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621, 778; 621, 982, 45; 539, 389, 4

## On the Roller Straightener\*

(2nd Report, Straightening of Round Bars, Pipe and Tubings)

By Haruo Tokunaga\*\*

The rotary straighteners for straightening round bars and tubes are discussed.

The author found that the residual stress distribution in the bar straightened by the rotary straightener represents an alternate vortex of tensile and compressive stress zones around the center of the bar, which is not affected by the repeated rotary bending. To obtain better straightness, therefore, the bar has to be bent more severely at the point under the rolls, and receive more multiple rotary bending near the outlet in the straightener. For the rotary plastic bending of the bar, the direction of the bending moment does not coincide with that of the bending of the bar, and there is some angle between both directions, and therefore power is consumed for the rotary plastic bending of the bar. From this point of view, the author derived a formula to calculate the power required for straightening of the bar.

#### 1. Introduction

For the straightening of the round bars and tubings, the rotary straighteners having oblique rollers are mainly used, because the bars and tubings can not obtain better straightness by the parallel roller straightener used for the straightening of the shapes and profiles, since on the parallel roller straightener, the round bars and tubings rotate on the rollers, when the bending moment is applied. There are many different types of the rotary straighteners according to

the arrangements of the rollers, number of rollers, the types of roller drivings and so on, and in all of the rotary straighteners, the rollers of concave hyperbolic contour are used, but in very special cases, the cylindrical or the drum type ones are used.

It is the principle of the rotary straightener that the bars and tubings advancing in the rotary straightener are applied with a rotary plastic bending moment, and very few scholars have hitherto, studied the fundamental of the process  $^{(1)^{\sim}(4)}$ .

The 5-roller straightener is the typical one of the rotary straighteners; it is shown in Fig. 1 (A). In the present paper, the author studies the

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plastic-elastic bending of the bars in the rotary straightener, and clarifies the mechanism of the power consumption and the distribution of the residual stress through theoretical treatment and experiment to obtain the necessary data for designing the rotary straighteners.

# 2. Deformation of the bars by the rotary plastic bendings in the rotary straightener

Now, to study the deformation of the bar in the rotary straightener, the following conditions are assumed to simplify the mathematical treatment.

- (1) The outside shapes of the sections of the bars and tubings are round and the pipe and tubings have a wall of uniform thickness.
- (2) The cross sections remain on a plane during the process of bending, and always make right angles to the axis of the material.
- (3) Every stress component other than the stress in the longitudinal direction is zero.
- (4) The material is perfectly elastic-plastic, and its stress-strain relations in uniaxial tension and compression are the same. It has the same modulus of elasticity and yield point in tension and compression.

Repeated tension and compression hysteresis curve of the material is deemed to be as Fig. 2,

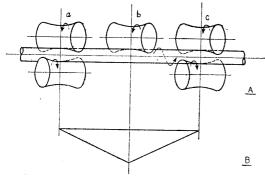
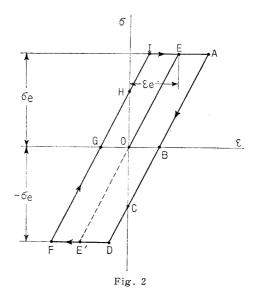


Fig. 1

according to the assumption (4). Here,  $\overline{\text{OE}}$  is the range of an elastic deformation, and  $\varepsilon_e$ ,  $\sigma_e$  denote the yielding strain and yielding stress respectively. In accordance with assumptions (1) and (2), the strain at a certain point on the cross section is in proportion to the distance between the axis of bending and that point, when the bar is bent. The bending moment of the bar straightened by the 5 roller rotary straightener is shown in Fig. 1, (B), and it can be understood that a certain section of the bar which rotates and advances in the straightener undergoes a gradually increasing bending moment for a distance from the point a to the point b, and also undergoes a gradually decreasing bending moment for a distance from the point b to the point c. Therefore, the strain of the section along the axis of the bar can be shown as in Fig.

In Fig. 3, zz' is the axis of the bar, and yy' is the axis perpendicular to the axis zz' through the center of gravity of the section of the bar and the surface of the paper represents the sectional area of the bar containing these two axes. The



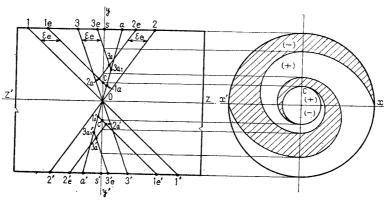


Fig. 3

strain that is caused when the initially bent material is first straightened, is represented by a0a' line.

