On the Role of Information in Contests (R)*

Pradeep Dubey†

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Abstract

Consider a contest for a prize in which each player knows his own ability, but may or may not know those of his rivals (the complete or incomplete information regimes). Our main result is that, if the value of the prize is high, more effort and output are engendered under incomplete information; whereas, if the value is low, that distinction goes to complete information. We also examine strategic behavior of a "contest manager" who is privy to information about the abilities of all the players. It turns out that, in order to inspire performance, it is often better for him neither to reveal all nor to conceal all, but to follow a middle path of partial revelation.

JEL Classification: C70, C72, C79, D44, D63, D82.

1 Introduction

Consider a contest in which a prize is awarded to the highest output. Each player's output depends upon his innate ability, over which he has no choice, as well as the effort he chooses to undertake. We suppose, in keeping with the tradition in much of the literature, that a player always knows his own ability. Beyond this, we consider two information regimes: "complete information", where each player also knows the abilities of his rivals; or "incomplete information", where each knows only the probability distribution on their abilities, not the actual realizations. A natural question arises: how does the information regime influence the contest? We show

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†Center for Game Theory, Department of Economics, Stony Brook University and Cowles Foundation for Research in Economics, Yale University

that the answer has to do with the value[1] of the prize. If the value is high then more effort, hence also output, is engendered under incomplete information. In contrast, if the value is low, then that distinction goes to complete information.

The intuition behind the result is extremely simple, and brought out with minimal fuss in a binary world[2] of two players, who could have either low or high ability, and who could "shirk" or "work". If both have similar abilities, then the contest will be evenly poised, and each player will find it worth his while to work since he gets half a shot at the prize. But if their abilities are *widely* disparate, then the weak (strong) will have a low (high) probability of winning the prize, regardless of the effort levels the two have chosen, with the upshot that neither will have incentive to work.[3] We conclude that, under complete information, both will work if they have similar abilities, and they will shirk if they are disparate. Now let us turn to incomplete information. Here a player must choose his effort level conditional only on his own ability, since he does not know those of his rivals. He ascribes positive probability to the event that the rival is similar to him, and — as was said — in this event it pays for him to work. Thus, if the value of the prize is high enough, his *expected* gain from effort overcomes the cost of that effort, even though the effort goes waste when the rival is disparate. Thus each player works unconditionally (regardless of his ability). By the same argument, the situation is reversed for a low prize: the expected gain is less than the cost incurred from work, so both shirk unconditionally.

Of course matters can become a bit more delicate when the disparity in abilities is not too wide, because in this case the players may have incentive to work, even when the prize is of low value and there is incomplete information. However, as the analysis below will show, our result is not marred when we average on a domain of ability pairs that is "reasonably" diverse.

Finally we ask what would happen if a "designer-manager" of the contest were privy to information regarding the abilities of the players[4]. How should he reveal

[1] We assume similarity in players' valuations, i.e., everyone accords a high value to the prize, or everyone accords a low value. ("Mixed" valuations are not considered in this paper, though our techniques can be applied to that case as well.)

[2] But we shall emphasize only those results, and methods of proof, that go beyond the binary world. (See the remarks in Section 3 for a heuristic discussion of various extensions of our model.)

[3] Put another way: the course of the contest is being determined mostly by innate abilities, so why bother to undertake effort that incurs significant cost but has only minor impact on the outcome? This problem is extremely important and is the raison de etre for handicaps, or other devices for creating "level playing fields", that in turn will spur competition. We bypass it only because it lies outside the focus of this paper (but see,e.g., (6), (5), (13)).

[4] A possible scenario for this could be as follows. Suppose the track record of any player's past performance is enough to guage his ability. The rules of the contest could *require* every player to (i) submit his track record to the manager, prior to the contest; and (ii) withhold this information

information about each to the other, in order to inspire more performance? It turns out that often he would do better not to reveal all or to reveal nothing, but to follow instead a middle path of partial revelation.

Related Literature. There is a vast literature on contests for prizes, under conditions of either complete or incomplete information (by way of a list that is indicative, but by no means exhaustive, see (15), (11), (17), (10),(1),(14),(16), (4) and the references therein). But to the best of our knowledge, there has been no comparison of the two information regimes from the standpoint of generating more competition and output. Nor have we come across the strategic revelation of information by a contest manager in this setting.

There is also work — related in spirit — on the value of public information in a Cournot duopoly, see in particular (7) and its references. Apart from the special structure of the Cournot model, the authors in (7) assume that the firms have *symmetric* (complete or incomplete) information, in order to ensure uniqueness and interiority of their NE. Otherwise they get "corner equilibia" for which their static analysis becomes problematic (see section 4 of (7)). In our scenario, ex ante symmetry of the players is not critical[5], and indeed most of our equilibria are in pure strategies, hence in the "corner". But perhaps the biggest difference between the Cournotian approach and ours lies in the point-of-view: they consider participants' payoffs, whereas we focus on total output, though of course the two goals may sometimes be in tandem (see Remark (5b)).

