(l)} N(f)df} \\ &= K_l \frac{\int_{B(l)} A^{-1}(l, f)df}{\int_{B(l)} N(f)df} - 1 \end{aligned} \quad (14)$$

The transmitted power is

$$P(l) = \int_{B(l)} S_l(f)df = K_l B(l) - \int_{B(l)} A(l, f)N(f)df \quad (15)$$

If the power is determined as the minimum needed to satisfy the SNR condition

$$SNR(l, B(l)) \geq SNR_0 \quad (16)$$

then the optimal energy distribution $S_l(f)$ can be obtained through the following numerical procedure.

For each distance l, we begin by finding the optimal frequency $f_o(l)$, and setting the initial value of the constant K_l to $K_l^{(0)} = AN_{min}(l)$. We then proceed iteratively, increasing K_l in each step by some small amount, until the condition (16) is met. In particular, if $K_l^{(n)}$ denotes the current value of the constant K_l, for which the SNR is still below the desired threshold, then the following operations are performed in the n-th step:

1. Determine $B^{(n)}(l)$ as that region of frequencies for which $A(l, f)N(f) \leq K_l^{(n)}$.
2. Calculate $SNR^{(n)}$ from (14) using the bandwidth $B^{(n)}(l)$ and the constant $K_l^{(n)}$.
3. Compare $SNR^{(n)}$ to SNR_0. If $SNR^{(n)} < SNR_0$, increase K_l by a small amount, and continue the procedure. For example, $K_l^{(n+1)} = (1 + \epsilon)K_l^{(n)}$ was used for numerical evaluation of results in Sec. IV, with ϵ=0.01.

When $SNR^{(n)}$ reaches (or slightly exceeds) SNR_0, the procedure ends. The current value of $K_l^{(n)}$ is set as the desired constant K_l, and the current value of the bandwidth $B^{(n)}(l)$ is set as the desired bandwidth $B(l)$. The optimal energy distribution is

$$S_l(f) = \begin{cases} K_l - A(l, f)N(f), & f \in B(l) \\ 0, & \text{otherwise} \end{cases} \quad (17)$$

and the total power is obtained from (15). Finally, the channel capacity is

$$C(l) = \int_{B(l)} \log_2 \left[\frac{K_l}{A(l, f)N(f)}\right] df \quad (18)$$

In comparison, the capacity (if it may be called that) of the heuristic scheme that uses equal energy distribution across the 3 dB bandwidth is

$$C_3(l) = \int_{B_3(l)} \log_2 \left[1 + \frac{P_3(l)/B_3(l)}{A(l, f)N(f)}\right] df \quad (19)$$

IV. Numerical results

The bandwidth, capacity, and transmission power were evaluated through numerical integration of the expressions presented in the previous section. Results are presented for both the 3 dB definition and the capacity-maximizing definition of bandwidth. For lack of better names, we shall refer to these two cases at the heuristic case and the optimal case, respectively. In both cases, the acoustic loss is modeled using practical spreading, $k = 1.5$, and the noise p.s.d. is that obtained for moderate shipping activity $s = 0.5$ and wind speed $w = 0$. The SNR threshold is set to SNR_0=20 dB.

Figure 5 illustrates the results obtained using the 3 dB bandwidth definition. The upper plot shows the bandwidth $B_3(l)$ and the corresponding capacity $C_3(l)$, evaluated numerically from the expression (19). The resulting bandwidth efficiency is 6.6 bps/Hz. The lower plot shows the transmission power $P_3(l)$, evaluated from the expression (11).

