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On the response of ocean currents to atmospheric cooling

By DORON NOF, *Division of Meteorology and Physical Oceanography, and the Cooperative Institute for Marine and Atmospheric Studies, Rosenstiel School of Marine and Atmospheric Science, 4600 Rickenbacker Causeway, Miami, Florida 33149, U.S.A.*[1]

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ABSTRACT

The response of steady ocean currents to forced atmospheric cooling is examined by a simplified two-layer model. The study focuses on currents, such as the Gulf Stream, which lose their heat to the atmosphere as they flow from one region to another. The model is inviscid and includes no coupling between the ocean and the atmosphere. Approximate solutions for specified cooling processes acting steadily on currents whose undisturbed speed is uniform are obtained analytically using a uniformly valid power series expansion.

It is found that upon encountering a region of cooling, the interface steepens and the whole current is displaced horizontally. The streamlines in the upper portion of the light layer are displaced to the right (looking downstream) whereas the streamlines in the lower portion of the upper layer are displaced to the left. These movements result from a combined effect of advection and "thermal wind" motion. For actual currents and heat losses in the ocean, the predicted interface steepening is of the same order as the slope upstream and the predicted horizontal displacements during strong cooling processes can be as high as ~ 100 km.

Possible application of this theory to the separated Gulf Stream which loses heat to the atmosphere as it flows from Cape Hatteras toward the northeast is discussed. The model predicts that during the late winter the position of the Gulf Stream front will be farther to the south than it is during the summer and that the slope of the interface will be larger in winter. Both processes agree qualitatively with the observed seasonal variability of the Stream.

1. Introduction

It has been recognized for a long time that differential heating plays an important role in ocean dynamics and in affecting the structure of the ocean. The highest rate of heat loss in the world ocean is found to the east of Cape Hatteras where its annual average reaches about 5×10^9 J m^{-2} (see e.g., Budyko, 1963; Worthington, 1976). This heat loss from the Gulf Stream and its neighboring waters is much larger than the heat loss in most other parts of the ocean. During the winter, its effect on the Stream is so large that after reaching about two thousand kilometers off-shore, the Gulf Stream no longer possesses its characteristics of high temperature and, consequently, it is no longer distinguishable from the neighboring water of the Sargasso Sea (Worthington, 1972; Fuglister, 1963, Mann, 1967).

Because of this significant change, it is of interest to examine the influence that a reduction in the temperature (see Fig. 1) might have on the structure of the flow. To do this we simplify the problem to that of an upper uniform flow which is subject to a specified cooling process. As the fluid flows into the region of cooling, it experiences gradual changes in its density which must be offset by local changes in the structure of the flow. Our aim is to study the general characteristics of this process and to examine the details of the oceanic response. We do not intend to simulate all the details of the real system; we merely hope to preserve enough analogy to the real system so that our results will give insight into natural phenomena.

[1] Present affiliation: Department of Oceanography, The Florida State University, Tallahassee, Florida 32306, U.S.A.

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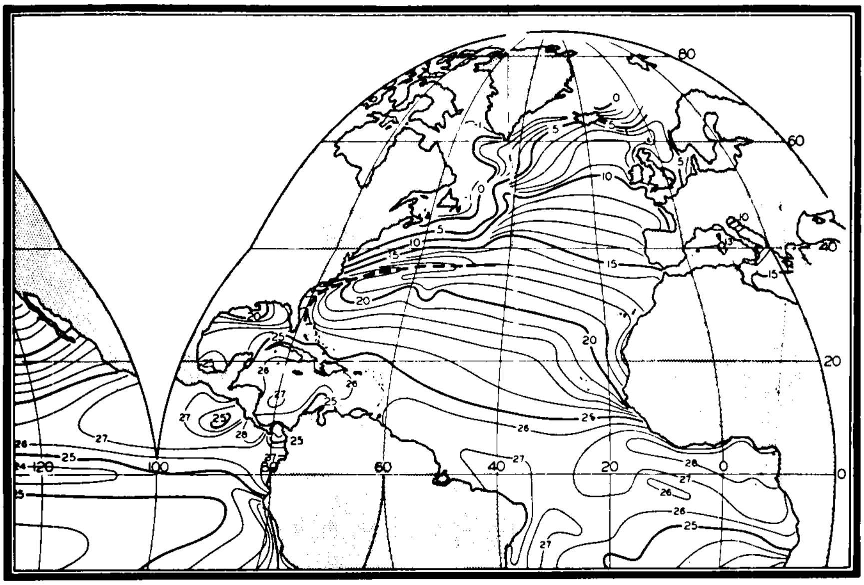


Fig. 1. Average surface temperatures for February [reproduced from Sverdrup *et al.* (1942)]. The dashed line denotes the approximate position of the Gulf Stream. Note that the stream loses a few degrees as it flows from Cape Hatteras toward the north-east.

There have been a number of previous investigations of ocean–atmosphere heat exchange processes (e.g., Veronis, 1976; Haney, 1974; Gill and Niiler, 1973, Stommel and Veronis, 1980; Bryan, 1965; Bryan *et al.*, 1975) but none of them have dealt directly with the problem considered in this study where ocean currents adjust themselves to an imposed cooling process. To simplify the analysis we shall consider a two-layer inviscid model whose motions are driven solely by remote forcing and local cooling (Fig. 2). In other words, the effects of local winds are not taken into account so that the specified upstream flow would have remained unaltered throughout the whole field had the cooling not been imposed.

The movement is confined to an upper layer which corresponds to the upper few hundred meters of the ocean. The lower layer is assumed to be motionless and its density is taken to be uniform everywhere. The upper layer is bounded by a front on the left[1] hand side and extends to infinity on the right. This front corresponds to the region where the interface strikes the free surface and can be thought of as representing the northern edge of the Gulf Stream. As the current enters the region of cooling, its density changes and consequently new pressure gradients, which were not present upstream, are generated. As we shall see, these pressure gradients drive a flow in a direction perpendicular to the current upstream; as a result of this flow, the interface steepens and the current is displaced to the right or left of its upstream position.

To obtain the solution to the problem, the amount of heat loss is specified and a perturbation scheme in $\varepsilon = R_d/l$, the ratio between the Rossby deformation radius (R_d) and the length scale of the cooling area (l), is applied. The parameter ε corresponds to the ratio between the scale of the flow in the cross-stream and long-stream directions, and is small for most problems of practical interest. The perturbation scheme reduces the problem from a complicated three-dimensional field to a solvable set of equations. As we shall see, the solution of the reduced equations indicates that, within our range of parameters, the flow field consists of advection and "thermal-wind" movements.

After presenting the detailed solution and its implications, the results are qualitatively applied to

[1] Hereafter, right and left are with reference to an observer looking downstream.

