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On The Robustness Of Multivariable Algebraic Loops With Sector Nonlinearities

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Abstract: This paper is motivated by multivariable 'anti-windup' schemes. Such schemes are often described in a form that includes a multivariable algebraic loop. Multivariable algebraic loops may be particularly problematic in terms of well-posedness (the existence and uniqueness of the solutions) and robust stability. Conditions for well-posedness of such loops have been analysed previously. Here we extend this analysis to consider robust stability of the algebraic loop with small time-delays. It is shown here that sufficient conditions for robustness of the algebraic loop can be directly incorporated into an LMI based static anti-windup controller synthesis. In addition, an alternative implementation of the algebraic loop is shown to have robustness without altering the LMI set up.

1. Introduction

The performance of a feedback system with a well-designed linear controller may degrade significantly and in some cases lead to instability in the presence of control input saturation. One of the techniques to deal with this control input constraint while optimising the closed loop linear performance is to add 'anti-windup' compensation that may be static or dynamic. A nonlinear algebraic loop may arise in such schemes that may make the feedback system suffer from problems related to the issues of well-posedness and robust stability. Algebraic feedback systems involving diagonal nonlinearities in the sector $[0,I]$ are considered here, including saturation nonlinearities which fall in this category.

Several previous authors have considered some issues on well-posedness, stability, and the effect of time delays for linear and nonlinear feedback systems. For example, a piecewise linear expression of a vector saturation function is used in [10]. In there, it is assumed that the induced L_2 norm of a gain matrix within a nonlinear algebraic loop in the anti-windup scheme is less than unity to ensure the existence of a well-defined nonlinear inverse operator. This operator must be causal for L_2 stability of the feedback loop [3]. In [1], conditions for well-posedness related to L_2 stability due to time delays are given for scalar linear distributed systems. Well-posedness and stability of interconnected dynamical systems has been presented in [12]. Using the notion of 'w-stability', it has also been pointed out in [4] that for a linear dynamic feedback system this assumption is also a necessary condition for robust stability for the linear feedback systems under small time delays. A similar illustration for scalar system with regard to well-posedness is also found in [6]. In [14], 'realizability conditions' for nonlinear feedback systems that were described as unrelated to stability was studied. For certain cases, time delays in the feedback loops were shown to be sufficient for realisability of the nonlinear systems.

In [5], the possibility of having algebraic loops in a dynamic anti-windup compensator can be practically avoided in the LMI-based synthesis only by restricting attention to a strictly proper anti-windup compensator. However, in [9] where a static anti-windup scheme is proposed, algebraic loops cannot be avoided. Since the saturation nonlinearity is memoryless and Lipschitz continuous, conditions for well-posedness in this case are obvious following the application of Lyapunov technique [6]. Alternatively, using the analysis [5], one may conclude that the well-posedness of the feedback system has actually been ensured by the LMI-based synthesis of the static anti-windup compensator. In addition, under the general anti-windup framework of [7], it is assumed in [9] that a certain matrix parameter (H_2) has to be square invertible. It has been suggested in [8] that this assumption can be included in a 'direct one-step' anti-windup synthesis.

Here, based on an incremental finite gain argument, we present a sufficient condition to ensure the existence of a well-defined nonlinear inverse operator within a nonlinear algebraic loop (well-posedness), which in addition also guarantees robustness to small time delays in not only a linear but also nonlinear algebraic loop. Hence, the L_2 incremental finite gain stability is preserved under small time delay in the loop. This condition is then shown to be able to be incorporated into an anti-windup synthesis problem by, for example, introducing an extra LMI in the existing LMI in [9]. A similar approach may also be applied to the synthesis of a dynamic compensator in [5] allowing algebraic loops. We also present a particular implementation of the algebraic loop that achieves the same robust stability. Simulation results from an example of the application are also given.