Now, when the strain caused by the first bending is represented by 303' line, the plastic strain  $3_{\epsilon}3_{a1}03_{a1}'3_{\epsilon}'$  left behind after the elastic strain  $\varepsilon_e$  is deduced from 303' line. According to the same considerations, when the second bending and the third bending, which are more and more heavy than the first bending, are applied in the opposite direction, the final plastic strain  $\overline{1_e 1_a 0 1_a' 1_e'}$  line is left and this final plastic strain is determined only in terms of the value of the heaviest final bending. Next, when the gradually decreasing bending represented by 202' and 303' line is applied repeatedly in the opposite direction, the final plastic strain  $\overline{3_e 3_a 2_a 1_a 0 1_a' 2_a' 3_a' 3_e'}$  line is left behind. The straightened bars and tubings are at an equilibrium state of the internal stress which is proportional to the distance of the plastic or elastic strain line from axis yy' and hold the straightness.

The residual stress distribution of the section of the bar is represented in Fig. 3, in which the hatched area and the other area are the zones left by the compressive residual stress and the tensile residual stress respectively and these two zones surround the center of the bar which undergoes no plastic deformation and retains the initial deformation. The residual stress of the plastic zone builds up the residual moment to the center of the bar. The direction of this bending moment rotates one revolution according as the taken up sections change between one pitch length which is the length advanced during one revolution of the bar in straightening by the rotary straightener. Therefore, the bar and tubing straightened by the rotary straightener, are a spring coil form which has a very small coil diameter and the above mentioned pitch lengths. But the center of this coil is affected by the core of the bar which is not straightened and retains the initial bend. Hence, we may say that the bar and tubing are straightened more exactly and have smaller residual stress, when the rotary straightener is used by a method which applies more severe bend and more multiple rotary plastic bend at the period of the reducing bending moment.

Now,  $\rho_s$  denotes the radius of the residual bend of the bar after the straightening and  $\rho_0$  denotes the radius of the initial bend of the bar. The moment denoted by  $M_e$  of the core of the bar which undergoes no plastic deformation is represented by the following equation:

$$M_e = EI_e \left( \frac{1}{\rho_0} - \frac{1}{\rho_s} \right) \cdots (1)$$

where

 $I_e$ : moment of inertia of the core

E: modulus of elasticity

After the straightening of the bar, assuming that the outer zone of the bar which undergoes the plastic deformation is exactly straightened, and its moment of inertia is  $I_a$ , and the bar has the radius  $\rho_s$  of residual bend, this outer zone has the moment  $M_a$  to the center of the bar represented by the following equation:

The relation between the maximum bending radius  $\rho_1$  and the radius of the elastic core of the bar is  $r_e/\rho_1 = \varepsilon_e$ .

Hence, we may calculate the value  $\rho_1$  by Eq. (3) for the necessary straightness  $\rho_s$ .

In the case of a solid bar, we obtain

$$\rho_1 = \frac{r_2}{\varepsilon_0} \sqrt{\frac{\rho_0}{\rho_s}} = \frac{ER}{\sigma_s} \sqrt{\frac{\rho_0}{\rho_s}} \cdots (4)$$

where  $r_2$  is the radius of the bar.

The distribution of the stress of the bar undergoing a rotary plastic bending is represented in Fig. 4. In this case, the bar bends about the xx' axis and a compressive and a tensile side of the bend are upper and lower halves of the section respectively. Here,  $r_1$  and  $r_2$  denote the outer and inner radii of the pipe and also  $r_s$  denotes the radius of the elastic core.

Making a circle with its radius r in the plastic zone, we take the points A, B, C, D, F, G, H and I as indicated in Fig. 4, then the state of the stress of these points are same as the state of the corresponding points of A, B, C, D, F, G, H and I of the hysteresis curve in Fig. 2. The loci of these points describe the lines  $B_2$   $B_1$  C,  $D_2$  DE',  $G_2$  GH, and  $G_2$   $G_3$   $G_4$   $G_4$   $G_5$   $G_6$   $G_7$   $G_8$   $G_8$   $G_8$   $G_8$   $G_8$   $G_8$   $G_9$   $G_9$ 

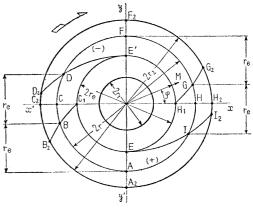


Fig. 4

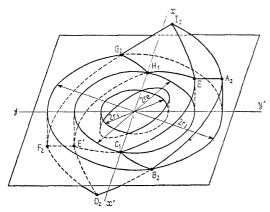


Fig. 5

 $F_2FE'ED_2$  are  $\sigma_e$  and  $-\sigma_e$  respectively, and the stresses decrease gradually outward from this zone and no stress is on the lines  $B_2BC$ , and  $G_2GH_1$ . The state of this stress distribution is represented cubically in Fig. 5.

