LBL-18095
Preprint c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Physics Division

Submitted for publication

ON THE SCALING OF THE AVERAGE MULTIPLICITY IN HADRON-HADRON COLLISIONS

J. Finkelstein

July 1984

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

DISCLAIMER

July 1984 LBL-18095

ON THE SCALING OF THE AVERAGE MULTIPLICITY IN HADRON-HADRON COLLISIONS

J. Finkelstein*

Department of Physics
San Jose State University
San Jose, California 95192

Abstract

We show that, under rather general assumptions, the ratio of the average multiplicity to the cross section (rather than the average multiplicity itself) is expected to grow as ℓn s. We see that this ratio is already linear in ℓn s at presently-available energy, and we extrapolate this linear dependence to predict the value of this ratio at higher energy.

---

*Participating Guest at Lawrence Berkeley Laboratory.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

I. Introduction

Two of the prominent features which have been found in high-energy pp and $\bar{p}p$ collisions are, first, that as s (the square of the total energy) increases, the total cross section σ_t rises, and second, that the average multiplicity $<n>$ increases faster than the ℓn s -increase which would be expected from Feynman scaling. In this paper we discuss the connection between these two features, through a scaling rule which states

$$[1/\ell n(s/s_o)]\,[\,<n>/\sigma_t] \to \text{const. as } s \to \infty. \qquad (1)$$

This rule means that it is the ratio $<n>/\sigma_t$, rather than $<n>$ itself, which is expected to increase as ℓn s. The fastest possible increase of $<n>$ is thus as $(\ell n\ s)^3$, corresponding to Froissart-bound saturation by σ_t.

In the next section of this paper we show that (1) can be derived from the following three assumptions: first, that σ_t does not decrease to zero as $s \to \infty$; second, that the average value of transverse momentum is bounded as $s \to \infty$; and third, that the angular-momentum singularities controlling the high-energy behavior of the six-point function (i.e. the Mueller diagram [Ref. (1)]) are the same as those for the four-point function. In the third section we illustrate why $<n>$ might be expected to increase more rapidly than ℓn s by considering the example of the multi-string parton model of Refs. [2-4], which has been successfully used to describe many features of high-energy hadron-

hadron scattering. We wish to emphasize, however, that the rule (1) is completely model-independent.

Since the ratio $\sigma_{inelastic}/\sigma_t$ must surely go to a constant as $s \to \infty$, we could replace σ_t in (1) by $\sigma_{inelastic}$ and get a statement of equivalent validity. In the fourth section of this paper we will confront our rule with data, and so we have to make a choice. We choose to use $\sigma_{inelastic}$; that is, we will test the relation

$$\langle n \rangle / \sigma_{inelastic} = A + B \,\ell n\, s \quad (A, B = \text{const.}) \qquad (2)$$

which we expect to be valid at sufficiently large energy. Because we are unable to calculate corrections to (1), we are unable to predict the value of the energy at which (2) should become valid. However, we will find that data on pp and $\bar{p}p$ scattering from FNAL, the ISR, and the CERN $S\bar{p}pS$ collider are consistent with (2). From a fit to these data, we will present our extrapolation of the ratio $\langle n \rangle/\sigma_{inelastic}$ to still-higher energies.

II. Derivation of the Scaling Rule

We will derive the rule (1) under the following three assumptions:

(i) σ_t does not approach zero as s increases; more precisely, there is a constant $c_1 > 0$ such that $\sigma_t(s) > c_1$ for all s. We note that, if σ_t behaves as a power of $\ell n\, s$, i.e. if $\sigma_t \sim (\ell n\, s)^\gamma$, then from our assumption and the Froissart bound we have $0 \leq \gamma \leq 2$; however, we will not have to assume that σ_t does behave as a power of $\ell n\, s$.

(ii) The average transverse momentum of the produced particles remains limited as s increases; this means that we can integrate over transverse momentum without introducing any additional energy dependence. Since the available rapidity increases as $\ell n\, s$, this in turn means that $\langle n \rangle$ will be proportional to $\ell n\, s$ times the height of the "central plateau" (which of course may be energy-dependent).