2 The Binary Model

There are two players who, for simplicity, are assumed to be risk-neutral and ex ante symmetric. Each **player** $i \in \{1, 2\}$ can have one of two **abilities**[6] $a \in \{\alpha, \beta\}$ and can choose one of two **effort levels** $e \in \{0, 1\} = \{shirk, work\}$. If i has ability a then the **output** produced by i via effort $0, 1$ is given by a, $k(a)a$ respectively for some $k(a) > 1$.(This serves to *define* a and $k(a)$.)

The abilities α, β are picked independently for the two players with probabilites $\pi, 1 - \pi$. For both players, the cost of effort 0 is 0 and that of effort 1 is $c > 0$. Both place the same value $v > 0$ on a **prize,** which is awarded to the player with the

from the other players. Indeed, unless the law of the land prohibits it, the manager has every incentive to design the contest in this manner, since he stands to benefit from strategic revelation of the information so obtained.

[5] We assume symmetry for the most part purely for ease of presentation, and show how to relax it in section 4.

[6] Both α and β are positive scalars.

higher output, or randomized equally in case of a tie.

Scaling c and v by the same positive factor is tantamount to a change of units in measuring players' payoffs, and leaves the contest unchanged, so w.l.o.g. we fix c and vary v. (We *could* vary the function k too, but do not do so in order to keep the exposition simple.)

This well-defines the **games** $\Gamma_C(\alpha,\beta,\pi,v)$, $\Gamma_I(\alpha,\beta,\pi,v)$ of **complete, incomplete information** respectively.

The game $\Gamma_C(\alpha,\beta,\pi,v)$ is made up of four constituent 2×2 bimatrix **subgames** corresponding to the pairs $(\alpha,\alpha),(\beta,\beta),(\alpha,\beta),(\beta,\alpha),(\beta,\beta)$ which occur with probabilities $\pi^2,(1-\pi)^2,\pi(1-\pi),(1-\pi)\pi$; with players similar in the first two subgames and disparate in the other two. Here a player's pure actions are to shirk or work, and so a pure strategy in $\Gamma_C(\alpha,\beta,\pi,v)$ is a map from the four ability pairs to the two actions.

As for $\Gamma_I(\alpha,\beta,\pi,v)$, it is a 4×4 bimatrix game, in which each player has the pure strategies $\{0,0\}$, $\{1,0\}$, $\{0,1\}$,$\{1,1\}$, where $\{e,f\}$ means that he chooses effort e when his ability is α, and f when β.

We shall vary all the parameters α,β,π,v of our model, since our interest lies not so much in any specific game as in the global behavior over a diverse domain of games. To this end, let $A\subset\mathbb{R}_{++}$ and $P\subset(0,1)$ be arbitrary closed intervals of positive length. And, for any v, let $(\alpha,\beta,\pi)\in A\times A\times P$ be distributed according to some measure λ which assigns positive probability to every open set in $A\times A\times P$.

Our assumptions on the binary model are as follows.

Axiom 1 *(Minimum Valuation)* $v>2c$.

This says that were the prize to be split equally, both players would work (since $v/2>c$). The axiom thus enables us to focus on the failure of work occasioned by strategic competition, rather than inadequacy of the prize.

Axiom 2 *(Monotonicity of Output) $k(a)a$ is strictly monotonic in $a\in A$.*

In other words, the output of work goes up as ability increases.

Axiom 3 *(Sufficient Disparity).There exists $(\alpha,\beta)\in A\times A$ such that $k(\alpha)\alpha<\beta$.*

The mild requirement here is that A be diverse enough to admit a "widely disparate" pair $(\alpha,\beta)\in A\times A$ where α is so weak relative to β that he lags behind β even if he works and β shirks.

3 Main Result

For a clean statement of our result, we trim $A \times A$ a bit, by removing a negligible subset. Let $D = \{(\alpha, \beta) \in A \times A : \alpha = \beta\}$, and $E_1 = \{(\alpha, \beta) \in A \times A : k(\alpha)\alpha = \beta\}$, and $E_2 = \{(\alpha, \beta) \in A \times A : k(\beta)\beta = \alpha\}$. Thus D is the "northeasterly" diagonal of the square $A \times A$, and E_1 is a monotonic curve above D (the intersection of $A \times A$ with the graph of the function $k(a)a$ defined on domain A), and E_2 is the reflection of E_1 around D. Removing these three curves from the square, we put $R = (A \times A) \diagdown (D \cup E_1 \cup E_2)$.

Let us define our **space of games** to be $\Sigma \equiv R \times P$, which differs from $A \times A \times P$ only by a $\lambda-$ null set.