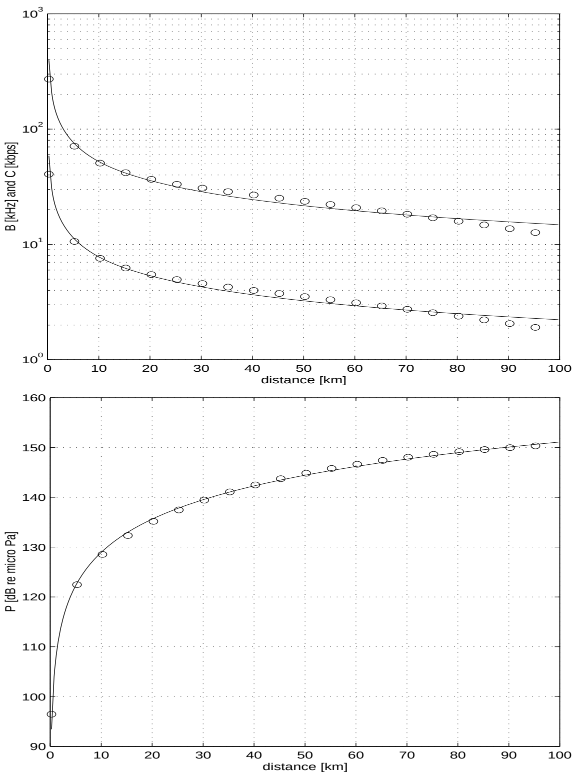

Figure 5: Bandwidth and capacity (upper plot) and transmission power (lower plot) needed to achieve SNR_0=20 dB. Equal energy distribution and the 3 dB bandwidth definition are used. Circles indicate results of numerical integration; solid curves represent closed-form approximations.

For the case of optimal resource allocation, we first find the transmitted signal p.s.d. for each distance and the desired threshold SNR. Fig.6 illustrates the attenuation-noise characteristic $A(l, f)\cdot N(f)$, and the optimal p.s.d. $S_l(f)$ obtained for l=5 km. Shown together with the AN characteristic is the value of K_l for which the total SNR reaches SNR_0=20 dB. The points on the frequency axis where K_l crosses the AN characteristic mark the optimal signal bandwidth for this distance and the chosen SNR threshold.

The results obtained using the optimal bandwidth definition are summarized in Fig.7. The upper plot shows the bandwidth $B(l)$ and the corresponding capacity $C(l)$, evaluated numerically from the expression (18). The resulting bandwidth efficiency is 8 bps/Hz. The improvement in bandwidth efficiency owes to the optimal energy-bandwidth allocation. The lower plot shows the transmission power $P(l)$, evaluated from the expression (15).

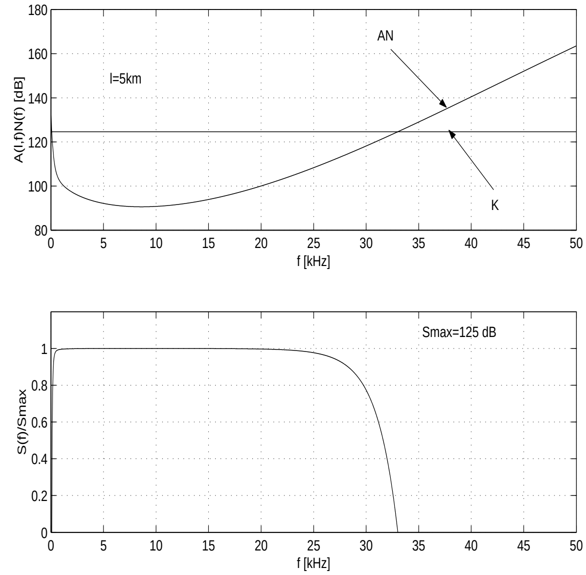

Figure 6: Finding the optimal p.s.d of the transmitted signal for transmission distance l=5km: upper plot shows $A(l, f)N(f)$ and the constant level K_l for which the received SNR equals SNR_0=20 dB; lower plot shows the resulting p.s.d. $S_l(f)$.