1.1 Notation

The following notations are used throughout this paper: For a function $f(.):[0,\infty)\to R^n$, we say $f(.)\in L^n_{pe}$ (the extended L_p space), if for every truncation time T>0 of $f(.)$ we have $f_T(.)\in L_p$. $\sigma(F)$ denotes singular values of F and

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$\lambda(F)$ denotes eigenvalues of F. $\bar{\sigma}(F)$ is the maximum singular values of F . $\rho(F) := \max_{1\le i\le n}|\lambda_i|$ is the spectral radius of F. The induced 2-norm of F is denoted by $\|F\|_{i2} = \bar{\sigma}(F)$.

1.2 Definitions

Definition 1:

Δ={diagonal nonlinearity in the sector [0,I]}

A memoryless nonlinearity $f:[0,\infty)\mathrm{x}R^n \to R^n$ with $f(t,0)=0\,\forall t \ge 0$ is a diagonal nonlinearity in the sector $[0,I]$ if f=diag(f_1, ..., f_n) and $0 \le y_i f_i(t, y_i) \le y_i^2$ for i=1, ...,n.

Definition 2: *L_p Incremental Finite Gain Stability* , [11]

A causal operator N: $L_{pe}^n \to L_{pe}^m$ is said to be incrementally finite gain L_p stable if $N(0)\in L_{pe}^m$, and for all T>0 and $u_a, u_b \in L_{pe}^n$, there exists some gain k>0 such that

$$\|N(u_a) - N(u_b)\|_T \le k\|u_a - u_b\|_T \qquad (1)$$

Note that L_p incremental finite gain stability is a Lipschitz continuity condition on the operator N. For example, any $N\in\Delta$ and N: $L_{pe}^n \to L_{pe}^n$ is L_2 incremental finite gain stable operator with a (Lipschitz) constant k=1. In the paper, the subscripts "T" in the norm notations are dropped for convenience.

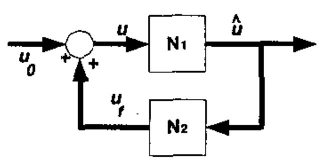

Fig.1 Multivariable loop

Definition 3: *Linear and Nonlinear Algebraic Loop, and Well-posedness*

(i) A linear algebraic loop is formed if each of N_1 and N_2 is a linear dynamic system that has a direct feed-through[1] from input to output. If in addition at least one of them has a nonlinear direct feed-through, then we have a nonlinear algebraic loop.

(ii) A linear/nonlinear multivariable algebraic loop is said to be well-posed if there exist unique solutions to the linear/nonlinear and multivariable implicit equation in its variables of the loop.

2. Motivating Example

We now turn into the specific case of a multivariable algebraic loop that arises in anti-windup schemes such as in [9]. In this scheme, the nominal controller is $K(s)$ with state space realisation (A_c,B_c,C_c,D_c), input e, and output $\bar{u}$.

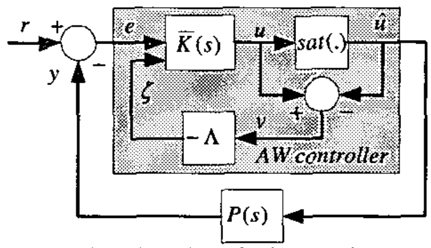

Fig.2 A static anti-windup scheme

The compensation is made by the signal

$$\zeta = \begin{bmatrix}\zeta_1^T & \zeta_2^T\end{bmatrix}^T = -\begin{bmatrix}\Lambda_1^T & \Lambda_2^T\end{bmatrix}^T (u - \hat{u}). \qquad (2)$$

ζ_1 and ζ_2 act on the controller's states x_c and on its output $\bar{u}$ respectively. The output $\hat{u}$ of the anti-windup controller, $\bar{K}(s)$, may be written as

$$u = H_2(C_c x_c + D_c e) + (I - H_2)\hat{u} = H_2\bar{u} + F\hat{u} = u_0 + F\hat{u}.$$
$$\hat{u} = sat(u) \qquad (3)$$

where $H_2 = (I + \Lambda_2)^{-1}$ is required to be an invertible square matrix [7]. For any given u_0, since (3) describes a nonlinear and multivariable implicit equation in $\hat{u}$, it is possible that this equation has none, one, some, many, or an infinite number of solutions.