All the games in the space Σ have unique **Nash equilibria** (NE). (Indeed, as we shall see, quite often the NE are in **strictly dominant strategies** (SD).) Taking this fact provisionally on faith, denote the average output in $\Gamma_C(\alpha, \beta, \pi, v)$ and $\Gamma_I(\alpha, \beta, \pi, v)$ at the NE by $\tau_C(\alpha, \beta, \pi, v)$ and $\tau_I(\alpha, \beta, \pi, v)$. Then the overall output on Σ engendered by a prize of value v under the complete and incomplete information regimes is given by

$$\tau_C(v) = \int_\Sigma \tau_C(\alpha, \beta, \pi, v) d\lambda(\alpha, \beta, \pi)$$

and

$$\tau_I(v) = \int_\Sigma \tau_I(\alpha, \beta, \pi, v) d\lambda(\alpha, \beta, \pi)$$

We are ready to state our main result.

Theorem There exist $2c < v_- < v_+ < \infty$ such that $\tau_C(v) > \tau_I(v)$ if $2c < v < v_-$ and $\tau_C(v) < \tau_I(v)$ if $v > v_+$

Proof Partition R into the two sets

$$R_1 = \{(\alpha, \beta) \in A \times A : \beta > k(\alpha)\alpha\} \cup \{(\alpha, \beta) \in A \times A : \alpha > k(\beta)\beta\}$$

and

$$R_2 = \{(\alpha, \beta) \in A \times A : \alpha < \beta, \beta < k(\alpha)\alpha\} \cup \{(\alpha, \beta) \in A \times A : \beta < \alpha, \alpha < k(\beta)\beta\}$$

(Each R_i consists of two regions which are mirror images of one another around the diagonal D, with R_2 adjacent to the diagonal. As mentioned earlier, one may think of R_1 as made up of ability pairs in R that are "widely disparate", and R_2 of the residual pairs that are "not-so-widely disparate".)

Denote

$$\sigma \equiv \frac{c}{v}$$

and observe that, on account of Axiom 1, $0 < 2\sigma < 1$. Also given any ability pair $(\alpha, \beta) \in R$, we shall say that α is **weak** and β is **strong** if $\alpha < \beta$.

A routine calculation reveals that, under complete information, regardless of v (and also necessarily of π, since π can have no effect on NE of the subgames of Γ_C), players work if they are similar; and, if dissimilar, they shirk in region R_1 and "half shirk" in region R_2. Precisely we have that, in the four constituent subgames of $\Gamma_C(\alpha, \beta, \pi, v)$, the unique NE are as follows:

(a) $(1, 1)$ is SD if players are similar and $(\alpha, \beta) \in R$

(b) $(0, 0)$ is SD if players are disparate and $(\alpha, \beta) \in R_1$

(c) the weak, strong shirk with probabilities $(1 - \sigma), \sigma$ respectively if players are disparate and $(\alpha, \beta) \in R_2$

For the analysis of the incomplete information game $\Gamma_I(\alpha, \beta, \pi, v)$, it will be useful to consider two cases.

Case A (Large v): $0 < 2\sigma < 1 - 2\sigma < 1$

Case B (Small v): $0 < 1 - 2\sigma < 2\sigma < 1$.

The NE of $\Gamma_I(\alpha, \beta, \pi, v)$, for the instances relevant for this proof, are as follows. (Their verification, also routine, is indicated in the Appendix).

(d) Case A and $2\sigma < \pi < 1 - 2\sigma \implies$ { work if weak &work if strong} is SD

(ea) Case B and $1 - 2\sigma < \pi < 2\sigma$ and $(\alpha, \beta) \in R_1 \implies$ {shirk if weak & shirk if strong} is SD

(eb) Case B and $1 - 2\sigma < \pi < 2\sigma$ and $(\alpha, \beta) \in R_2 \implies$ {shirk with probability $x = (1 + \pi - 2\sigma)/2\pi$ if weak & shirk with probability $y = (2\sigma - \pi)/2(1 - \pi)$ if strong} is the unique NE

Now notice that, as $v \uparrow \infty$, we have $2\sigma \downarrow 0$ and $1 - 2\sigma \uparrow 1$. Therefore $P \subset (2\sigma, 1 - 2\sigma)$ for large enough v,say $v > v^*$. But then by (a), (b),(c) and (d), in conjunction with the fact that players are chosen disparate with probability $2\pi(1 - \pi) > 0$ in the game $\Gamma_C(\alpha, \beta, \pi, v)$, we have $\tau_C(\alpha, \beta, \pi, v) < \tau_I(\alpha, \beta, \pi, v)$ for *every* $(\alpha, \beta) \in R$ and $\pi \in P$ (i.e., pointwise in Σ, not just on average) for $v > v^*$.So, by continuity of the unique NE, there exists $v_+ < v^*$ such that $\tau_C v) < \tau_I(v)$ if $v > v_+$.as asserted in the theorem.