(iii) The angular-momentum singularities controlling the high-energy behavior of the six-point function at zero momentum transfer are the same (i.e. are the same type with the same position) as those for the four-point function. This assumption is discussed in Ref. [1], and is motivated by the fact that the four- and six-point functions are related by unitarity equations, and so it is difficult to see how they could have different singularities. It implies that at high energy the six-point function will be proportional to the same function of the appropriate sub-energy that the total cross section is of the total energy. Factorization, which we do not assume, would imply in addition relationships among the coefficients of these singularities.

Given these asumptions, we will show that there is a constant

$c > 0$ and a sequence s_k with $\lim s_k = \infty$ such that

$$\lim_{k\to\infty} [1/\ln s_k][<n>/\sigma_t] = c \tag{3}$$

Unless the ratio $<n>/\sigma_t$ were to oscillate, this is the same as (1).

To establish (3), consider $E_c\, d\sigma/d^3p_c$, the single-particle inclusive cross section for the reaction $a + b \to c + X$; the multiplicity (of particle type c) can be written

$$<n> = [1/\sigma_t] \int [d^3p_c/E_c]\, E_c\, d\sigma/d^3p_c. \tag{4}$$

Now let $f(y_c, s)$ be the integral over transverse momentum of $E_c\, d\sigma/d^3p_c$ for some center-of-mass rapidity y_c ; if the energy-dependence of f is independent of y_c (as will be true below), then since the available rapidity increases as $\ln s$, we have, for large s,

$$<n> = [1/\sigma_t]\, f(y_c = 0, s) \ln s. \tag{5}$$

Mueller has pointed out (Ref. [1]) that $E_c\, d\sigma/d^3p_c$ can be related to a discontinuity A of the six-point function $a + \bar{c} + b \to a + \bar{c} + b$; this relation is depicted by the equality in Fig. (1). Furthermore, Mueller argued that at large values of $|y_a-y_c|$ and $|y_b-y_c|$, A would be dominated by angular-momentum singularities of the same type as govern the total cross section (as is also depicted in Fig (1)); combining this with our assumption that the integration over transverse momentum does not change the energy dependence, we can write, for large s,

$f(y_c, s) = c_2\, \sigma_t(|y_a-y_c|)\, \sigma_t(|y_b-y_c|)$, where c_2 is a constant. Since $|y_a| = |y_b| \approx \frac{1}{2} \ln s$, we can re-write (5) as

$$<n> = c_2 \left\{[\sigma_t(\tfrac{1}{2} \ln s)]^2 / \sigma_t(\ln s)\right\} \ln s. \tag{6}$$

Suppose, for illustration, that σ_t grows as $\ln s$ [this case has previously been considered in Ref (5)]; then (6) implies that $<n>$ grows as $(\ln s)^2$, and so $<n>/\sigma_t$ does indeed grow as $\ln s$. In general, we need only use the fact that σ_t will be a continuous function of $\ln s$, and our assumption (i) which, together with the Froissart bound, says that there are positive constants c_1 and c_3 such that, for all s,

$$c_1 < \sigma_t < c_3(\ln s)^2. \tag{7}$$

In the appendix we show that for any continuous function $\sigma(y)$ satisfying (7), there is a constant $c_4 > 0$ and a sequence y_k with $\lim y_k = \infty$ and with $\lim [\sigma(\frac{1}{2} y_k)/\sigma(y_k)] = c_4$. Since (6) can be written

$$<n>/\sigma_t(\ln s) = c_2[\sigma_t(\tfrac{1}{2} \ln s)/\sigma_t(\ln s)]^2 \ln s \tag{8}$$

this establishes (3), with $c = c_2 c_4^2$.