A static anti-windup controller may lead to non unique solutions. Ideally we want to have a unique solution for any value of u_0 including zero. Since, in this case, the nonlinearities are piecewise linear, an exhaustive set of tests based on each linear region may be conducted. Consider the case of a two input system with $u=[u_1\ u_2]^T$, $e_1=[1\ 0]^T$, and $e_2=[0\ 1]^T$, then there will be 3^2=9 possible solutions for u and $\hat{u}$ (the combinations of $u_i \le 1$, $u_i \ge 1$, and $|u_i| \le 1$ for i=1,2).

For Example, consider the numerical result taken from [9] with Λ_2=[210.4 211.7;170.0 168.3]^T and assume first that the solution is in the linear region, i.e.$|u|\le 1$, then the only solution will be $u = \hat{u} = 0$ provided the linear system is well-posed i.e. $(I-F)=H_2$ is invertible and proper [15], where $F=I-H_2=\Lambda_2(I+\Lambda_2)^{-1}$. However, in the saturation (nonlinear) region, the following two are also possible solutions,

- $u_1 \ge 1$ and $u_2 \le -1$: $\hat{u} = [1\ \ -1]^T$
  $\Rightarrow u=F\hat{u}$ =[2.9148 -2.9168]^T;
- $u_1 \le -1$ and $u_2 \ge 1$: $\hat{u} = [-1\ \ 1]^T$
  $\Rightarrow u=F\hat{u}$ =[-2.9148 2.9168]^T.

Hence, for some values of u_0 the nonlinear multivariable algebraic loop in the anti-windup controller gives more

[1] A linear operator with a state space realisation (A,B,C,D) has a direct feed-through if $D\neq 0$.

than one solution. In other words, this particular anti-windup compensator is not well-posed[2].

3. Well-posedness and Robust Stability

In order to analyse the relationship between well-posedness and robust stability the following results shall be utilized. We first examine stability robustness for small signals near $u = \hat{u} = 0$. In this case, the saturation nonlinearity becomes the identity map. In N_2 we add, in addition to linear term F, a small multiplicative delay, $e^{-\varepsilon s}$, to represent computational or other delays in the implementation of the anti-windup scheme.

Theorem 1: ***Linear Stability Robustness***
Let $N_1=I$ and $N_2=e^{-\varepsilon s}F$ with $\varepsilon>0$ and $F\in R^{n\times n}$. The feedback system in Fig.1 (asymptotically) stable if and only if

$$\rho(F) < 1. \tag{4}$$

Proof: Let $F = T\Lambda_F T^{-1}$ where Λ_F=complex Schur form of F and λ_i = the i-th eigenvalue of F. The characteristic equation of the system is

$$\det(I - e^{-\varepsilon s}F) = \prod_{i=1}^{n}(1-\lambda_i e^{-\varepsilon s}) = 0$$

For i=1, 2, …,n and k=0, ±1, ±2,…, the closed loop poles are

$$p_{i,k} = \frac{1}{\varepsilon}\ln\lambda_i = \frac{1}{\varepsilon}\ln|\lambda_i| + j\frac{\arg(\lambda_i)+2k\pi}{\varepsilon}.$$

The condition (4) is obtained by requiring $\mathrm{Re}\{p_{i,k}\}<0$. ■

A similar condition in terms of $\|F\|_{i2}$ to Theorem 1 for dynamic systems is found in [4] that allows different time delays in different channels of N_2. We now wish to extend the previous results to consider the large signal behaviour of the saturation nonlinearity. To do this, we use the well known Incremental Small Gain Theorem, e.g. in [13] or [11].

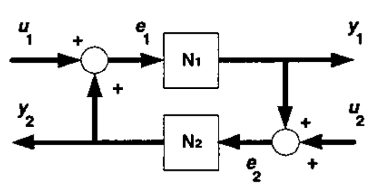

Fig.3 Multivariable feedback system

Theorem 2: ***Incremental Small Gain Theorem: Well-posedness and L_p Stability***
Consider the multivariable feedback system of Fig.3. Let the operators $N_1 : L^n_{pe} \to L^m_{pe}$ and $N_2 : L^m_{pe} \to L^n_{pe}$ be L_p incremental finite gain stable with gains k_1 and k_2 respectively. Then if $k_1k_2<1$, for a given $u_1 \in L^n_{pe}$ and $u_2 \in L^m_{pe}$ there exist unique solutions $e_1, y_2 \in L^n_{pe}$ and $e_2, y_1 \in L^m_{pe}$, i.e. the feedback system is well-posed. Moreover, the feedback system is L_p incremental finite gain stable from u_1, u_2 to y_1, y_2. ■