While there is no closed-form solution for the system bandwidth as a function of distance, a closer examination of the numerical results reveals that the bandwidth decays almost linearly with distance on a logarithmic scale. A similar observation can be made for the capacity. The power increases with distance, also following a linear trend on the logarithmic scale. Such trends are observed for both the heuristic and the optimal bandwidth definition. Hence, the following approximations are proposed:

$$\begin{array}{lll} \hat{B}_3(l) = b_3 l^{-\beta_3}, & \hat{C}_3(l) = c_3 l^{-\gamma_3}, & \hat{P}_3(l) = p_3 l^{\pi_3} \\ \hat{B}(l) = b_o l^{-\beta_o}, & \hat{C}(l) = c_o l^{-\gamma_o}, & \hat{P}(l) = p_o l^{\pi_o} \end{array} \tag{20}$$

where the coefficients b, c, p, and the exponents β, γ, π are positive constants that can be determined by curve-fitting. Least-squares approximation by a first-order polynomial on a logarithmic scale provided the values of these parameters that were used to plot the results of Figs.5 and 7. Solid curves represent the closed-form approximations, while circles indicate the actual values obtained through numerical integration. Clearly, there is a very good agreement between the numerical results and the approximate closed-form solutions. Hence, the closed-form expressions offer an efficient way of estimating the system resources (available bandwidth and capacity, required power) for a given distance. They may thus prove to be a useful tool in the design and analysis of underwater acoustic networks, where it might be cumbersome to evaluate numerically the link capacities and powers for every different topology.

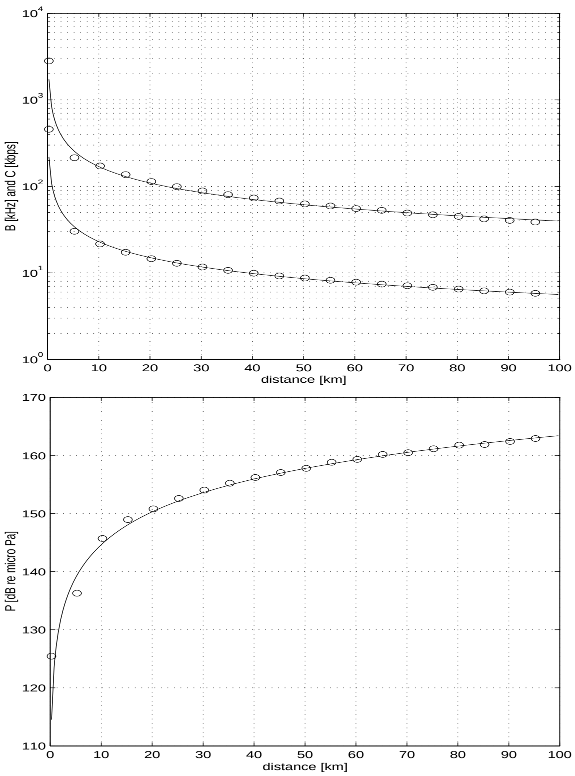

Figure 7: Bandwidth and capacity (upper plot) and transmission power (lower plot) needed to achieve SNR_0=20 dB. Capacity-maximizing energy distribution and the corresponding optimal bandwidth definition are used. Circles indicate results of numerical integration; solid curves represent closed-form approximations.

The results of Figs. 5 and 7 correspond to the SNR threshold of 20 dB. For a different SNR threshold, different values of bandwidth, capacity, and transmission power will be obtained. The effect of varying SNR on the link capacity and bandwidth is summarized in Fig.8. Shown in the figure is the bandwidth efficiency, i.e. the ratio between the system capacity and bandwidth, $C(l)/B(l)$ in bps/Hz, for several values of transmission distance, $l = 5, 15, 25, \ldots 75$ km. The capacity-maximizing definition of bandwidth is used, and the system parameters are evaluated for SNR_0 between -15 dB and 45 dB. The first plot (top) provides a relationship between the bandwidth efficiency and the transmission power. The bandwidth efficiency increases with transmission power, following a similar pattern for various distances. The second plot illustrates the bandwidth efficiency as a function of SNR. Although one might expect the C/B curves to collapse into a single curve, this is not the case, except at low SNR. At a moderate SNR around 10 dB, the C/B curves start to diverge slightly, showing a greater bandwidth efficiency for a greater distance. However, with a further increase in the SNR, the curves cross each other, yielding higher bandwidth efficiency to shorter distances. As a benchmark, the plot also shows the bandwidth efficiency of an equivalent AWGN channel,