Next notice that as $v \downarrow 2c$ we have $1 - 2\sigma \downarrow 0$ and $2\sigma \uparrow 1$. Therefore $P \subset (1 - 2\sigma, 2\sigma)$ for small enough $v > 2c$. Also observe that as $v \downarrow 2c$ we have $\sigma \uparrow 1/2$, hence , by (c) and (eb), we see that for $(\alpha, \beta) \in R_2$ the shirk probability of both the weak and the strong converge to $1/2$ as $v \downarrow 2c$ in each of the games $\Gamma_I(\alpha, \beta, \pi, v)$ and $\Gamma_C(\alpha, \beta, \pi, v)$,i.e., in the limit players behave the same, and so produce the same, in the two games whenever their abilities are in R_2 (although we might note in passing that, throughout the convergent process, the shirk probability of the strong in Γ_C $= \sigma > (2\sigma - \pi)/2(1-\pi) =$ shirk probability of the strong[7] in Γ_I, preventing us from showing pointwise dominance[8] of Γ_C over Γ_I, analogous to Case A.) But by (a), (b), (ea) much more is produced in $\Gamma_C(\alpha, \beta, \pi, v)$ than in $\Gamma_I(\alpha, \beta, \pi, v)$ for $(\alpha, \beta) \in R_1$. Moreover R_1 has positive $\lambda-$ probability (since Axiom 3 implies that R_1 contains an open set, and since λ assigns positive probability to every open set). Thus, again appealing to the continuity of unique NE, we infer the existence of v_- as in the theorem.■

3.1 Remarks

(1) (**Complete Characterization of NE)** We list the (unique) NE for all the incomplete information games in $\Sigma \times (2c, \infty)$ that were ignored by us during the proof of the theorem, being irrelevant there.

(f) In Case A, $0 < \pi < 2\sigma \Longrightarrow$ {shirk if weak & work if strong} is NE

(g) In Case A, $1 - 2\sigma < \pi < 1, (\alpha, \beta) \in R_1 \Longrightarrow$ {work if weak & shirk if strong} is NE

(h) In Case A, $1 - 2\sigma < \pi < 1, (\alpha, \beta) \in R_2 \Longrightarrow$ {work if weak & work if strong} is NE

(i) In Case B, $0 < \pi < 1 - 2\sigma \Longrightarrow$ {shirk if weak & work if strong} is NE

(j) In Case B, $2\sigma < \pi < 1, (\alpha, \beta) \in R_1 \Longrightarrow$ {work if weak & shirk if strong} is NE

(k) In Case B, $2\sigma < \pi < 1, (\alpha, \beta) \in R_2 \Longrightarrow$ {work if weak & work if strong} is NE

Notice the equalities (f) "=" (i), (g) "=" (j), (h) "="(k), which hold in the sense that players' strategies are the same, and σ lies on the same side of the relevant "central interval" $(2\sigma, 1 - 2\sigma)$, $(1 - 2\sigma, 2\sigma)$ depending on whether Case A, Case B applies.

Based upon this characterization, one could strengthen the theorem by showing the existence of a *single* threshold, i.e., $v_- = v_+$, under appropriate circumstances. For instance suppose that λ assigns very little probability to R_2, i.e., ability pairs are widely disparate most of the time. Then, by (a), (b), (c), $\tau_C(v)$ will rise very slowly

[7]The inequality is equivalent to $\sigma - \pi\sigma > \sigma - (1/2)\pi$, which follows from the fact that $\sigma < 1/2$ on account of Axiom 1.

[8]Indeed, with $\alpha < \beta$, and $(\alpha, \beta) \in R_2$, suppose $k(\alpha)$ is *very slightly* above 1,while $k(\beta)$ is *much* above 1. Then even though the weak α shirks *less* in Γ_C compared to Γ_I (i.e., $1-\sigma < (1+\pi-2\sigma)/2\pi$ as one may easily check), this is outweighed by the *much* stronger β shirking more in Γ_C compared to Γ_I. Thus more output will be produced in Γ_I than in Γ_C throughout the convergent process, i.e., for all $v \downarrow 2c$.

as we increase v; indeed one can make its slope as small as desired, by reducing λ on R_2.In contrast, using the above characterization, $\tau_I(v)$ will rise at a good pace with v till, *after* overtaking the nearly horizontal output of $\Gamma_C(\alpha, \beta, v)$ at a single crossing $v_{\_} = v_+ = v^*$, it will also become nearly horizontal much beyond v^*.