We now use Theorem 2 to study the nonlinear algebraic loop in the the feedback system in in Fig.1 as follows:

Theorem 3: ***Nonlinear Robustness and Well-posedness***
Let $N_1\in\Delta$ and $N_2=e^{-\varepsilon s}F$ with $\varepsilon>0$ and $F\in R^{n\times n}$. If

$$\|F\|_{i2} < 1 \tag{5}$$

then the feedback system in Fig.1 is L_2 incremental finite gain (robust) stable and well-posed, i.e. there exist unique solutions $u, u_f, \hat{u}$ for any given u_o.

Proof: N_1 is L_2 incremental finite gain stable operator with the constant k_1=1. Since $\|e^{-\varepsilon s}\|_{i2} \le 1$, for N_2, it follows that

$$\|N_2(\hat{u}_a) - N_2(\hat{u}_b)\|_2 = \|e^{-\varepsilon s}F(\hat{u}_a - \hat{u}_b)\|_2$$

$$\|N_2(\hat{u}_a) - N_2(\hat{u}_b)\|_2 \le k_2\|\hat{u}_a - \hat{u}_b\|_2 ;$$

where $k_2 = \|F\|_{i2}$. Hence, N_2 is also L_2 incremental finite gain stable with the gain k_2. Then, condition (5) follows by applying Theorem 2.■

The same conditions will be obtained for the cases if $N_1\in I$ or $N_1\in\Delta$ and $N_2=F\in R^{n\times n}$. As in [2], the condition in Theorem 3 can be extended by introducing an appropriate scaling matrix D=diag(d_1, …,d_n)>0 as follows:

Corollary 1: Nonlinear Robustness with D-scaling Matrix
Let $N_1\in\Delta$, $N_2=e^{-\varepsilon s}F$ with $\varepsilon>0$, and $F\in R^{n\times n}$. The feedback system in Fig.1 is L_2 incremental finite gain robustly stable and well-posed, i.e. there exist unique solutions $u, u_f, \hat{u}$ for any given u_o if

$$\|DFD^{-1}\|_{i2} < 1 \tag{6}$$

where D=diag(d_1, …,d_n)>0 is the scaling matrix.
Proof: Let $N_{1x}:=D\circ N_1\circ D^{-1}$ and $N_{2x}:=D\circ N_2\circ D^{-1}=N_2$. Since N_1=diag(N_{11}, ..., N_{1n})$\in\Delta$ with Lipschitz constant k_1=1, we have

$$\|N_{1x}(z_a) - N_{1x}(z_b)\|_2^2 = \sum_{i=1}^{n}\left[d_iN_{1i}(d_i^{-1}z_{a,i}) - d_iN_{1i}(d_i^{-1}z_{b,i})\right]^2$$

$$= \sum_{i=1}^{n} d_i^2 d_i^{-2}\|(z_{a,i} - z_{b,i})\|_2^2 = \|z_a - z_b\|_2^2 .$$

Hence $N_{1x}\in\Delta$. Similar to the proof in Theorem 3, condition (6) yields an L_2 incremental finite gain stability argument.■

[2] We suspect there is a typographical or numerical error in [9]; since the LMI approach espoused there does guarantee well-posedness.

4. Application of Robust Stability Conditions

Consider again the static anti-windup controller in [9]. The equivalent structure of the part of the controller involving a nonlinear multivariable algebraic loop is shown below.