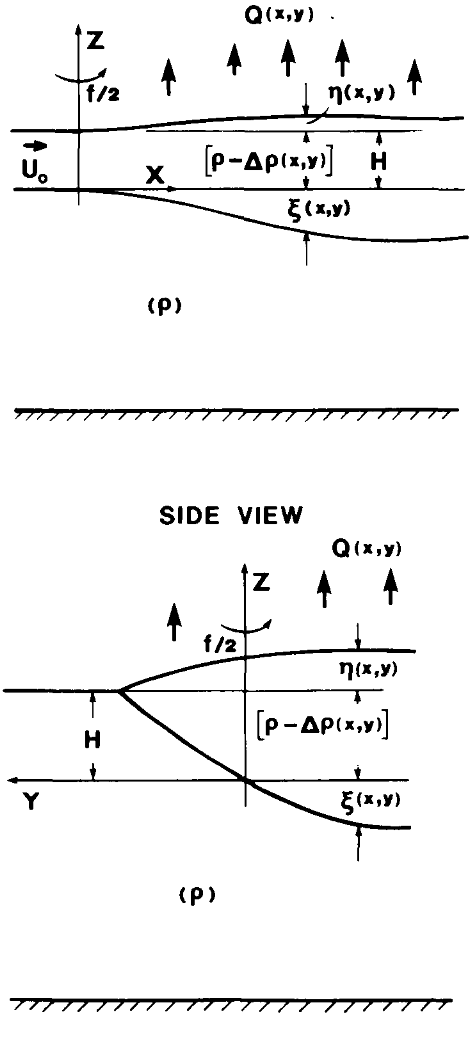


Fig. 2. Schematic diagram of the model under study. The free surface vertical displacement $\eta(x,y)$ is measured upward from the undisturbed free surface; the interface displacement ξ is measured downward [i.e., $\xi(0,0) = 0$]. H is the upper layer undisturbed depth, $Q(x,y)$ denotes the heat loss to the atmosphere, and U_0 is the uniform upstream flow. Upstream the upper layer density is uniform so that any cooling from above produces an instantaneous sinking of surface water to the lower portion of the upper layer (see text). Consequently, the upper layer density is independent of depth [$\Delta\rho = \Delta\rho(x,y)$].

the North Atlantic and its corresponding heat-exchange. It will be seen that the horizontal displacements of the front can be as large as 100 km and that these displacements are expected to vary during the year according to the seasonal variability of the heat loss. The predicted seasonal movement of the front (in the north–south direction) is then qualitatively compared to the observed mean variability of the Gulf Stream path and a general agreement, in both the direction and magnitude, is noted.

This paper is organized as follows: The formulation of the problem is presented in Section 2 and the perturbation analysis in Section 3. The detailed solution is given in Section 4 and its applicability to the Gulf Stream is considered in Section 5. The results of the study are summarized in Section 6.

2. Formulation

As an idealized formulation of the problem, consider the two-layer system shown in Fig. 2. The lower layer is taken to be deep and motionless and its density is uniform (ρ). The upper layer contains a uniform upstream flow (U_0) and its uniform upstream density ($\rho - \Delta\rho_0$) increases in the field in response to the heat exchange with the atmosphere [($Q(x,y)$)].

We shall focus our attention on cooling processes which are moderate in the sense that the corresponding density increase of the upper layer is always smaller than $\Delta\rho_0$ so that breaking of the interface and sinking of upper fluid into the lower layer does not occur. Since the undisturbed upper layer density is uniform, any cooling of the free surface will immediately produce local convection which will mix the fluid within the upper layer. Consequently, the cooling is quickly distributed to the whole upper layer column. The time scale associated with this fast distribution corresponds to a free fall of a fluid parcel (i.e.,

$$\left(2H/g\,\frac{\Delta\rho^*}{\rho}\right)^{1/2},$$

where $\Delta\rho^*$ is the density anomaly caused by the cooling and H is the depth of the upper layer) which would take a few minutes.

This time scale is, obviously, much shorter than the advection time scale and the adjustment time scale (f^{-1}, where f is the Coriolis parameter) so that $\Delta\rho$ can be taken to be independent of depth $[\Delta\rho = \Delta\rho(x,y)]$. Thus, our model contains an upper layer whose density varies horizontally but does not vary vertically. It will be shown later that this representation of the vertical structure by two layers is qualitatively adequate for the cooling processes under discussion. It should be stressed, however, that the model cannot be used to describe heating processes because they lack convection so that their vertical mixing time scale is not necessarily small compared to the advection and the adjustment time scale.

The origin of our coordinate system is located upstream at the intersection of the sloping interface with the upper layer undisturbed depth. This undisturbed depth (H) corresponds to the depth that the upper layer would have in the absence of any motion; it is equivalent to the upper layer depth in the absence of both rotation ($f = 0$) and cooling. The x and y axes are directed along and across the upstream flow (U_0). The z axis is directed upward and the system is rotating uniformly at an angular speed $\frac{1}{2}f$ about the vertical axis. The region of cooling has a length scale l so that the time that a parcel of fluid spends in the cooling area is l/U_0; this time scale is assumed to be much shorter than a season so that one may consider the cooling to be steady. It is easy to show that this assumption is adequate for many cases of practical interest because $U_0 \sim 0.5$ m s^{-1} and $l \sim 1000$ km so that the advection time scale is of O(20 days).

For steady and hydrostatic motions, the deviation of the pressure field from the pressure associated with a state of rest is:

$$p = g\rho\eta(x,y) + g\delta(x,y)\,\Delta\rho_0(H - z) + \mathrm{O}\left(g\frac{\Delta\rho_0}{\rho}\eta\right), \tag{2.1}$$

where $\eta(x,y)$ is the free surface vertical displacement which is measured upward from the free surface (see Fig. 2), $\Delta\rho_0$ is the upstream density difference between the upper and lower layer, and δ is defined by:

$$\Delta\rho = \Delta\rho_0[1 - \delta(x,y)]. \tag{2.2}$$

Note that $\delta(x,y)$ is always positive since it corresponds to an upper layer density which increases downstream.

The condition of no flow in the lower layer gives:

$$\eta(x,y) = [\xi(x,y) + H]\frac{\Delta\rho_0}{\rho}(1 - \delta) + 0\left(\eta\frac{\Delta\rho_0}{\rho}\right), \tag{2.3}$$

where $\xi(x,y)$ is the interface vertical displacement which is measured downward from the origin [i.e., $\xi(0,0) = 0$]. Substitution of (2.3) into (2.1) yields:

$$p = g\Delta\rho_0\xi(x,y)[1 - \delta(x,y)] - g\delta(x,y)\Delta\rho_0 z + gH\Delta\rho_0 \tag{2.4}$$

indicating that, since δ is a function of x and y, the pressure gradients are depth dependent so that the horizontal velocity components (u,v) are depth dependent as well [i.e., $u = u(x,y,z)$; $v = v(x,y,z)$]. With the aid of (2.4), the equations of motion and continuity for a frictionless Boussinesq fluid can be written in the form:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -g'(1 - \delta)\frac{\partial \xi}{\partial x} + g'\frac{\partial \delta}{\partial x}(\xi + z) \tag{2.5}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -g'(1 - \delta)\frac{\partial \xi}{\partial y} + g'\frac{\partial \delta}{\partial y}(\xi + z) \tag{2.6}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2.7}$$

where g' is defined by $g' = (\Delta\rho_0/\rho)g$ so that it does not vary in the field.