To investigate higher moments of the multiplicity distribution, one could study multi-particle inclusive cross sections. The naive generalization of Ref. [1] to the two-particle inclusive cross section for the reaction $a + b \to c + d + X$ would be to suppose that this cross section would be given by a diagram similar to the right-most one in

Fig. (1), with both produced particles c and d coming from the central vertex, and to interpret that to mean that the energy dependence of $f_2(y_c = 0, y_d = 0, s)$ would be the same as that of $f(y_c = 0, s)$. However, this could not be correct in the interesting case in which f/σ_t increases as s increases, since the bound

$$(f_2/\sigma_t) \geq (f/\sigma_t)^2 - (f/\sigma_t) \tag{9}$$

then implies that f_2/σ_t must increase at least as fast as $(f/\sigma_t)^2$. (A somewhat similar argument appears in Ref. [6] which, however, assumes factorization.) Independent evidence for the need for a more complicated generalization of Ref. [1] is provided by the study of inclusive distributions in the Reggeon calculus by Abramovskii, Kancheli, and Gribov (Ref. [7]). They find that, for the single-particle inclusive distribution f, all of the "cut Reggeon" diagrams cancel except for the one that corresponds to the rightmost diagram of Fig. (1) (see Fig. (36) of Ref. [7]). However, for the two-particle inclusive distribution f_2 the surviving diagrams include, in addition to one corresponding to both produced particles coming from the central vertex of the rightmost diagram of our Fig. (1) [see their Fig. (43)] another diagram [their Fig. (45)] which could restore the bound (9).

III. The Multi-string Parton Model

The multi-string parton model has been sucessfully used (Ref. [2-4]) to describe many feature of high-energy hadronic scattering processes, and can furnish a good example in which to study the growth of $<n>$. In this model, produced particles are assumed to come from exchanged Pomerons, each of which consists of two "strings" (or "chains"); the multiplicity of particles coming from any single string increases as the ℓn of the energy available to that string. In earlier versions of this model (for example, Ref. [8]) only the exchange of a single Pomeron was considered, and so it was simple to see that $<n>$ would increase as ℓn s. In Refs. [2-4] the exchange of many Pomerons are permitted; in making detailed fits one has to determine how the total available energy is shared among the various strings (here is where the parton distributions come in), but since in all cases considered the energy per string grows as some power of the total energy, it is still true that $<n>$ grows as (ℓn s) times the average number of Pomerons.

The cross section σ_k corresponding to the produced particles coming from k Pomerons was derived in Ref. [9] from the Reggeon calculus, and can be written

$$\sigma_k(s) = g[\ell n(s/s_0)/k]\,[1 - e^{-z} \sum_{i=0}^{k-1} (z^i/i!)],$$

where $z \equiv \lambda s^{\Delta}/\ell n(s/s_0)$, with s_0, g, λ, and Δ positive constants. (10)

To find the average number of Pomerons $<k>$, we have to calculate

$\Sigma\, k\sigma_k$; this sum can be done explicitly from (10) to give $\Sigma\, k\sigma_k = g\lambda s^{\Delta}$. Since in this model σ_t grows as $(\ell n\, s)^2$, we have (as was pointed out in Ref. [10]) $<k> \sim (s^{\Delta})/(\ell n\, s)^2$, and so $<n> \sim (s^{\Delta})/(\ell n\, s)$. One can get the same answer somewhat more easily if one replaces σ_k as given in (10) with the following approximation:

$$\sigma_k(s) = \begin{cases} g\, \ell n\, (s/s_0)/k & \text{for } k \leq z \\ 0 & \text{for } k > z \end{cases} \qquad (11)$$

From (11) it is evident that $\Sigma \sigma_k$ grows as $(\ell n\, s)^2$ and that $\Sigma\, k\sigma_k$ grows as s^{Δ}; again this means that $<n>$ grows as $(s^{\Delta})/(\ell n\, s)$.