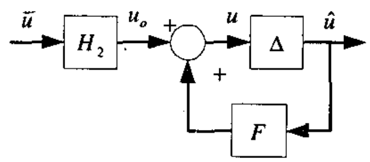

Fig.4 A multivariable algebraic loop in the static anti-windup controller [eq.(3)]

Referring to Fig.1, in this case $N_1 \in \Delta$ and $N_2 = F = \Lambda_2(I+\Lambda_2)^{-1}$, it follows from Theorem 3 that the sufficient condition for the feedback system to be well-posed and L_2 incremental finite gain robustly stable under small time delay in the algebraic loop is

$$\|F\|_{i2} = \left\|\Lambda_2(I+\Lambda_2)^{-1}\right\|_{i2} < 1$$

or equivalently the LMI

$$\Lambda_2 + \Lambda_2^T + I > 0. \qquad (7)$$

This LMI then can be included into the existing one to gain robust stability under small time delay in the algebraic loop.

To match with the existing LMI for designing the static anti-windup controller, one may need to impose a certain structure of some variables, which is not necessary since there exist a scaling matrix such that Theorem 3 is satisfied.

Remark 1:
Since $F = \Lambda_2(I+\Lambda_2)^{-1}$ and using Corollary 1, then we have

$$(D\Lambda_2 D^{-1})^T + (D\Lambda_2 D^{-1}) + I > 0 \qquad (8)$$

Take $D = M^{-1/2}$, since $\Lambda = XM^{-1} = \left[\Lambda_1^T \quad \Lambda_2^T\right]^T$, then, while preserving the structure of variables (M and X) of the existing LMI (see [9]) the sufficient condition will be

$$X_2 + X_2^T + M > 0, \qquad (9)$$

where $X_2 = [0 \quad I]X$.■

Remark 2:
To allow a compromise between time responses and robust stability of the algebraic loop with small time delays, a more restrictive condition than (6) may be stated as

$$\left\|DFD^{-1}\right\|_{i2} < \alpha \qquad (10)$$

where $0 < \alpha \le 1$. As in (8), using the same structure of the variables in the existing LMI, the corresponding LMI of (10) will be

$$(1-\alpha^{-2})X_2^T M^{-1} X_2 + X_2 + X_2^T + M > 0. \qquad (11)$$■

Remark 3:
The LMIs, (9) or (11), allow D scaling, but only by choosing $D = M^{-1/2}$ which is a matrix that occurs elsewhere in the LMIs found in [9]. Unfortunately, at this point in time, we do not see any way of allowing a free search for diagonal $D>0$ whilst retaining the LMI structure. ■

As an alternative to applying additional LMIs to ensure robust stability, we now give a different implementation of the nonlinear algebraic loop. Consider the implementation as shown in Fig.5 by rewriting (3) as

$$\hat{v} = \hat{u} - 0.5u = sat(u) - 0.5u := \Phi(u) \qquad (12)$$

$$u = F.(\hat{v} + 0.5u) + u_o \qquad (13)$$

where $G = (I + 0.5\Lambda_2)^{-1}\Lambda_2$ and $H = (I + 0.5\Lambda_2)^{-1}$.

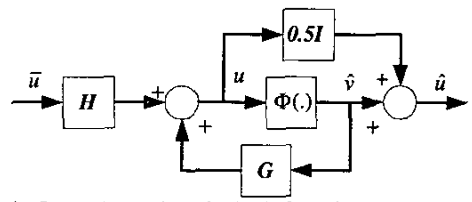

Fig.5 An alternative algebraic loop implementation

Noting that Φ is in the sector $[-0.5I, 0.5I]$ and applying Theorem 3, the condition for well-posedness and robust stability will be

$$\|G\|_{i2} < 2, \qquad (14)$$

which is equivalent to

$$\Lambda_2 + \Lambda_2^T + 2I > 0. \qquad (15)$$

Similar to Remark 1, the D-scaled version of this LMI is

$$X_2 + X_2^T + 2M > 0, \qquad (16)$$

which has already been ensured by the LMI in [9].

5. Numerical Example & Simulation Results

We take the example in [9] and consider the four cases below. The first three cases use the algebraic loop implementation in Fig.4 while the last case uses the implementation in Fig. 5. The constant γ is the L_2 gain performance bound obtained from LMIs optimisation.

The Case A is calculated based on the work of [9], whereas Cases B and C use the results obtained by extending Mulder et.al.'s LMI approach to include the additional LMIs (9) and (11) respectively. Case D is the same as Case A but with the alternative algebraic loop implementation of Fig.5.