If the dependence of the density on the temperature field is taken to be linear, i.e.,

$$\Delta\rho = \rho\alpha(T - T_l) \tag{2.8}$$

where T and T_l are the temperatures of the upper and lower layers (respectively) and α is the coefficient of thermal expansion, then in view of (2.2) one finds:

$$\delta = (T_0 - T)/(T_0 - T_l), \tag{2.9}$$

where T_0 is the temperature of the upstream flow.

Because the density within the upper layer is independent of depth, it is convenient to consider the vertically integrated equation for conservation of heat. With the aid of (2.9), this equation can be written as:

$$\frac{\partial \delta}{\partial x}\int_{z=-\xi(x,y)}^{H} u\,dz + \frac{\partial \delta}{\partial y}\int_{z=-\xi(x,y)}^{H} v\,dz = \frac{Q(x,y)}{C_p(T_0 - T_l)} \tag{2.10}$$

where $Q(x,y)$ is the vertical heat flux through the free surface (which will be specified later) and C_p is the water heat capacity. Note that in obtaining (2.10) we have used the approximation $h \approx H + \xi$ (since $\Delta\rho/\rho \ll 1$ so that $\eta \ll \xi$), neglected horizontal diffusion of heat and, assumed that there is no heat flux through the interface $[z = -\xi(x,y)]$.

The system (2.5–2.7) and equation (2.10) are subject to the following boundary conditions:

$$u = U_0; \quad x = 0; \quad -\infty < f \leqslant g'H/fU_0 \tag{2.11a}$$

$$v = w = 0; \quad x = 0; \quad -\infty < y \leqslant g'H/fU_0 \tag{2.11b}$$

$$\xi = -fy/g'; \quad x = 0; \quad -\infty < y \leqslant g'H/fU_0 \tag{2.11c}$$

$$Q = Q(x,y); \quad 0 \leqslant x \leqslant l; \quad -\infty < y \leqslant \gamma(x) \tag{2.11d}$$

$$w(x,y,z) = \left[u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y}\right]_{z=H+\eta} \approx 0; \quad z = H + \eta(x,y) \tag{2.11e}$$

$$w(x,y,z) = \left[u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y}\right]_{z=-\xi(x,y)}; \quad z = -\xi(x,y) \tag{2.11f}$$

$$\xi(x,y) = -H; \quad y = \gamma(x) \tag{2.11g}$$

$$(u\mathbf{i} + v\mathbf{j})\cdot\nabla_H[\gamma(x) - y] = 0; \quad y = \gamma(x) \tag{2.11h}$$

where $\gamma(x) - y = 0$ denotes the left edge of the current $[\gamma(0) = g'H/fU_0]$ and ∇_H is the horizontal two-dimensional del-operator. Relations (2.11a–c) reflect the upstream conditions and the assumption that the flow remains unaltered until the region of cooling is reached. Condition (2.11d) represents the specified cooling function and (2.11e–f) correspond to the condition of no normal flow through the free surface and the interface. Relation (2.11g) states that the interface strikes the free surface ($\xi = -H$) along a curve which is not known in advance $[y = \gamma(x)]$ while (2.11h) requires that this curve will be a streamline. These two conditions correspond to the fact that the location and shape of the current's edge are not known *a priori* but rather must be determined as part of the problem.

As we shall see in the coming Sections, these conditions are sufficient for determining the solution; it is not necessary to impose any limitations on the flow downstream.

3. Perturbation analysis

To simplify the structure of the governing equations we shall restrict ourselves to those cases where the heat exchange causes only small changes in the structure of the flow. That is, we shall focus our attention on processes whose heat-exchange driven velocity components are small compared to the mean upstream flow. In order to obtain the corresponding equations, the following scales and non-dimensional variables are introduced:

$$\left.\begin{aligned}
&\hat{x} = x/l; \quad \hat{y} = y/R_d; \quad \hat{z} = z/H; \quad \hat{u} = u/U_0\\
&\hat{v} = v/U_0\left(\frac{R_d}{l}\right); \quad R_0 = \frac{U_0}{fR_d}; \quad F_r = U_0^2/g'H;\\
&R_d = (g'H)^{1/2}/f\\
&\hat{w} = w/U_0\left(\frac{H}{l}\right); \quad \hat{\xi} = \xi/H; \quad \hat{\eta} = \eta/H;\\
&\varepsilon = R_d/l\\
&\hat{Q} = \frac{Ql}{\varepsilon C_p H U_0(T_0 - T_l)}; \quad \hat{\gamma} = \gamma/R_d;\\
&\nabla = R_d\nabla_H
\end{aligned}\right\} \tag{3.1}$$

Here, R_0 and F_r are the Rossby and Froude numbers which are not necessarily small. R_d, the deformation radius, is typically ~30–50 km since $g' \sim 2 \times 10^{-2}$ m s^{-2}, $H \sim 500$ m and $f \sim 10^{-4}$ s^{-1}; the parameter ε is of O(0.01) because the length scale of the cooling region is typically ~1000 km.

Note that in defining the non-dimensional variables, it has been assumed that the cross-stream displacements of the flow will have a length scale of the deformation radius (R_d). While this assumption is plausible, it is not *a priori* obvious under what conditions, the assumption is adequate because the displacement will, in general, depend on δ which is not represented in R_d. We shall see later, however, that for the range of parameters considered in this study, the horizontal displacements are of $O(R_d)$ despite the fact that R_d is independent of δ.