The multi-string dual model is an example of how one can start with a basic mechanism which yields a $\ell n\, s$ -increase in $<n>$ (i.e. single Pomeron exchange) and yet wind up with $<n>$ increasing faster than $\ell n\, s$. Also, we shall see in the next section that data for values of $\sqrt{s}$ ranging from 13 GeV to 546 GeV do agree with Eq. (2), and since this model can fit these same data, it must be numerically compatible with (2) for this range of energy. However, because the model predicts that $<n>$ ultimately increases like a power of s, at some energy it must deviate from (2). The reason for this apparent conflict is that the authors of Refs. [2-4] were interested in constructing a model to predict data at attainable energies, without necessarily getting correct the asymptotic behavior at super-high energies; they therefore ignored a class of diagrams (called Pomeron interaction diagrams) which the Reggeon calculus (on which Eq. (10) is based) implies should be present. Presumably these additional diagrams would moderate the eventual

growth of $<n>$ so as to restore compatibility with Eq. (1). In Ref. [10] it is shown that, at least in a certain regime of the Reggeon calculus, this is exactly what does happen: σ_t increases as a power of $\ell n\, s$ [$\sigma_t \sim (\ell n\, s)^{\gamma}$ with $\gamma < 2$] and $<n> \sim (\ell n\, s)^{\gamma+1}$, so that $<n>/\sigma_t \sim \ell n\, s$.

Ignoring once again these Pomeron interaction diagrams, one can also obtain the large-s behavior of higher moments of the multiplicity distribution in the multi-string model. From Eq. (10) [or else more simply from Eq. (11)] one can see that, for $m \geq 1$, $\Sigma\, k^m \sigma_k \sim (s^{m\Delta})/(\ell n\, s)^{m-1}$; this means that $<n^m> \sim (s^{m\Delta})/(\ell n\, s)$. Since this implies that, for $m \geq 2$, $<n^m>$ increases faster than $(<n>)^m$, one is predicting a violation of KNO scaling. It would be interesting to know whether this feature [that $<n^m>$ increases faster than $(<n>)^m$] survives the inclusion of Pomeron interaction diagrams.

IV. Comparism with Experiment

We wish to confront the relation

$$R(s) \equiv \langle n \rangle / \sigma_{inelastic} = A + B \, \ell n \, s \qquad (A, B = \text{const.}) \quad (12)$$

with high-energy data. The UA5 group has reported (Ref. [11]) a measurement of the multiplicity in non-single diffractive (NSD) $\bar{p}p$ events at $\sqrt{s}$ = 546 GeV. At lower energies, the ratio of $\langle n \rangle_{NSD}$ to $\langle n \rangle_{inelastic}$ is essentially constant, and if this same ratio is maintained at $\sqrt{s}$ = 546 GeV, it would make no difference to our analysis whether we use $\langle n \rangle_{NSD}$ or $\langle n \rangle_{inelastic}$ in (12): we would merely get different values of A and B. However, since $\langle n \rangle_{inelastic}$ has not been directly measured at $\sqrt{s}$ = 546 GeV, we will choose to interprete $\langle n \rangle$ in (12) as the (charged) multiplicity in NSD events. Also, we have chosen to use $\sigma_{inelastic}$ rather than σ_t in (12) since we guess that the ratio with $\sigma_{inelastic}$ might achieve its asymptotic linear dependence on $\ell n \, s$ at a lower energy than would the ratio with σ_t, but since the ratio $\sigma_{inelastic}/\sigma_t$ changes very little in the energy range we consider, our results would have been quite similar had we used σ_t.

Values of R at various energies are displayed in Fig. 2; these values were obtained by dividing measured values of the charged multiplicity in NSD events by measured values of the inelastic cross sections, and combining errors in quadrature. All but the highest-energy point refer to pp scattering; the two lowest-energy points are from FNAL experiments at p_{lab} = 100 GeV/c (Ref. [12]) and at p_{lab} = 205 GeV/c (Ref. [13]), while the next three points are from the ISR, with

data on $\langle n \rangle$ from Ref. [14] and on $\sigma_{inelastic}$ from Ref. [15]. The highest-energy point is for the $\bar{p}p$ reaction at the $Sp\bar{p}S$ (but we would expect essentially identical results for pp); $\langle n \rangle$ is from Ref. [11] and $\sigma_{inelastic}$ is from Ref. [16].