(3) (**Randomness)** Output could be be subject to random noise. As long as the size ε of the noise is small compared with the diversity of abilities, i.e., $\varepsilon/(\text{length } A)$ is small, it would seem that our analysis essentially remains intact. One needs to substitute the notion of "winning" the prize, (or "losing" it, or getting it "with probability 1/2") with the looser notions of obtaining the prize with "high probability" (or "low probability", or "middling probability").

(4) (**Multiple effort levels)** In a similar vein, one could generalize the model to allow for multiple effort levels, substituting "work", "shirk" with "high","low" effort levels; and suitably extending our hypotheses to this broader setting.

(5a) (**Variable prize**) In many applications, the prize (and hence its value) may depend upon the abilities of both agents as well as their effort levels. This clearly does not affect our analysis, provided the variable value remains uniformly high or uniformly low.

(5b) (**Maximizing players' payoffs)** Suppose v depends on the effort levels, and that its value is considerably enhanced when both work. It is evident that, under incomplete information, not only is $(1, 1)$ the SD yielding maximal output, it is also the unique payoff-maximal point for the players.

(6) (**High vs. Low Prizes)** The reader will have noticed that the "high prize" half of our theorem (about the existence of v_+) is more robust than the "low prize" half (about v_-). In the "high prize" case (i.e., for $v > v_+$ and $\pi \in P$) both players work, in the game $\Gamma_I(\alpha, \beta, \pi, v)$, at *every* point $(\alpha, \beta) \in R = R_1 \cup R_2$; while, if $(\alpha, \beta) \in R_1$, both shirk in the subgames of $\Gamma_C(\alpha, \beta, \pi, v)$ when abilities are disparate, which happens with probability $2\pi(1 - \pi) > 0$. So in order to conclude that $\Gamma_I(\alpha, \beta, \pi, v)$ induces no less effort than $\Gamma_C(\alpha, \beta, \pi, v)$, it really does not matter how players in behave in $\Gamma_C(\alpha, \beta, \pi, v)$ for the remaining ability pairs $(\alpha, \beta) \in R_2$. The situation, however, becomes tricky for "low prizes" (i.e., for $v < v_-$ and $\pi \in P$). Recall that here, for $(\alpha, \beta) \in R_1$, both players shirk in $\Gamma_I(\alpha, \beta, \pi, v)$ whereas, in $\Gamma_C(\alpha, \beta, \pi, v)$, both work when similar and both shirk otherwise. But this outperformance of $\Gamma_C(\alpha, \beta, \pi, v)$ over $\Gamma_I(\alpha, \beta, \pi, v)$) in the region R_1, albeit by a significant amount that is bounded away from zero, may be undone by its underperformance on the complementary region R_2. We needed to do a precise calculation to show that the underperformance went to zero as $v \downarrow 2c$. But is this zero limit an artifact of our binary model?

Let us step beyond the binary model and contemplate a more general setting. One fact is worthy of note: if the diversity of abilities becomes sufficiently pronounced (i.e., length A is large), and if the function $k(a)a$ does not grow fast with $a \in A$ (e.g., it is strongly concave, evincing "sharply decreasing returns to ability"), then R_1 will be a much "bigger" region than R_2, and it might then be reasonable to postulate that λ ascribes considerably higher probability to R_1 than to R_2. This setting enables us to essentially ignore R_2, and bolsters the plausibility of the theorem for the "low prize" case as well.

(7) **(Multiple Prizes)** Though we envisaged a *single* prize, that is given to the highest output, this is not critical. For instance, if the prize were to be split in proportion to the output produced (as is supposed in the vast literature on lobbying[9]), a similar result could be articulated. One would have to break R into different pieces Q_1 and $Q_2 = R \backslash Q_1$, where Q_1 is made up of pairs (α, β) where $k(\alpha)\alpha/\beta$ is "suitably small" (denoting, as ever, the weak by α), and assume that there is enough diversity of abilities as to guarantee that Q_1 occurs with significant probability. After that the argument would proceed along similar lines

4 Ex Ante Asymmetry: Extensions of the Model

So far we took the model to be ex ante symmetric, partly because that is the time-honored tradition, but mostly because it made for a simpler presentation. Now we show how to accomodate ex ante asymmetry, especially in terms of players' information. To this end, we shall need to consider somewhat more complex games Γ^*, Γ^{**} than uptil now. It will turn out nevertheless that Γ^*, Γ^{**} are amenable to analysis via our theorem from the simple binary world. This is essentially because, as shown in the theorem, players have the same fixed dominant action across varying ability pairs (α, β).