$$\left(\frac{C}{B}\right)_{AWGN} = \log_2(1 + SNR_0) \qquad (21)$$

We observe that the bandwidth efficiency of an acoustic Gaussian channel tends to that of an equivalent AWGN channel at low SNR regardless of the distance, but then deviates from it as the SNR increases. For the considered model of a time-invariant single-path acoustic channel, the bandwidth efficiency is greater than that of an equivalent AWGN channel in the SNR range between about 10 dB and 30 dB, but falls below it as the SNR further increases.

It may also be interesting to present the bandwidth efficiency as a function of the bit SNR, a figure of merit commonly used in the study of communication systems. For a channel corrupted by the AWGN, the bit SNR is the ratio of the bit energy E_b to the noise p.s.d. N_0. The noise in the acoustic channel is not white, but one can define the p.s.d. of an equivalent white noise as

$$N_0(l) = \frac{1}{B(l)} \int_{B(l)} N(f)df \qquad (22)$$

The dependence of the equivalent noise p.s.d. on the distance is caused by that of the bandwidth. The received bit energy is

$$E_b(l) = \frac{1}{C(l)} \int_{B(l)} S_l(f)A^{-1}(l,f)df \qquad (23)$$

Hence, we define the equivalent bit SNR as

$$\frac{E_b}{N_0} = \frac{B(l)}{C(l)} SNR_0(l, B(l)) \qquad (24)$$

It may be interesting to note that although both the bit energy and the equivalent white noise p.s.d. depend on the distance, their ratio does not. The third plot of Fig.8 shows the bandwidth efficiency as a function of the equivalent bit SNR E_b/N_0. As a calibration benchmark, the plot also shows the bandwidth efficiency of the equivalent AWGN channel, which obeys

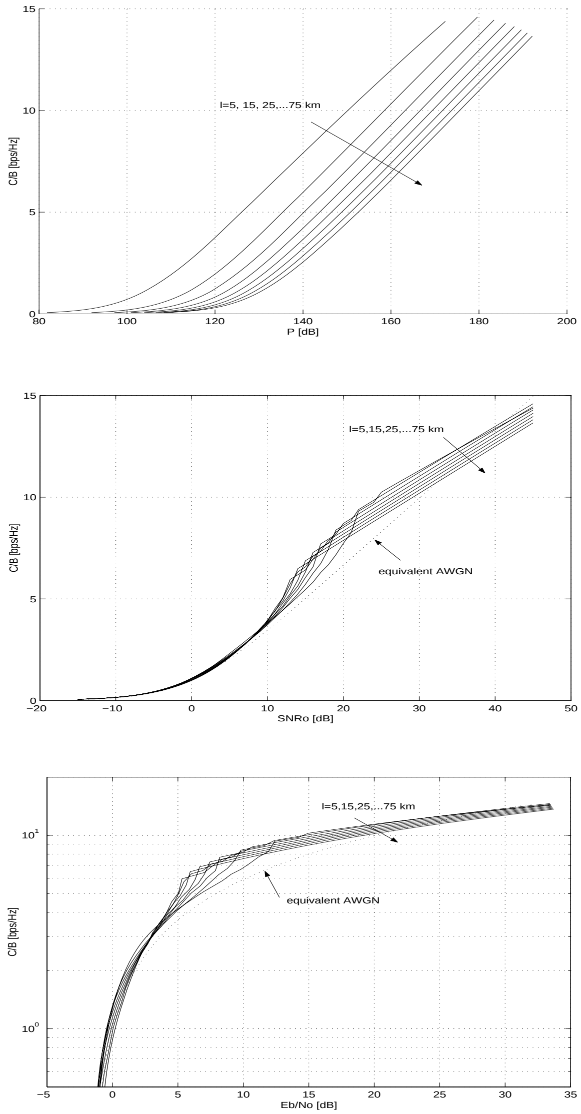

Figure 8: Bandwidth efficiency as a function of transmission power, SNR and equivalent E_b/N_0.