The straight line in Fig. 2 is a best fit of the form R(s) = A + B ℓn s to the pp points; the $Sp\bar{p}S$ point at $\sqrt{s}$ = 546 GeV was not included in the fit. The best values are (with s measured in GeV^2) A = -.0283 mb^{-1} and B = .0494 mb^{-1}. As can be seen from the figure, the fit is quite good ($\bar{\chi}^2$ = 1.8 for 3 degrees of freedom), and the "prediction" at $\sqrt{s}$ = 546 GeV is well satisfied, and so our conclusion is that the data in this energy range are comfortably compatible with Eq. (12).

It is not at all obvious in models such as the one discussed in the previous section that an asymptotic relation such as (12) should be relevant at such conparatively low energies, and of course the agreement with the data that we have found could have been fortuitous. Nevertheless, it is interesting to examine the extrapolation of our linear fit for R to higher energies. In the table we list the extrapolated values of R at several (hopefully) soon-to-be-available energies. As an illustration, if we assume that $\sigma_{inelastic}$ = 65 mb at $\sqrt{s}$ = 2 TeV, then we predict from the table a charged multiplicity of about 47 at that energy.

Acknowledgments

I wish to acknowledge the hospitality of the Lawrence Berkeley Laboratory. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Appendix

To get from Eq. (8) to Eq. (3), we need the following

Theorem. Let $\sigma(y)$ be a continuous function on $y\epsilon[1, \infty]$ with constants c_1 and c_3 such that $0 < c_1 < \sigma(y) < c_3y^2$. Define $r(y) \equiv \sigma(\frac{1}{2}y)/\sigma(y)$. Then there exists $\ell \epsilon [\frac{1}{4}, 1]$ and a sequence y_k with $\lim_{k\to\infty} y_k = \infty$ and $\lim_{k\to\infty} r(y_k) = \ell$.

Proof. Suppose $\exists$ a sequence y_k with $\lim y_k = \infty$ and $r(y_k) = 1$. The theorem is then obviously true, with $\ell = 1$. So we only have to consider the possibility that $\exists\, y_0$ such that $y \geq y_0 \Rightarrow r(y) \neq 1$. Since σ and hence r is continuous, this implies either $r(y) > 1$ for $y \geq y_0$ (which we call case I) or $r(y) < 1$ for $y \geq y_0$ (case II).

Case I. $r(y) > 1$ for $y \geq y_0$. If for some $\varepsilon > 0$ there were a y_ε such that $r(y) > 1 + \varepsilon$ for $y \geq y_\varepsilon$, then we would have, for every positive integer k, $\sigma(\frac{1}{2} y_\varepsilon) > (1 + \varepsilon)^k \sigma(2^{k-1}y_\varepsilon)$, which would contradict the condition $c_1 < \sigma(y)$. Therefore for every $\varepsilon > 0$ there is a y arbitrarily large with $r(y) \leq 1 + \varepsilon$. Take $\varepsilon = 1/k$, and construct a sequence y_k such that $1 < r(y_k) \leq 1 + (1/k)$. Then $\lim r(y_k) = 1$.

Case II $r(y) < 1$ for $y \geq y_0$. If for some $\varepsilon > 0$ there were a y_ε such that $r(y) < \frac{1}{4} - \varepsilon$ for $y \geq y_\varepsilon$, then the condition that $\sigma(y) < c_3y^2$ would be contradicted. So far any $\varepsilon > 0$ there is a y arbitrarily large with $r(y) \geq \frac{1}{4} - \varepsilon$. Then we can construct a sequence y_k with $\frac{1}{4} - (1/k) \leq r(y_k) < 1$. This sequence must have a subsequence with limit $\ell \epsilon [\frac{1}{4}, 1]$.