Suppose (for $i = 1, 2$) that player $i's$ information regarding the abilities of his rival is given by a partition Π_i of A. Suppose further that there is a probability measure ξ on A, with positive density everywhere on A; and that the ability of each player is picked according to ξ, independently of his rival's ability and of $\pi \in P$. For simplicity, let us continue to stipulate that each player knows his own ability precisely. This well-defines a game $\Gamma^*$with incomplete, asymmetric[10] information (in

[9] See, e.g., (18), (12), (8),(2), (3),(9)

[10] Note that in Γ^* a player i chooses an action contingent on two variables: his own ability α and his information $W \subset \Pi_i$ about the ability of his rival j. In an NE of Γ^*, each such action must be optimal against the map from W to actions that his rival j is playng. That map itself is indexed (conditioned) by $X \subset \Pi_j$ where X is the unique element of Π_j that contains α.(It might help to think of A as a *finite* set, in which case NE exist by Kuhn's theorem and our analysis remains true

which of course the exogenous data Π_i, ξ, etc.of the game is taken to be common knowledge). Assume that Π_i is not "too coarse" in the sense that it robustly admits widely disparate ability-pairs, precisely: there exists $\alpha^* \in A$ and $S \in \Pi_i$ such that $\xi(S) > 0$ and $(\alpha^*, \beta) \in R_1$ for all $\beta \in S$. Since R_1 is an open set, we have also $(\alpha, \beta) \in R_1$ for all $(\alpha, \beta) \in U \times S$ for some neighbourhood U of α^*.But then, from (b) (in the proof of the theorem), it is a *strictly dominant* strategy[11] for player i to shirk whenever his ability is α and the rival's is β, for $(\alpha, \beta) \in U \times S$. Thus shirking will occur with positive probability in the game Γ^*. On the other hand, when we move to the game Γ^{**} of incomplete information, with $\Pi_i = A$ for both i, it follows immediately[12] from (d) (in the proof of the theorem) that it is strictly dominant for both players to always work in Γ^{**}, provided v is large enough. This establishes that there is more output in Γ^{**} than in Γ^*, for large enough v. The dual result regarding the efficacy of complete information for small v also remains intact under suitable hypotheses that we leave to the reader.

Next we show how to vary the other characteristics of the players. For this we return to our basic binary model (the extension to the complex games being straightforward). The players could have differences in their cost c_i of work and valuation v_i of the prize. By independent affine transformations of utilities, we can w.l.o.g. take $c_1 = c_2 = c$. Then our theorem would hold with the amendment that $v_i > v_+$ for both i or $v_i < v_-$ for both i. Next suppose the probability π_i, ascribed by i to his rival being weak, also differed for $i = 1, 2$. In this case one would need to constrain the π_i by requiring that either $2c/v_i < \pi_i < 1 - 2c/v_i$ for both i or else $1 - 2c/v_i < \pi_i < 2c/v_i$ for both i. Finally if even the ability-pair α_i, β_i attributed by i to his rival were to be different across i, the constraint that either $(\alpha_i, \beta_i) \in R_1$ for both i or $(\alpha_i, \beta_i) \in R_2$ for both i, would keep the theorem intact as stated. Otherwise a more nuanced version of our theorem would need to be developed, using Facts 1 and 2 in the Appendix below. Indeed, using those two Facts (plus others, in the same vein, about best replies in different circumstances), one could analyze what happens when there is asymmetric information acquisition, e.g.,when one player learns about the other's abilities but not vice-versa (the fact that such learning has taken place being common knowledge). Finally, nothing stops us from considering different ambient sets A, B from which the abilities of players $1, 2$ are drawn, since our analysis turns — at bottom — not on symmetry between the players but on

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once we postulate that R_1 has positive probability. We took A to be a continuum not because of mathematical necessity but because it seemed more natural.)

[11] i.e., it is optimal for i of ability $\alpha \in U$ to shirk, *regardless of the ability* $\beta \in S$ *and of the action of his rival*

[12] noting that (d) applies for *all* $(\alpha, \beta) \in R$ and *all* $\pi \in P$ (for large enough v).

their being disparate with decent probability. We leave the precise formulation of these extensions for future work.

5 Strategic Revelation of Information

Suppose there is a "manager" of the contest who knows the abilities of both players, while each player knows only his own. The question confronting the manager is whether to make the information public or not. His aim, of course, is to ensure that the contest be keen so that maximal output is produced.

Our analysis so far has shown that if the option is limited to revealing all or nothing, he should reveal all if the prize is small, and reveal nothing if it is big. But there are strategies of partial revelation, in between these two extremes, which may often stand the manager in better stead. We shall show a typical example of this in our binary model.

First fix $(\alpha, \beta) \in R_1$ with β "much stronger" than α. Also suppose $0 < 2\sigma < 1 - 2\sigma < \pi < 1$ and that $\pi \equiv$ probability of α, is "not too far" above $1 - 2\sigma$. (The words within quotes shall become clear shortly.)