Case A: well-posed and not robustly stable

$$\Lambda_2 = \begin{bmatrix} 3.4541 & 4.3733 \\ 3.4763 & 2.4167 \end{bmatrix} \Rightarrow F = \begin{bmatrix} -229.1 & 294.6 \\ 234.1 & -299 \end{bmatrix}$$

$\|F\|_{i2} = 532.4379 > 1$, $\left\|DFD^{-1}\right\|_{i2} = 528.999 > 1$, and $\gamma = 1.5543$

Case B: Case A with additional LMI (9); well-posed and robustly stable

$$\Lambda_2 = \begin{bmatrix} 1536.7 & 1967.6 \\ 1199.7 & 1536.2 \end{bmatrix} \Rightarrow F = \begin{bmatrix} 0.3332 & 0.8535 \\ 0.5204 & 0.3332 \end{bmatrix}$$

$\|F\|_{i2} = 1.0595 > 1$, $\|DFD^{-1}\|_{i2} = 0.99967 < 1$, and γ=1.5543

Case C: Case A with additional LMI (11) and α=0.95; well-posed and robustly stable

$$\Lambda_2 = \begin{bmatrix} 9.2564 & 12.4779 \\ 7.6085 & 9.2564 \end{bmatrix} \Rightarrow F = \begin{bmatrix} 0 & 1.2166 \\ 0.7418 & 0 \end{bmatrix}$$

$\|F\|_{i2} = 1.2166 > 1$, $\|DFD^{-1}\|_{i2} = 0.95 < 1$, and γ=10.5373.

Case D: the same as Case A but with the implementation in Fig.5; well-posed and robustly stable

$$\|DGD^{-1}\|_{i2} = 1.9925 < 2$$

The input reference used in the simulation is $[0.63, 0.79]^T$ time t=1 sec. Case A shows that the condition (5) is a sufficient (not necessary) condition for well-posedness that can be proved by the calculation shown in Section 2. Since (5) is also a sufficient condition for robust stability, robustly stable results may be obtained from Case A, e.g. by setting a different value of feasibility radius in the LMI optimisation. To ensure the robust stability, however, one may need to consider the condition in the LMI-based design regardless the chosen value of the optimisation parameter.

Figures 7 and 8 clearly show that Cases B and C have a robust stability property that Case A does not have. Without any time delay in the algebraic loops, the controller designed in Case B has maintained the output responses as of Case A (see Fig.6). The higher performance cost for Case C shows the price for better responses under small time delays but slower ones without the delay (see Fig.8).

Fig.9 shows that the implementation in Fig.5 ensures the robust stability and is able to produce a quite good transient performance. Based on our experience, one may need to tune the LMI optimisation's parameter such that the calculated parameter Λ_2 of the anti-windup compensator is not too large. Otherwise, (although it gives robustly stable results) the settling time could be long.

6. Concluding Remarks

Motivated by an anti-windup scheme, we have extended sufficient conditions for well-posedness to a closely related concept of robust stability of multivariable algebraic loops involving multivariable diagonal nonlinearities in the loops. Such robust stability ensures that the anti-windup controller implementation is simplified by avoiding "infinite" sensitivity to unmodelled dynamics. A sufficient condition for robust stability of multivariable algebraic loops can be easily included in LMI-based synthesis of an anti-windup controller. It has also been shown that provided the nonlinear algebraic loop in the anti-windup compensator is implemented in a particular structure, the need for additional LMIs to ensure robust stability may be obviated.