Components of moments of the stress about x and y axes are represented by the following equations. Here,  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$  denote sums of the moments of the zones  $A_2AEII_2$  and  $F_2FE'DD_2$ , the zones  $I_2$  IEH<sub>1</sub>GG<sub>2</sub> and  $D_2DE'C_1BB_2$ , the zones  $B_2BC_1EAA_2$  and  $G_2GH_1E'FF_2$  and the zone of the elastic core respectively. Then we obtain the components about x and y axes of these moments as follows.

$$M_{1x} = 2 \int_{r_{e}}^{r_{2}} \int_{0}^{\cos^{-1}(2r_{e}-r)/r} \sigma_{e} r^{2} \cos\theta d\theta dr, \quad M_{1y} = 2 \int_{r_{e}}^{r_{2}} \int_{0}^{\cos^{-1}(2r_{e}-r)/r} \sigma_{e} r^{2} \sin\theta d\theta dr \qquad (5)$$

$$M_{2x} = 2 \int_{r_{e}}^{r_{2}} \int_{\cos^{-1}(2r_{e}-r)/r}^{\cos^{-1}(r_{e}-r)/r} \sigma_{e} \left( \frac{r + r \cos\theta - r_{e}}{r_{e}} \right) r^{2} \cos\theta d\theta dr$$

$$M_{2y} = 2 \int_{r_{e}}^{r_{2}} \int_{\cos^{-1}(2r_{e}-r)/r}^{\cos^{-1}(r_{e}-r)/r} \sigma_{e} \left( \frac{r + r \cos\theta - r_{e}}{r_{e}} \right) r^{2} \sin\theta d\theta dr$$

$$M_{3x} = 2 \int_{r_{e}}^{r_{2}} \int_{0}^{\cos^{-1}(r_{e}-r)/r} \sigma_{e} \left( \frac{r \cos\theta - r + r_{e}}{r_{e}} \right) r^{2} \cos\theta d\theta dr$$

$$M_{3y} = 2 \int_{r_{e}}^{r_{2}} \int_{0}^{\cos^{-1}(r_{e}-r_{e})/r} \sigma_{e} \left( \frac{r \cos\theta - r + r_{e}}{r_{e}} \right) r^{2} \sin\theta d\theta dr$$

$$M_{4x} = \frac{I_{e}}{r_{e}} \sigma_{e}, \quad M_{4y} = 0 \qquad (8)$$

Integrating Eqs. (5) (6) and (7), we obtain

$$M_{1x} = \frac{2\sigma_e}{r_e} \sqrt{r_e(r_2 - r_e)} \left\{ \frac{4}{5} r_e(r_2 - r_e)^2 + \frac{4}{3} r_e^2(r_2 - r_e) \right\} \qquad (9)$$

$$M_{2x} = \frac{2\sigma_e}{r_e} \left\{ \sqrt{r_e (2r_2 - r_e)} \cdot I_1 + \sqrt{r_e (r_2 - r_e)} \cdot I_2 + \frac{1}{8} r_2^4 (\theta_1 - \theta_2) + \left(\frac{1}{15} - \frac{\pi}{16}\right) r_e^4 \right\} \dots (10)$$

$$M_{3x} = \frac{2\sigma_e}{r_e} \left\{ \sqrt{r_e (2r_2 - r_e)} \cdot I_1 + \frac{1}{8} r_2^4 \theta_3 - \left(\frac{1}{15} + \frac{\pi}{16}\right) r_e^4 \right\}$$
 (11)

$$M_{4x} = \frac{2\sigma_e}{r_a} \frac{\pi}{8} (r_e^4 - r_1^4) \qquad (12)$$

where

$$I_{1} = \frac{1}{64} (2r_{2} - r_{e})^{3} - \frac{3}{320} r_{e} (2r_{2} - r_{e})^{2} - \frac{11}{192} r_{e}^{2} (2r_{2} - r_{e}) - \frac{1}{64} r_{e}^{3}$$