Recall that nature picks the four pairs $(\alpha, \alpha), (\alpha, \beta), (\beta, \alpha), (\beta, \beta)$ with probabilities $\pi^2, \pi(1-\pi), (1-\pi)\pi, (1-\pi)^2$. Let the manager reveal players' abilities with probability q if, and only if, they are different.We shall show that with a judicious choice of q the manager can get them to produce more than in either $\Gamma_C(\alpha, \beta, \pi, v)$ or $\Gamma_I(\alpha, \beta, \pi, v)$.

Suppose the manager's "$q-$ revelation" policy, described above, is in place. When the players are revealed to be disparate by the announcement of the manager (an event which occurs with probability $q2\pi(1-\pi)$), both will shirk since the ability-pair is in R_1 (by (b) in the proof of the Theorem). Otherwise the situation is as follows.

When his own ability is α, a player will impute probabilities $\pi, (1-q)(1-\pi)$ to his rival having abilities α, β and there being no announcement (instead of the probabilities $\pi, 1-\pi$ that he imputed in $\Gamma_I(\alpha, \beta, \pi, v)$); which, after normalization, is tantamount to conditional[13] probabilities $\xi, 1-\xi$ of the rival being of type α, β for some $\xi > \pi$ (i.e., the effective probability of the rival being the weak α is *raised)*. Since $\pi > 2\sigma$ to start with, we have $\xi > 2\sigma$. But then by Fact 1 in the appendix, a player will work whenever he is of ability α.

Next when his own ability is β, a player will impute probabilities $(1-q)\pi, (1-\pi)$ to his rival having abilities α, β and there being no announcement (instead of the probabilities $\pi, 1-\pi$ that he imputed in $\Gamma_I(\alpha, \beta, \pi, v)$); which, after normalization,

[13] i.e., conditional on there being no announcement

is tantamount to probabilities probabilities $\delta, 1-\delta$ of α, β where $\delta < \pi$ (i.e., the effective probability of the rival being the weak α is *lowered).* Since π is not too far above $1-2\sigma$ to start with, we can arrange for $\delta < 1-2\sigma$ by a suitable choice of q. But then by Fact 2 in the appendix, a player will work whenever he is of ability β.

Thus under the manager's policy of $q-$ revelation, both agents will shirk when they are revealed different, i.e., they will both shirk with probability $q2\pi(1-\pi)$; and will both work with the complementary probability. By (b) of the proof of the Theorem, they both shirk with higher probability $2\pi(1-\pi)$ in the game $\Gamma_C(\alpha, \beta, \pi, v)$. Thus the $q-$ revelation policy induces more output than $\Gamma_C(\alpha, \beta, \pi, v)$. Also we see from (g) of Remark 1 that the weak α works and the strong β shirks in $\Gamma_I(\alpha, \beta, \pi, v)$. By taking β much larger than $k(\alpha)\alpha$ it is evident that $q-$ revelation also induces more output than $\Gamma_I(\alpha, \beta, \pi, v)$ since it gets the much-more-productive β to work with significant probability.

It is clear that a similar maneuver of partial revelation can be carried out, with the attendant benefits, at many other points in our space Σ of games.

6 Appendix

We indicate the routine calculation of NE of $\Gamma_I(\alpha, \beta, \pi, v)$, by working out one case (the rest being similar). As before, we denote $\alpha < \beta$, i.e., α is weak and β is strong.

Fact 1 If a player i is of ability α, and $i's$ rival is α, β with probabilities $\pi, (1-\pi)$ respectively, then 1 (work) dominates 0 (shirk) for i if $2\sigma < \pi$ and $(\alpha, \beta) \in R$ (and, somewhat more weakly: 0 dominates 1 for i if $2\sigma > \pi$ and $(\alpha, \beta) \in R_1$).

(In the above statement, interpret π as the probability assigned *by* **i** that his rival is the weak α. The analogous assignment by his rival has clearly no bearing on the matter. Also, needless to say, $\sigma \equiv c/v$ refers to **i's** cost c and **i's** valuation v.)

It suffices to check that 1 dominates 0 when $(\alpha, \beta) \in R_1$ (because 1 does even better for i compared to 0, against any pure strategy of the rival, when $(\alpha, \beta) \in R_2$). Suppose the rival chooses the pure strategy $\{0, 0\}$. Then if i chooses 0, he gets $v/2$ with probability π and 0 with probability $(1-\pi)$, hence he gets $\pi v/2$ on average. On the other hand, if i chooses 1, he gets $v - c$ with probability π and $-c$ with probability $(1-\pi)$, hence $\pi v - c$ on average. Since $\pi > 2\sigma = 2c/v$, we see that 1 does better for i than 0, if the rival plays $\{0, 0\}$. (The argument also shows, but *only* for $(\alpha, \beta) \in R_1$, that 0 does better for i than 1, against the rival's $\{0, 0\}$, if $\pi < 2\sigma$.) Similar calculations confirm that the superiority of 1 over 0 (or 0 over 1 under the appropriate clauses) remains true if the rival plays $\{1, 0\}$, $\{0, 1\}$, or $\{1, 1\}$, establishing Fact 1.