7. Acknowledgements

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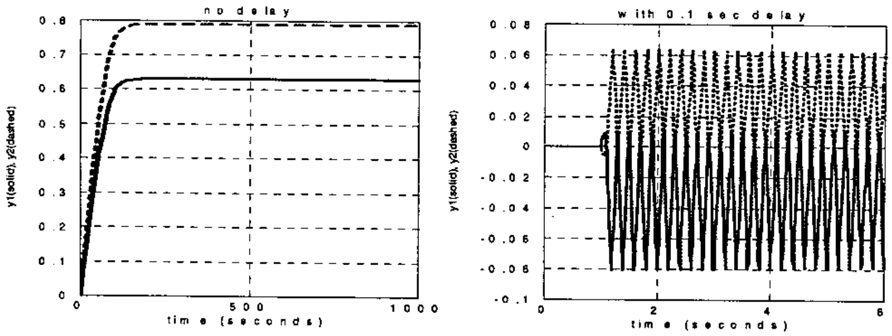

Fig. 6 Case A: LMI Anti-windup design in [9]

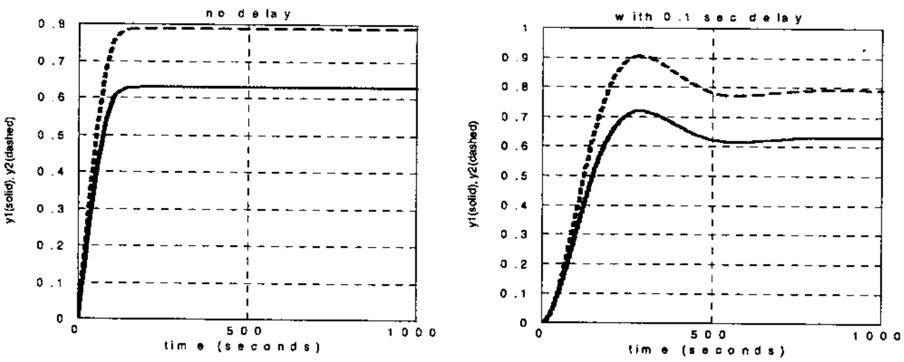

Fig.7 Case B: LMI Anti-windup design plus eq.(9)

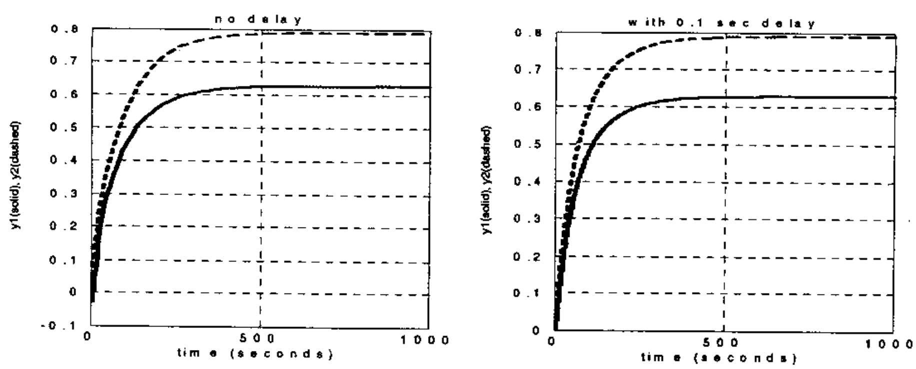

Fig.8 Case C: LMI Anti-windup design plus eq.(11)

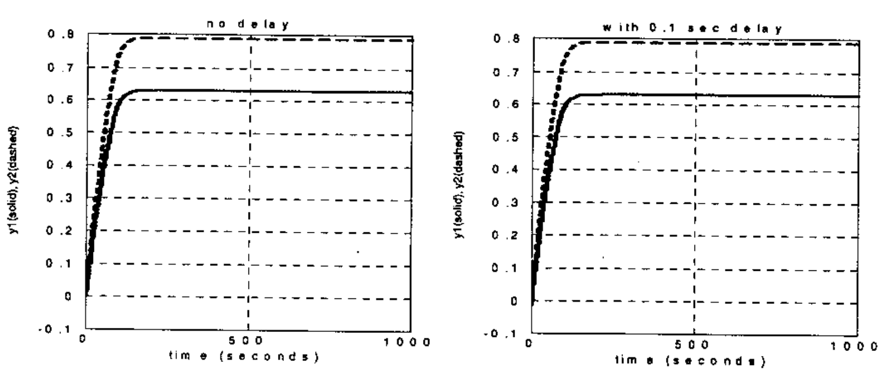

Fig.9 Case D: the same as Case A with the implementation in Fig.5