In terms of the scaled variables defined in (3.1), the governing equations (2.5–2.8) and (2.10) are:

$$F_r\left(\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} + \hat{w}\frac{\partial \hat{u}}{\partial \hat{z}}\right) - \frac{F_r}{R_0}\hat{v} = -(1-\delta)\frac{\partial \hat{\xi}}{\partial \hat{x}} + \frac{\partial \delta}{\partial \hat{x}}(\hat{\xi} + \hat{z}) \tag{3.2}$$

$$\varepsilon^2 F_r\left(\hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} + \hat{w}\frac{\partial \hat{v}}{\partial \hat{z}}\right) + \frac{F_r}{R_0}\hat{u} = -(1-\delta)\frac{\partial \hat{\xi}}{\partial \hat{y}} + \frac{\partial \delta}{\partial \hat{y}}(\hat{\xi} + \hat{z}) \tag{3.3}$$

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} + \frac{\partial \hat{w}}{\partial \hat{z}} = 0 \tag{3.4}$$

$$\int_{\hat{z}=-\hat{\xi}}^{1}\left(\hat{u}\frac{\partial \delta}{\partial \hat{x}} + \hat{v}\frac{\partial \delta}{\partial \hat{y}}\right)d\hat{z} = -\varepsilon\hat{Q} \tag{3.5}$$

The corresponding non-dimensional boundary conditions are:

$$\hat{u} = 1; \quad \hat{x} = 0; \quad -\infty < \hat{y} \leqslant R_0/F_r \tag{3.6a}$$

$$\hat{v} = \hat{w} = 0; \quad \hat{x} = 0; \quad -\infty < \hat{y} \leqslant R_0/F_r \tag{3.6b}$$

$$\hat{\xi} = -\frac{F_r}{R_0}\hat{y}; \quad \hat{x} = 0; \quad -\infty < \hat{y} \leqslant R_0/F_r \tag{3.6c}$$

$$\hat{Q} = \hat{Q}(\hat{x},\hat{y}); \quad 0 \leqslant \hat{x} \leqslant 1; \quad -\infty < \hat{y} \leqslant \hat{\gamma}(\hat{x}) \tag{3.6d}$$

$$\hat{w}(\hat{x},\hat{y},\hat{z}) = \left[\hat{u}\frac{\partial \hat{\eta}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{\eta}}{\partial \hat{y}}\right]_{\hat{z}=1+\hat{\eta}} \approx 0;$$

$$\hat{z} = 1 + \hat{\eta}(\hat{x},\hat{y}) \tag{3.6e}$$

$$\hat{w}(\hat{x},\hat{y},\hat{z}) = \left[\hat{u}\frac{\partial \hat{\xi}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{\xi}}{\partial \hat{y}}\right]_{\hat{z}=-\hat{\xi}};$$

$$\hat{z} = -\hat{\xi}(\hat{x},\hat{y}) \tag{3.6f}$$

$$\hat{\xi}(\hat{x},\hat{y}) = -1; \quad \hat{y} = \hat{\gamma}(\hat{x}) \tag{3.6g}$$

$$(\hat{u}\mathbf{i} + \hat{v}\mathbf{j}) \cdot \nabla|\hat{\gamma}(\hat{x}) - \hat{y}| = 0; \quad \hat{y} = \hat{\gamma}(\hat{x});$$

$$\hat{z} = 1 \tag{3.6h}$$

It is further assumed that the dependent variables possess power series expansions in ε, e.g.,

$$\left.\begin{aligned} &\hat{u} = u^{(0)} + \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \ldots; \quad \hat{v} = v^{(0)} + \varepsilon v^{(1)} + \varepsilon^2 v^{(2)} + \ldots \\ &\delta = \delta^{(0)} + \varepsilon\delta^{(1)} + \varepsilon^2\delta^{(2)} + \ldots \end{aligned}\right\} \tag{3.7}$$

Similarly, the position of the front $\hat{y} = \hat{\gamma}(\hat{x})$ is expanded:

$$\hat{y} = \gamma^{(0)}(\hat{x}) + \varepsilon\gamma^{(1)}(\hat{x}) + \varepsilon^2\gamma^{(2)}(\hat{x}) + \ldots \tag{3.7a}$$

Substitution of (3.7) into (3.2–3.5) and collecting terms of order unity gives the zeroth-order equations:

$$F_r\left(u^{(0)}\frac{\partial u^{(0)}}{\partial \hat{x}} + v^{(0)}\frac{\partial u^{(0)}}{\partial \hat{y}} + w^{(0)}\frac{\partial u^{(0)}}{\partial \hat{z}}\right) - \frac{F_r}{R_0}v^{(0)} = -(1-\delta^{(0)})\frac{\partial \xi^{(0)}}{\partial \hat{x}} + \frac{\partial \delta^{(0)}}{\partial \hat{x}}(\xi^{(0)} + \hat{z}) \tag{3.8}$$

$$\frac{F_r}{R_0}u^{(0)} = -(1-\delta^{(0)})\frac{\partial \xi^{(0)}}{\partial \hat{y}} + \frac{\partial \delta^{(0)}}{\partial \hat{y}}(\xi^{(0)} + \hat{z}) \tag{3.9}$$

$$\frac{\partial u^{(0)}}{\partial \hat{x}} + \frac{\partial v^{(0)}}{\partial \hat{y}} + \frac{\partial w^{(0)}}{\partial \hat{z}} = 0 \tag{3.10}$$

$$\int_{-\xi^{(0)}}^{1}\left(u^{(0)}\frac{\partial \delta^{(0)}}{\partial \hat{x}} + v^{(0)}\frac{\partial \delta^{(0)}}{\partial \hat{y}}\right)d\hat{z} = 0. \tag{3.11}$$

These equations correspond to a basic state where no-cooling is taking place because the right hand side of (3.11), which represents the cooling, equals zero. The solution is, therefore, identical to the structure upstream, i.e.,

$$\left.\begin{aligned} &u^{(0)} = 1; \quad v^{(0)} = w^{(0)} = \delta^{(0)} = 0; \\ &\xi^{(0)} = -\frac{F}{R_0}\hat{y}; \quad \gamma^{(0)} = R_0/F_r \end{aligned}\right\} \tag{3.12}$$

The first-order balances are:

$$F_r\left(\frac{\partial u^{(1)}}{\partial \hat{x}} - R_0^{-1}v^{(1)}\right) = -\frac{\partial \xi^{(1)}}{\partial \hat{x}} + \frac{\partial \delta^{(1)}}{\partial \hat{x}} \times \left(-\frac{F_r}{R_0}\hat{y} + \hat{z}\right) \tag{3.13}$$

$$\frac{F_r}{R_0}u^{(1)} = -\frac{\partial \xi^{(1)}}{\partial \hat{y}} - \frac{F_r}{R_0}\delta^{(1)} + \frac{\partial \delta^{(1)}}{\partial \hat{y}} \times \left(-\frac{F_r}{R_0}\hat{y} + \hat{z}\right) \tag{3.14}$$

$$\frac{\partial u^{(1)}}{\partial \hat{x}} + \frac{\partial v^{(1)}}{\partial \hat{y}} + \frac{\partial w^{(1)}}{\partial \hat{z}} = 0 \tag{3.15}$$

$$\frac{\partial \delta^{(1)}}{\partial \hat{x}}\left(1 - \frac{F_r}{R_0}\hat{y}\right) = -\hat{Q} \tag{3.16}$$

and the corresponding first-order boundary conditions can be written as:

$$u^{(1)} = 0;\quad \hat{x} = 0;\quad -\infty < \hat{y} \leqslant R_0/F_r \tag{3.17a}$$