the relationship

$$\frac{E_b}{N_0} = \frac{2^{(C/B)_{AWGN}} - 1}{(C/B)_{AWGN}} \tag{25}$$

This plot presents the same results as the second one, but perhaps in a more familiar framework, which clearly shows the Shannon's limit.

Finally, we note that the closed-form approximations for bandwidth, capacity and transmission power depend on the SNR threshold, as do their true values (Fig.8 showed the actual values obtained through numerical integration). Hence, the coefficients and the exponents of the approximations (20) are functions of the SNR threshold SNR_0. Figs.9 and 10 show the approximation parameters for the 3 dB bandwidth definition, while Figs.11 and 12 correspond to the the optimal definition. The coefficients are given in dB relative to 1 kHz, 1 kbps, and 1 μPa, for the bandwidth, capacity, and power, respectively, and the exponents are given in dB per km. We recall that these results correspond to a single path propagation model with practical spreading and a particular ambient noise profile.

V. Conclusions

It is well known that the frequency-dependency of the acoustic path loss imposes a bandwidth limitation on an underwater communication system, such that a greater bandwidth is available for a shorter transmission distance. This fact has a significant implication on the design of an acoustic network: if a greater bandwidth is available for a shorter distance, then the total network throughput can be increased by placing relay nodes between the information-generating ones. In designing a network, one will thus inevitably ask how many relays to use, where to place them, and what is the overall throughput improvement; or, more generally, what is the optimal resource allocation and what is the network capacity. To answer these questions, link capacity must be known as a function of distance.

This paper offers an insight into the relationship between an acoustic link capacity and distance. As a first approximation, a simple model of a time-invariant acoustic channel was considered, taking into account the physical laws of acoustic propagation and the ambient noise. The bandwidth, capacity, and transmission power needed to achieve a pre-specified SNR were evaluated analytically as functions of distance. Numerical results were shown to admit simple closed-form approximations. These semi-analytical solutions provide the needed functional dependence between the acoustic link capacity and transmission distance.

The basic principles used in this paper can be applied to more accurate acoustic channel models that take into account both multipath propagation and time-variability. Future research should focus on using these results to assess the capacity of multi-hop acoustic systems.

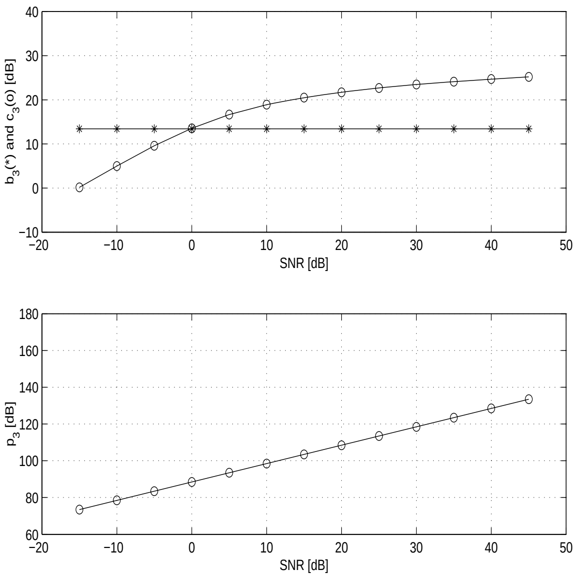

Figure 9: Model parameters as functions of SNR under the 3 dB definition: bandwidth and capacity coefficients b_3, c_3, and power coefficient p_3.