$$I_{2} = -\frac{1}{4} (r_{2} - r_{e})^{3} - \frac{13}{20} r_{e} (r_{2} - r_{e})^{2} - \frac{5}{12} r_{e}^{2} (r_{2} - r_{e}) + \frac{1}{4} r_{e}^{3}$$

$$\theta_{1} = \cos^{-1} \frac{r_{e} - r_{2}}{r_{2}} \qquad \left(\frac{\pi}{2} \le \theta_{1} \le \pi\right)$$

$$\theta_{2} = \cos^{-1} \frac{2r_{e} - r_{2}}{r_{2}}, \quad \theta_{3} = \cos^{-1} \frac{r_{2} - r_{e}}{r_{2}} \qquad \left(0 \le \theta_{3} \le \frac{\pi}{2}\right)$$

$$M_{x} = \sum_{s=1}^{4} M_{ix} = \frac{2\sigma_{e}}{r_{s}} \left\{ \sqrt{r_{e} (r_{2} - r_{e})} \cdot I_{3} + \frac{1}{8} r_{2}^{4} (\pi - \theta_{2}) - \frac{\pi}{8} r_{1}^{4} \right\} \qquad (14)$$

Here we denote

$$I_{3} = -\frac{1}{4} (r_{2} - r_{e})^{3} + \frac{3}{20} r_{e} (r_{2} - r_{e})^{2} + \frac{11}{12} r_{e}^{2} (r_{2} - r_{e}) + \frac{1}{4} r_{e}^{3}$$

$$M_{1y} = \frac{2\sigma_{e}}{r_{e}} \left( \frac{2}{3} r_{e} r_{2}^{3} - r_{e}^{2} r_{2}^{2} + \frac{1}{3} r_{e}^{4} \right)$$

$$M_{2y} = M_{3y} = \frac{2\sigma_{e}}{r_{e}} \left( \frac{1}{4} r_{e}^{2} r_{2}^{2} - \frac{1}{4} r_{e}^{4} \right)$$

$$(15)$$

$$M_{4y} = 0 \cdots (18)$$

$$M_{y} = \sum_{i=1}^{4} M_{iy} = \frac{2\sigma_{e}}{r_{e}} \left( \frac{2}{3} r_{e} r_{2}^{3} - \frac{1}{2} r_{e}^{2} r_{2}^{2} - \frac{1}{6} r_{e}^{4} \right) \cdots (19)$$

These results are obtained for the case of  $r_1 \le r_e$ . For the relation of  $r_1 \ge r_e$ , we may calculate these moments as a difference of the moments of the bars with their diameters  $2r_2$  and  $2r_1$ .

It is a noteworthy point that the axis of the bend and the direction of the bending moment make an angle  $\varphi$  to each other and is represented in the following equation:

$$\tan \varphi = M_y/M_x$$
 .....(20)

## 3. Measurements of the residual stress of the bar straightened by the rotary straightener

A residual stress is left in the bar after a straightening by the rotary straightener, and this residual stress is the state of equilibrium to hold the straightness of the bar. The residual stress distribution in the bar straightened by the rotary straightener is theoretically as Fig. 3. To confirm this consideration, we measure the residual stress of a pipe straightened by the Sutton type 7-rolls The pipe used is a seamless rotary straightener. one made by Stiefel-Mannesmann process. The data of the straightener and pipe are shown in Table 1. A short length test piece of pipe is cut off from the long pipe straightened, and slits are made at the ends of the test piece to form four tongues. We attach an electric resistance gauge on

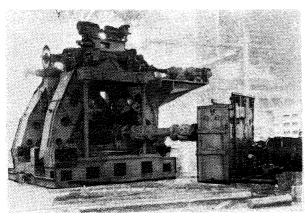


Fig. 6



Fig. 7

the inner side of the tongue in parallel with the axis of the pipe and measure the change of the electric resistance of the gauge, while the outer side of the tongues is gradually being cut off. To investigate the residual stress, the curvatures of the plate are usually measured<sup>(5)</sup> but it is very difficult, so we take the above mentioned method.

Now, in Fig. 8, assume that AA and BB are the outer and inner surface of the pipe and the wall thickness is h. The  $\sigma_a$  denotes the increased stress at the surface as which is bared by cutting off the wall thickness from the outside of the pipe. Then we obtain

$$\sigma_a = -\varepsilon E + \frac{2}{h-a} \int_0^a \sigma da \cdots (21)$$

where

 $\varepsilon$ : the strain of the electric resistance gauge

 $\sigma$ : the residual stress

moreover,  $\sigma_b$  denotes the stress of the surface aa. When the surface aa is cut off from the elementary thickness da, then we obtain the increase of the strain at the surface BB as follows.