$$v^{(1)} = w^{(1)} = 0;\quad \hat{x} = 0;\quad -\infty < \hat{y} \leqslant R_0/F_r \tag{3.17b}$$

$$\xi^{(1)} = 0;\quad \hat{x} = 0;\quad -\infty < \hat{y} \leqslant R_0/F_r \tag{3.17c}$$

$$\hat{Q} = \hat{Q}(\hat{x},\hat{y});\quad 0 \leqslant \hat{x} \leqslant \infty;\quad -\infty < \hat{y} \leqslant \hat{\gamma}(\hat{x}) \tag{3.17d}$$

$$w^{(1)} = 0;\quad \hat{z} = 1 \tag{3.17e}$$

$$w^{(1)} = \frac{\partial \xi^{(1)}}{\partial \hat{x}} - \frac{F_r}{R_0} v^{(1)}|_{\hat{z}=-\xi^{(0)}};$$

$$\hat{z} = -\frac{F_r}{R_0}\hat{y} \tag{3.17f}$$

$$-\frac{F_r}{R_0}\gamma^{(1)} + \xi^{(1)} = 0;\quad \hat{y} = R_0/F_r \tag{3.17g}$$

$$\frac{\partial \gamma^{(1)}}{\partial \hat{x}} - v^{(1)} = 0;\quad \hat{y} = R_0/F_r \tag{3.17h}$$

We see that the perturbation scheme has removed the non-linearity from the problem and simplified considerably the structure of the governing equations.

Because the lower layer is taken to be motionless, we must require that no cooling is taking place in regions where this layer is in direct contact with the atmosphere. Thus, our cooling function $\hat{Q}$ should satisfy:

$$\hat{Q} = 0;\quad 0 \leqslant \hat{x} < \infty;\quad \hat{\gamma}(\hat{x}) \leqslant \hat{y} < \infty \tag{3.18}$$

In order to satisfy this condition, we choose $\hat{Q}$ to be of the form:

$$\left.\begin{aligned} &\hat{Q} = (1 - \hat{\xi})\, G(\hat{x});\quad -\infty < \hat{y} \leqslant \hat{\gamma}(\hat{x}) \\ &\hat{Q} = 0;\quad \hat{\gamma}(\hat{x}) < \hat{y} < \infty \end{aligned}\right\} \tag{3.19}$$

where $G(\hat{x})$ will be specified later. Relation (3.19) states that the cooling function decreases toward the left and vanishes at the current's edge as required. This choice of $\hat{Q}$ does not only satisfy (3.18) but also simplifies the problem considerably because, as we shall see, it corresponds to a density field which is independent of $\hat{y}$. It will be demonstrated later that this choice of $\hat{Q}$ is supported by considerations related to the actual heat-exchange process.

Since (3.16) represents a balance of terms of $O(\varepsilon)$ and the right-hand side of (3.5) is of $O(\varepsilon)$, we should, for consistency, neglect terms of $O(\varepsilon)$ in (3.19). In other words, terms of $O(\varepsilon)$ in $\hat{Q}$ contribute to the second-order balances and should be ignored as small. Thus, we neglect the contribution of $\varepsilon\xi^{(1)}$ to $\hat{Q}$ and approximate (3.19) by:

$$\left.\begin{aligned} &\hat{Q} = \left(1 - \frac{F_r}{R_0}\hat{y}\right) G(\hat{x});\quad 0 \leqslant \hat{x} < \infty; \\ &\quad -\infty < \hat{y} \leqslant \hat{\gamma}(\hat{x}) \\ &\hat{Q} = 0;\quad 0 \leqslant \hat{x} < \infty;\quad \hat{\gamma}(\hat{x}) < \hat{y} < \infty \end{aligned}\right\} \tag{3.20}$$

4. Solution

To obtain the solution to the problem we proceed as follows. First, we insert (3.20) into (3.16) and integrate in $\hat{x}$ to find:

$$\delta^{(1)} = \int_0^{\hat{x}} G(\hat{x})\, \mathrm{d}\hat{x} + F(\hat{y}) \tag{4.1}$$

where the integration constant $F(\hat{y})$ is to be determined. Since $\delta^{(1)} = 0$ at $\hat{x} = 0$ for all $y \leqslant R_0/F_r$, it follows that $F(\hat{y}) = 0$ so that $\delta^{(1)}$ is independent of $\hat{y}$, i.e.,

$$\delta^{(1)} = \int_0^{\hat{x}} G(\hat{x})\, \mathrm{d}\hat{x} \tag{4.2}$$

In view of this, the third term on the right hand side of (3.14) vanishes and one concludes that $u^{(1)}$ is not a function of $\hat{z}$ [i.e., $u^{(1)} = u^{(1)}(\hat{x},\hat{y})$]. Therefore, the only term which can balance the term which includes $\hat{z}$ in (3.13) is the term which includes $v^{(1)}$. Consequently, $v^{(1)}$ is of the form:

$$-\frac{F_r}{R_0} v^{(1)} = \frac{\partial \delta^{(1)}}{\partial \hat{x}}\hat{z} + M(\hat{x},\hat{y}), \tag{4.3}$$

where $M(\hat{x},\hat{y})$ is to be determined. By substituting (4.3) into (3.13), (3.14) and (3.15), inspecting the new resulting set of equations and the boundary conditions, one finds that $M = 0$ and:

$$\left.\begin{aligned} &v^{(1)} = -\frac{R_0}{F_r}\frac{\partial \delta^{(1)}}{\partial \hat{x}}\hat{z};\quad u^{(1)} = 0; \\ &\xi^{(1)} = -\frac{F_r}{R_0}\delta^{(1)}\hat{y} \\ &w^{(1)} = 0;\quad \gamma^{(1)} = -(R_0/F_r)\,\delta^{(1)} \end{aligned}\right\} \tag{4.4}$$

This solution satisfies the governing equations and the boundary conditions for all $\hat{x}$ and $\hat{y}$ and thus is a valid solution to the problem.