Fact 2 If a player i is of ability β, and $i's$ rival is α, β with probabilities $\pi, (1-\pi)$ respectively, then 1 (work) dominates 0 (shirk) for i if $\pi < 1 - 2\sigma$ and $(\alpha, \beta) \in R$ (and, somewhat more weakly: 0 dominates 1 for i if $\pi > 1 - 2\sigma$ and $(\alpha, \beta) \in R_1$)

Again it suffices to check that 1 dominates 0 for $(\alpha, \beta) \in R_1$ (because 0 does even worse for i compared to 1, against any pure strategy of the rival, when $(\alpha, \beta) \in R_2$). Suppose the rival chooses the pure strategy $\{0, 0\}$. Then if i chooses 0, he gets v with probability π and $v/2$ with probability $(1-\pi)$, hence he gets $\pi v + ((1-\pi)v/2)$ on average. On the other hand, if i chooses 1, he gets $v - c$ with probability π and $v - c$ with probability $(1-\pi)$, hence $v - c$ on average. Since $\pi < 1 - 2\sigma = 2c/v$, we see that 1 does better for i than 0, if the rival plays $\{1, 0\}$.(The argument also shows, but *only* for $(\alpha, \beta) \in R_1$, that 0 does better for i than 1, against the rival's $\{0, 0\}$, if $\pi > 1 - 2\sigma$). Similar calculations confirm that these claims remain true if the rival plays $\{1, 0\}$, $\{0, 1\}$, or $\{1, 1\}$, establishing Fact 2

The two facts together establish claim (d) in the proof of the theorem, as well as (g), (j) of Remark 1.

Claims (ea) and (eb) in the proof, and (f), (h), (i), (k) of Remark 1, are established in a similar fashion. Finally the computation of the mixed strategy NE, and the verification that there are no other NE, is also completely standard.

References

[1] Barut, Y. and Kovenock, D. (1998). The symmetric multiple prize all-pay auction with complete information. *European Journal of Political Economy.* 14:627-644.

[2] Baye, M., Kovenock, D. and De Vries, C.G. (1993). Rigging the lobbying process: An application of the all-pay auction. *American Economic Review* 83:289-294.....

[3] Che, Y.K. and Gale, I. (1998). Caps on political lobbying. *American Economic Review* 88:643-651..

[4] Dubey, P., and Sahi, S. (2012). The Allocation of a Prize (Expanded), Working Paper, Department of Economics, Stony Brook University

[5] Dubey, P., and Geanakoplos, J. (2010). Grading exams: 100,99,98 .... or A,B,C ? *Games and Economic Behavior*, Vol 69, Issue 1,pp 72-94, Special Issue in Honor of Robert Auman

[6] Dubey, P., and Wu, C. (2001). When less scrutiny induces more effort. *Journal of Mathematical Economics.* 36(4):311-336.

[7] Einy, E. and Moreno, D. and Shitovitz, B. (2002). The value of public information in a Cournot duopoly. *Games and Economic Behavior*, Vol 44, Issue 2,pp 272-285.

[8] Ellingsen, T. (1991). Strategic buyers and the social cost of monopoly. *American Economic Review* 81:648-657.

[9] Fang Hanming (2002) "Lottery versus All-pay Auction Models of Lobbying". Public Choice, pp 351-371.

[10] Glazer, A., and Hassin, R. (1988). Optimal contests. *Economic Inquiry.* 26:133-143.

[11] Green, J., and Stokey, N. (1983). A comparison of tournaments and contracts. *Journal of Political Economy.* 91(3):349-364.

[12] Hillman, A.L. and Riley, J.G. (1989) Politically contestable rents and transfers. *Economics and Politics* 1:17-39.

[13] Kirkegaard, R. (2012). Favoritism in asymmetric contests: Head starts and handicaps, *Games and Economic Behavior*, Vol 76, Issue 1,pp 226-248.

[14] Krishna, V., and Morgan, J. (1998). The winner-take-all principle in small tournaments. *Advances in Applied Microeconomics.* 7:61-74.

[15] Lazaer, E., and Rosen, S. (1981). Rank order tournaments as optimum labor contracts. *Journal of Political Economy.* 89:841-864.

[16] Moldovanu, B. and Sela, A. (2001). The optimal allocation of prizes in contests. *American Economic Review.* 91(3):542-558.

[17] Rosen, S. (1986). Prizes and incentives in elimination tournaments. *American Economic Review.* 76:701-715.

[18] Tullock, G. (1975) On the efficient organization of trails. *Kyklos* 28:745-762.