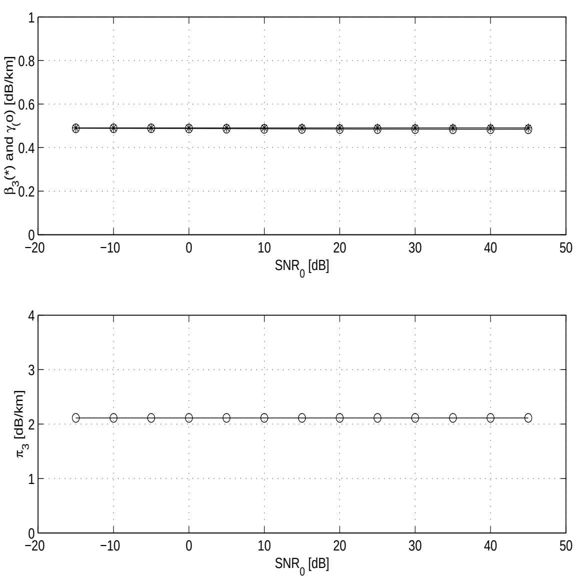

Figure 10: Model parameters as functions of SNR under the 3 dB definition: bandwidth and capacity exponents β_3, γ_3, and power exponent π_3.

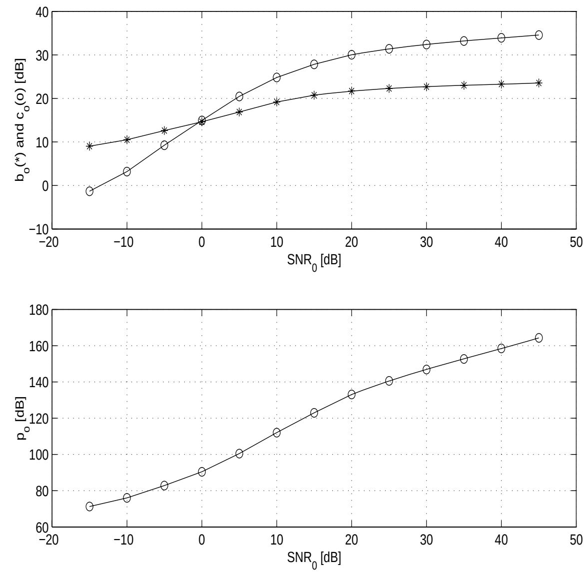

Figure 11: Model parameters as functions of SNR under the optimal definition: bandwidth and capacity coefficients b_o, c_o, and power coefficient p_o.

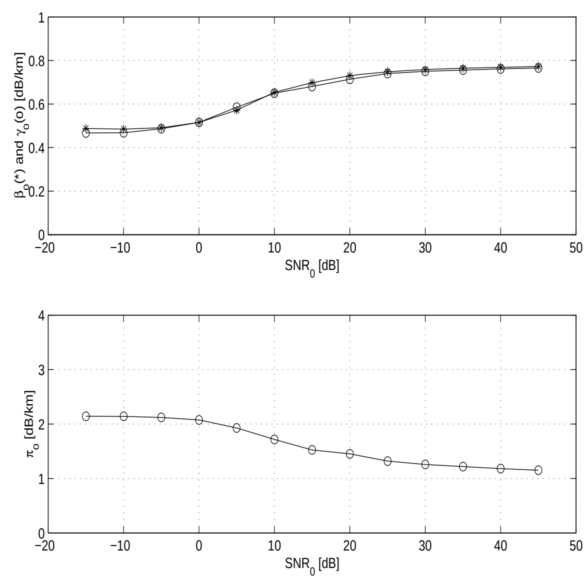

Figure 12: Model parameters as functions of SNR under the optimal definition: bandwidth and capacity exponents β_o, γ_o, and power exponent π_o.

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