With the aid of (3.7), (3.12), (4.2) and (4.4), the total solution can be written in the form:

$$\hat{u} = 1 + O(\varepsilon^2); \quad \hat{v} = -\varepsilon \frac{R_0}{F_r} G(\hat{x})\,\hat{z} + O(\varepsilon^2) \tag{4.5}$$

$$\hat{\xi} = -\frac{F_r}{R_0}\left(1 + \varepsilon \int_0^{\hat{x}} G(\hat{x})\,\mathrm{d}\hat{x}\right)\hat{y} + O(\varepsilon^2) \tag{4.6}$$

$$\hat{\gamma} = \frac{R_0}{F_r}\left(1 - \varepsilon \int_0^{\hat{x}} G(\hat{x})\,\mathrm{d}\hat{x}\right) + O(\varepsilon^2) \tag{4.7}$$

$$\hat{w} = 0. \tag{4.8}$$

In terms of the density field $\delta(\hat{x})$, these variables are:

$$\left.\begin{aligned} &\hat{u} \approx 1; \quad \hat{v} \approx -\frac{R_0}{F_r}\frac{\partial \delta}{\partial \hat{x}}\hat{z}; \quad \hat{w} \approx 0 \\ &\hat{\xi} \approx -\frac{F_r}{R_0}(1 + \delta)\hat{y}; \quad \hat{\gamma} \approx \frac{R_0}{F_r}(1 - \delta) \end{aligned}\right\} \tag{4.9}$$

We see that while the interface depth is altered due to the cooling, the flow in the $\hat{x}$ direction remains unchanged. The cross-stream velocity ($\hat{v}$) is generated by the cooling and its corresponding balance of forces is similar to the so-called "thermal wind" relationship.

To illustrate the properties of (4.9) and to analyze its various effects we shall consider a cooling function $[\hat{Q}(\hat{x},\hat{y})]$ whose $\hat{x}$ dependency $[G(\hat{x})]$ and corresponding density field are:

$$\left.\begin{aligned} G(\hat{x}) &= \begin{cases} A\hat{x}; & 0 \leqslant \hat{x} \leqslant 1 \\ A; & 1 < \hat{x} < \infty \end{cases} \\ \delta(\hat{x}) &= \begin{cases} \varepsilon A(\hat{x})^2/2; & 0 \leqslant \hat{x} \leqslant 1 \\ \varepsilon A(\hat{x} - \tfrac{1}{2}); & 1 \leqslant \hat{x} < \infty \end{cases} \end{aligned}\right\} \tag{4.10}$$

where A is a known constant of order unity (see Fig. 3). Under these conditions, the detailed solution is:

$$\hat{u} \approx 1; \quad 0 \leqslant \hat{x} < \infty \tag{4.11}$$

$$\hat{v} \approx \begin{cases} -\varepsilon \dfrac{R_0}{F_r} A\hat{x}\hat{z}; & 0 \leqslant \hat{x} \leqslant 1 \\ -\varepsilon \dfrac{R_0}{F_r} A\hat{z}; & 1 < \hat{x} < \infty \end{cases} \tag{4.12}$$

$$\hat{w} \approx 0: \quad 0 \leqslant \hat{x} < \infty \tag{4.13}$$

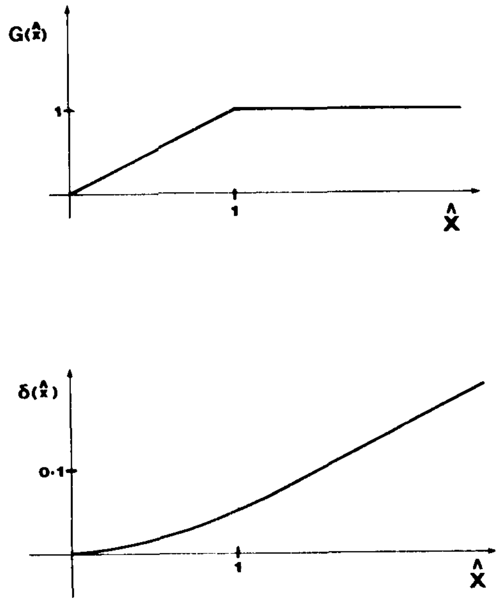


Fig. 3. The function $G(\hat{x})$ (top panel) and the corresponding density function $\delta(\hat{x})$ (lower panel) adopted for the model ($A = 1$, $\varepsilon = 0.1$).

$$\hat{\xi} \approx \begin{cases} -\dfrac{F_r}{R_0}[1 + \varepsilon A(\hat{x})^2/2]\hat{y}; & 0 \leqslant \hat{x} \leqslant 1 \\ -\dfrac{F_r}{F_0}[1 + \varepsilon A(\hat{x} - \tfrac{1}{2})]\hat{y}; & 1 < \hat{x} < \infty \end{cases} \tag{4.14}$$

$$\hat{\gamma} \approx \begin{cases} \dfrac{R_0}{F_r}[1 - \varepsilon A^2(\hat{x})^2/2]; & 0 \leqslant \hat{x} \leqslant 1 \\ \dfrac{R_0}{F_r}[1 - \varepsilon A(\hat{x} - \tfrac{1}{2})]; & 1 < \hat{x} < \infty \end{cases} \tag{4.15}$$

The interface depth ($\hat{\xi}$), as a function of $\hat{x}$ and $\hat{y}$, is shown in Fig. 4. This figure indicates that the cooling increases the interface tilt and shifts the position of the front toward the right (as also shown in Fig. 5). The velocity vector ($\hat{u}\mathbf{i} + \hat{v}\mathbf{j}$) rotates counterclockwise with depth as shown in Fig. 6; at $\hat{z} = 0$ the horizontal component $\hat{v}$ vanishes so that the velocity vector is directed along the $\hat{x}$ axis.

Before concluding the present discussion, it is appropriate to examine the validity of the pertur-

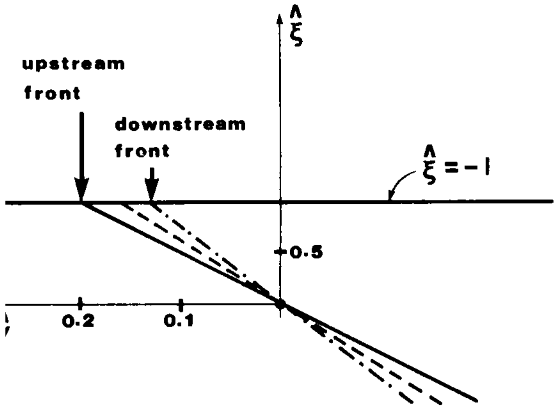
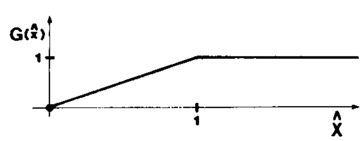


Fig. 4. The predicted interface depth as a function of $\hat{x}$ and $\hat{y}$. The solid line (——) denotes the interface at $\hat{x} = 0$; the dashed line (––––) and dashed-dotted line (–·–·–·) correspond to the interface at $\hat{x} = 2$ and $\hat{x} = 4$ respectively ($A = 1$, $\varepsilon = 0.1$, $F_r/R_0 = 5$). The line $\hat{\xi} = -1$ corresponds to the undisturbed free surface. Note that the interface steepens due to the cooling; this steepening results from cross-stream velocities which are generated by the horizontal density gradients induced by the cooling.