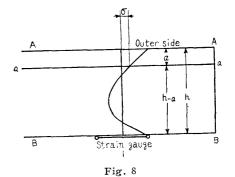
$$d\varepsilon = \frac{\sigma_b da}{Eh} - \frac{3\sigma_b da}{E(h-a)}$$

Solving about  $\sigma_b$ , we obtain

$$\sigma_b = -E \frac{h-a}{2} \frac{d\varepsilon}{da} \dots (22)$$

Table 1

Pipe used	Dimension mm			216O.D.×194I.D.
	Material %			C 0.08~0.14, Si 0.15~0.35, Mn 0.30~0.60
Operating conditions	Angle of	roll setting	Inlet driven roll	22.6°
			Outlet driven roll	22.8°
			Middle roll	36.8°
			Idle roll	25°
	No.of revolutions of roll r.p.m.			54
	Deflection of pipe by middle roll mm			5.4
	Pitch of advance of pipe mm			288
	Diameter of roll mm			500



Hence, the initial residual stress  $\sigma$  may be obtained by the following equation:

$$\sigma = \sigma_b - \sigma_a = -E \frac{h - a}{2} \frac{d\varepsilon}{da} + \varepsilon E - \frac{2}{h - a} \int_0^a \sigma da$$
....(23)

But this equation is not convenient to calculate the stress, since  $\sigma$  is implicitly represented. Therefore, deriving both sides of the equation, we obtain

$$\frac{d\sigma}{da} + \frac{\sigma}{h-a}$$

$$= \frac{1}{h-a} \frac{d}{da} \left\{ -E \frac{(h-a)^2}{2} \frac{d\varepsilon}{da} + E\varepsilon(h-a) \right\} \cdots (24)$$

Integrating the equation, we obtain

$$\sigma = (h-a) \left\{ -\frac{E}{2} \frac{d\varepsilon}{da} + \frac{E\varepsilon}{h-a} + \int_{0}^{a} \left( \frac{E}{h-a} \frac{d\varepsilon}{da} - \frac{2E\varepsilon}{(h-a)^{2}} \right) da + C \right\} \cdots (25)$$

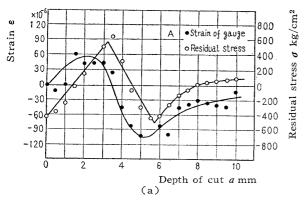
Here,  $\sigma_b = \sigma$  at a = 0. Hence, comparing Eqs. (22) and (25) in the case of a = 0, we obtain the integrating constant C as follows.

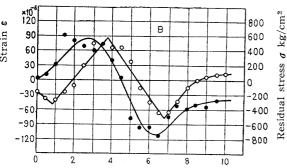
$$C = -\left(\frac{E\varepsilon}{h}\right)_{\text{at }a=0} \cdots (26)$$

Applying partial integration for Eq. (25), we obtain

$$\sigma = (h-a) \left\{ -\frac{E}{2} \frac{d\varepsilon}{da} + \frac{2E\varepsilon}{h-a} -3 \int_{0}^{a} \frac{E\varepsilon}{(h-a)^{2}} da - \left(\frac{2E\varepsilon}{h}\right)_{\text{at } a=0} \right\} \dots (27)$$

The stress  $\sigma$  is explicitly represented, and then we





Depth of cut a mm (b)
Fig. 9

may calculate the residual stress by measuring the strain of the gauge. Before slitting the end of the test pieces, the strain gauge has to be attached, and we measure the strain of the gauge as in Eq. (26) immediately after the slits are made.

In Fig. 9, the strain and also the stress of the wall calculated by Eq. (27) are represented with reference to the two points which make a rectangular angle to the center of the pipe. The phase of the point of Fig. 9 (a) advances by 90° farther in the direction of rotating of the pipe than that of Fig. 9 (b). Hence the phase of the stress in Fig. 9 (a) advances also by 90° farther than that of Fig. 9 (b). From Fig. 3, we may understand that the residual stress distributes in a wavy form at the section of the wall in parallel with the pipe axis. The results in Fig. 9 coincide with this theoretical study. In Fig. 9, the wave of the stress and strain gets out of shape slightly at the inner side of the wall. This appears to be due to the effect of the cutting force on an extremely thin wall of the pipe.

## 4. Power consumption

For the repeated rotary plastic bending of the bar, some power is consumed because the axes of the bend and the bending moment make an angle to each other owing to the plastic deformation of the material.