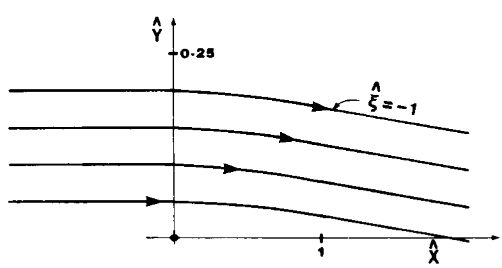


Fig. 5. The displacements of the surface streamlines [and the front ($\xi = -1$)] as a result of the cooling ($A = 1$, $\varepsilon = 0.2$ and $F_r/R_0 = 5$). The migration of the streamlines to the right of their upstream position results from cross-stream velocities which are generated by a downstream pressure gradient caused by the heat loss.

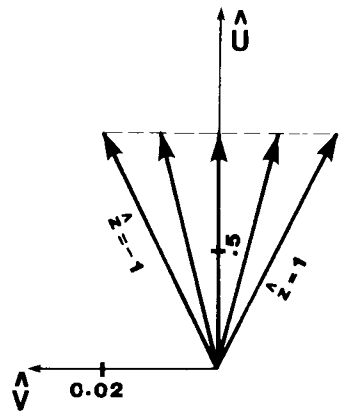


Fig. 6. The velocity vector ($u\mathbf{i} + v\mathbf{j}$) as a function of depth ($A = 1$, $\varepsilon = 0.2$ and $F_r/R_0 = 5$). The counterclockwise rotation with depth results from the fact that the pressure gradients are depth dependent even though the density is not (see text and eq. 2.4).

bation scheme by considering the second-order balances:

$$F_r\left[\frac{\partial u^{(2)}}{\partial \hat{x}} - \frac{v^{(2)}}{R_0}\right] = \delta^{(1)}\frac{\partial \xi^{(1)}}{\partial \hat{x}} - \frac{\partial \xi^{(2)}}{\partial \hat{x}} + \xi^{(1)}\frac{\partial \delta^{(1)}}{\partial \hat{x}} + \frac{\partial \delta^{(2)}}{\partial \hat{x}}(\xi^{(0)} + z)$$

$$\frac{F_r}{R_0}u^{(2)} = \delta^{(2)}\frac{\partial \xi^{(0)}}{\partial \hat{y}} + \delta^{(1)}\frac{\partial \xi^{(1)}}{\partial \hat{y}} - \frac{\partial \xi^{(2)}}{\partial \hat{y}} + \frac{\partial \delta^{(2)}}{\partial \hat{y}}(\xi^{(0)} + \hat{z})$$

$$\frac{\partial u^{(2)}}{\partial \hat{x}} + \frac{\partial v^{(2)}}{\partial \hat{y}} + \frac{\partial w^{(2)}}{\partial \hat{z}} = 0$$

$$\xi^{(1)}\frac{\partial \delta^{(1)}}{\partial \hat{x}} + (1 + \xi^{(0)})\frac{\partial \delta^{(2)}}{\partial \hat{x}} = 0$$

These equations show that the second correction to the velocity field is of $O(\varepsilon^2)$ because $v^{(2)} \sim O(v^{(1)})$. In view of this, it is expected that the perturbation scheme will be valid as long as ε is small compared to unity. That is, the perturbation scheme is uniformly valid and gives the correct solution everywhere.

As mentioned earlier, the scaling that we have used (3.1) is adequate as long as the displacement of the flow in the y direction is of $O(R_d)$ or smaller. Using the solution (4.9) in a dimensional form, it is easy to show that this will be the case as long as $\delta \sim O(F_r)^{1/2}$. This condition is satisfied by many flows of practical interest since δ, the ratio between the horizontal temperature gradient and the vertical gradient, is typically of $\sim O(0.1)$ and $(F_r)^{1/2}$ is of the same order for $U_0 \sim 0.5$ H m s^{-1}, $\Delta\rho/\rho \sim O(10^{-3})$ and $H \sim 500$ m.

5. Gulf Stream displacements

In this section, we shall qualitatively apply our results to the separated Gulf Stream and examine the influence of cooling on its mean position and the position of its front. As mentioned earlier, we shall focus our attention on the region east of Cape Hatteras, where the Gulf Stream flows toward the northeast and gradually loses its heat to the atmosphere (see Fig. 1).

Before discussing the numerical predictions of the model and their relationship to the behavior of the Gulf Stream, it is appropriate to comment on the applicability of the model to the actual flow. Obviously, the model is too simple to account for all the details of the actual flow, and for giving quantitative predictions. This results from our approximations, the most important one being the representation of the vertical structure by two layers.

As we saw earlier, when atmospheric cooling operates on such a system, the resulting upper layer density is independent of depth. Clearly, the actual decrease in temperature is not necessarily depth independent because the actual upper layer is stratified. However, various observations of the Gulf Stream suggest that the effects of winter cooling on the Stream penetrate as deep as 600 m (see e.g., Fig. 7 and the temperature maps presented by Gorshkov (1978)) so that the two-layer representation is probably qualitatively adequate. This is not to say that the actual mixing processes are identical to the cooling and resulting mixing considered in this model. Rather, we merely wish to point out that because the actual cooling penetrates deeply into the ocean, the two-layer approximation of the cooling area is as good as the two-layer approximation upstream where no cooling is taking place.

An additional approximation which has been used and should be discussed in some detail is the choice of a cooling function which decreases toward the left. Presently, there are no sufficiently accurate heat flux measurements which could support this choice but a qualitative examination of the actual mechanism involved in the heat loss process shows that such a decrease is adequate. To illustrate this point, consider a flow similar to the two-layer model shown in Fig. 2 but with an upper layer which is continuously stratified. Under such conditions, all the isopycnals intersect the free surface so that the sea surface temperature increases toward the right. If the temperature of the atmosphere is uniform and the heat loss is proportional to the temperature difference between the air and the sea surface, then the heat loss increases toward the right as has been assumed. These considerations support, therefore, the choice of a heat loss function which decreases toward the left.

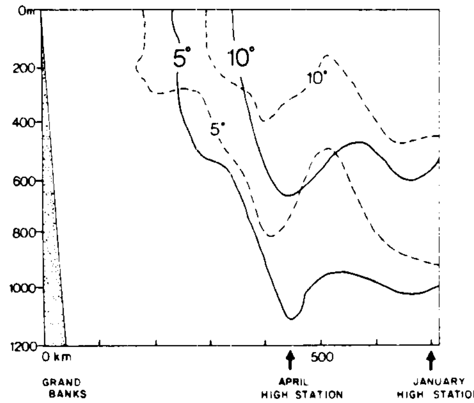


Fig. 7. Vertical section showing the 5 °C and 10 °C isotherms east of the Grand Banks in January 1964 (dashed lines) and April 1964 (solid lines). Reproduced from Worthington (1976). Note that the interface steepens due to cooling as predicted theoretically [Fig. 4 and eq. (4.9) with fixed F_r/R_0 and $\delta \sim 0.5$ which is typcial for the Stream].

We shall now address the relationship between the predictions of the model and the Gulf Stream. So far, there have not been any sufficiently detailed estimates of the actual heat loss which could be adequately used to specify the heat loss function (Q). Therefore, we shall use the observed density gradient (δ) as an input for our model. Density maps prepared by Gorshkov (1978) for the sea surface, 100 m depth and 200 m depth suggest that a horizontal density increase of $\sim 1 \times 10^{-3}/1200$ km and $\sim 0.5 \times 10^{-3}/1200$ km is typical for the late winter and summer, respectively. Note that these density gradients are of the same order as those that one would obtain by considering the actual heat loss (5×10^9 m^{-2} yr^{-1}) operating on a current similar to the Stream.

If the upstream density difference between the two layers is taken to be, say, 2×10^{-3}, $H \sim 400$ m and $f \sim 10^{-4}$ s^{-1}, then in view of (4.9), the horizontal density gradients will produce cross-stream velocities of $\sim$3 cm s^{-1} (late winter) and $\sim$1.5 cm s^{-1} (summer). With a mean flow of $\sim$0.40 m s^{-1}, these velocities will cause off-shore southward deflections (Fig. 8) of $\sim$90 km (winter) and $\sim$45 km (summer). Hence, our model predicts that during the late winter, the position of the Gulf Stream northern edge will be $\sim$45 km farther to the south than it is during the summer. This prediction agrees with Fuglister's (1972) analysis of the mean position of the Gulf Stream edge which is shown in Fig. 9. This figure shows that during the late winter, when the heat loss is largest, the mean position of the Stream edge is located farther to the south as

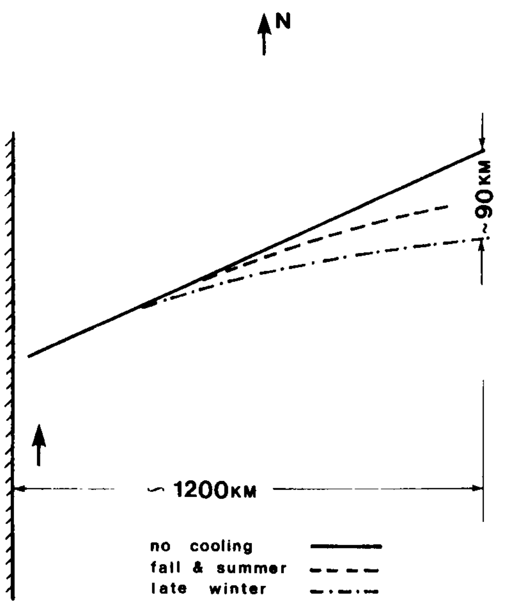


Fig. 8. Schematic diagram of the model applicability to the seasonal variability of the Gulf Stream northern edge. As shown in Figs. 4 and 5, the edge migrates to the right of its upstream position due to cooling. Consequently, the model predicts a southern position during the late winter [when the cooling is the strongest (Worthington, 1972)].

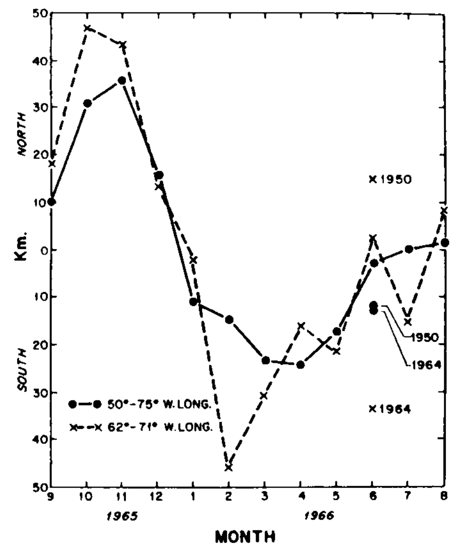


Fig. 9. North to South displacement of the Gulf Stream during 1965–66 [reproduced from Fuglister (1972)]. Note that the observed southward position during late winter (when the cooling is the strongest) agrees with the theoretical prediction (Figs. 5 and 8).

predicted theoretically. Note that the analysis of Fuglister was performed for a wide portion of the stream ($\sim$2000 km) so that the effects of meanders are probably filtered out.

This process, regarding the seasonal variability of the Gulf Stream edge, has been previously considered by both Iselin (1940) and Worthington (1976). Iselin (1940) suggested that the seasonal variability of the Gulf Stream path is related to the seasonal variability of the wind field and Worthington (1976) proposed that it is related to the formation of the 18 °C water south of the Stream. While both processes are possible, our present study indicates that the seasonal movement will be present without any of the previously proposed mechanisms being active. In other words, the present model suggests that the observed behavior may simply result from cooling and loss of heat.

In addition to the prediction regarding the position of the Stream edge, the model predicts a steeper thermocline during the late winter. Using the numerical values given earlier, one finds that the steepening due to cooling is of the same order as the interface slope upstream. This prediction is also supported observationally as shown in Fig. 7 which clearly illustrates that during the late winter the slope of the 5 °C and 10 °C isotherms is considerably larger than it is during the early winter when the heat loss is small.

6. Summary and conclusions

Prior to listing our conclusions, it is appropriate to mention again the main limitations involved in the analysis. The most important assumptions which have been made throughout the study are that the flow is frictionless and non-diffusive, that the vertical structure can be represented by two layers and that the cooling decreases toward the left edge of the current. Scaling arguments and considerations of the observed density field support the validity of these assumptions for the cases under discussion, but it should be kept in mind that under different circumstances the assumptions may not be valid. The results of the study can be summarized as follows:

(i) When a uniform current (flowing above an infinitely deep fluid) enters a region of cooling, horizontal movements in a direction perpendicular to the undisturbed flow are generated.

(ii) These cross-stream movements result from down-stream density gradients and are, in some sense, similar to the "thermal wind" effect. They are depth dependent and, consequently, the velocity vector rotates counterclockwise with depth.

(iii) The imposed heat loss causes a displacement of the front (and the upper portion of the light layer) to the right of the corresponding upstream position. In contrast, the fluid in the lower portion of the upper layer is displaced to the left of its corresponding upstream position.

(iv) The cross-stream velocities (induced by the cooling) cause steepening of the interface; they affect the position of the current but have no effect on the total transport which remains unaltered.

Prediction (iii) and the first part of (iv) were applied to the separated Gulf Stream which flows toward the northeast and gradually loses heat to the atmosphere. The model results agree qualitatively with the observed behavior. On this basis, it is suggested that the observed seasonal movement of the mean Gulf Stream position may be related to the seasonal variability of the heat loss. It is also suggested that the observed steepening of the thermocline during the late winter may result from the associated heat loss.

7. Acknowledgements

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