The bar with an elementary length dl is fixed at ts bottom, and attached with a rigid arm of length l such as shown in Fig. 10. Now assuming that  $d\theta$  is the bent angle of the bar by the bending force P at right angles to the arm at its top, and the directions of the bend and of the force P make the angle  $\varphi$ , the work dW done when the top of the arm point A rotates one revolution on the circle with its radius  $ld\theta$  is obtained by the following equation.

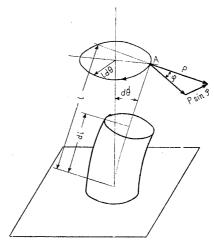


Fig. 10

 $dW=2\pi ld\theta\cdot P\sin\theta=2\pi M\sin\theta d\theta$  ......(28) where M=lP is a bending moment applied on the bar. Therefore, for the 5-rolls rotary straightener as shown in Fig. 11, assuming that

zaxis: the axis of the bar in the straightener

x axis: the axis of the bend of the bar

y axis: the axis perpendicular to x axis

x' axis: the axis of a resistant bending moment of the bar

W: the load by the middle roll

n: number of revolutions of the bar

 $\Delta x'$ : the deflection of the bar in direction of axis x' at the point of the middle roller

 $M_f$ : the fixed moment at both ends of the

l: the distance between inlet and outlet roller

N: the power consumption

 $M_y$ : component of the bending moment M about the y axis

 $\Delta i'$ : the difference of the inclination of the center of the bar in the x'-z plane

we obtain

$$N = 2\pi n \int_{0}^{t} M_{y} \frac{d^{2}y}{dz^{2}} dz = 2\pi n \int_{0}^{t} M \frac{d^{2}x'}{dz^{2}} dz$$

$$= 2\pi n \int_{0}^{t/2} 2\left(M_{f} - z \frac{W}{2}\right) \frac{d^{2}x'}{dz^{2}} dz$$

$$= 2\pi n (\Delta i' M_{f} + \Delta x' W) \cdots (29)$$

 $\Delta i' M_f$  is negligibly small compared with  $\Delta x' W$ . Hence, we obtain

$$N = 2\pi n \Delta x' W \cdots (30)$$

The power consumption is required due to the fact that the load by the middle roll exerts a moment in the opposite direction to the rotation of the bar. The value of  $\Delta x'$  and also the power consumption may be calculated by the theory in the previous article.

The Eqs. (29), (30) are in conformity with the material of the bar with any form of the hysteresis curve of the plastic deformation.

Now, assuming that the hysteresis curve is such as shown in Fig. 2, we obtain

$$dW = \int_{0}^{2\pi} \int_{r_{e}}^{r_{2}} 4\sigma_{e} \left(\frac{r}{\rho} - \varepsilon_{e}\right) r dr d\theta dl \quad \cdots \cdots (31)$$

where r,  $\theta$  denote the polar coordinate of which the pole is in the center of the bar, and  $\rho$  is the bending radius.

Integrating the Eq. (31), we obtain

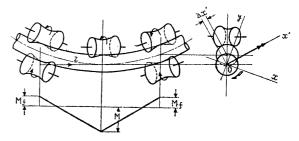


Fig. 11

$$N = 8\pi n\sigma_e \left[ \sum \frac{\Delta l}{\rho} \left\{ \left( \frac{r_2^3 - r_1^3}{3} \right) - \frac{\varepsilon_e \rho (r_2^2 - r_1^2)}{2} \right\} \right] \cdots (32)$$

in the case of  $r_e \leq r_1$ 

$$N = 8\pi n\sigma_e \sum \frac{\Delta l}{\rho} \left\{ \frac{r_2^3 - r_e^3}{2} - \frac{r_e(r_2^2 - r_e^2)}{2} \right\} \cdots (33)$$

in the case of  $r_e \ge r_1$ 

### 5. Conclusion

By studies and experiments on the rotary straightener, the following conclusions could be drawn:

- (1) The straightening mechanism of the rotary straightener is clarified and the residual stress distribution in the bar straightened by the rotary straightener is found to represent an alternated vortex of tensile and compressive stresses in the plastic zone around the elastic core.
- (2) To obtain better straightness, the bar has to be bent more severely and undergo a more multiple bend at the period in which the bending moment reduces near the outlet of the straightener.
- (3) The power consumed in the rotary straightener is the work done to rotate the bar overcoming the resultant moment of the load and reaction of the roller, about center of the bar.

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