References

1. A. H. Mueller, *Phys. Rev.* D2, *2963 (1970).*
2. A. Capella and J. Tran Thanh Van, *Phys. Lett.* 114B, *450 (1982); Z. Phys. C* 23, *165 (1984).*
3. A. B. Kaidalov and K. A. Ter-Martirosyan, *Phys. Lett.* 117B, *247 (1982).*
4. P. Aurenche, F. W. Bopp, and J. Ranft, *Z. Phys. C* 23, *67 (1984).*
5. N. G. Antoniou, C. B. Kouris, and G. M. Papaioannou, *Nuovo Cimento Lett.* 9, *509 (1974).*
6. N. G. Antoniou, C. B. Kouris, and G. M. Papaioannou, *Phys. Lett.* 52B, *207 (1974).*
7. V. A. Abramovskii, O. V. Kancheli, and V. N. Gribov, Proc. XVI Int. Conf. on High Energy Physics, Vol. 1, p. 389 (Batavia 1972).
8. A. Capella, U. Sukhatme, and J. Tran Thanh Van, *Z. Phys, C* 3, *329 (1980).*
9. K. A. Ter-Martirosyan, *Phys. Lett.* 44B, *377 (1973).*
10. M. S. Dubovikov, B. Z. Kopeliovich, L. I. Lapidus, and K. A. Ter-Martirosyan, *Nucl. Phys. B* 123, *147 (1977).*
11. UA5 Collaboration: G. J. Alner et al., *Phys. Lett.* 138B, *304 (1984).*
12. W. M. Morse et al., *Phys. Rev.* D15, *66 (1977).*
13. S. Barish et al., *Phys. Rev.* D9, *2689 (1974).*
14. ABCDHW Collaboration: A. Breakstone et al., preprint CERN/EP 83-165 (1983).
15. V. Amaldi et al., *Phys. Lett.* 44B *112 (1973).*
16. UA4 Collaboration: F. Cervelli, presented at the Fourth Workshop on pp Collider Physics, Bern (1984).

Table

Predicted values of $R \equiv \langle n \rangle / \sigma_{inelastic}$

$\sqrt{s}$ (TeV)	R (mb^{-1})
1	0.65
2	0.72
10	0.88
40	1.02

Figure Captions

Fig. 1. The Mueller analysis of the inclusive cross section.

Fig. 2. Values of R vs $\sqrt{s}$. Data from Refs. [11-16].

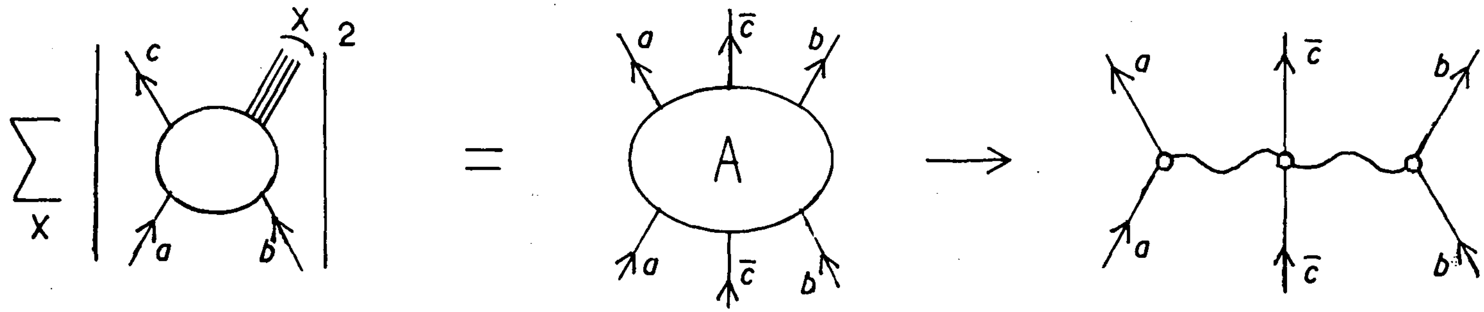

Figure 1

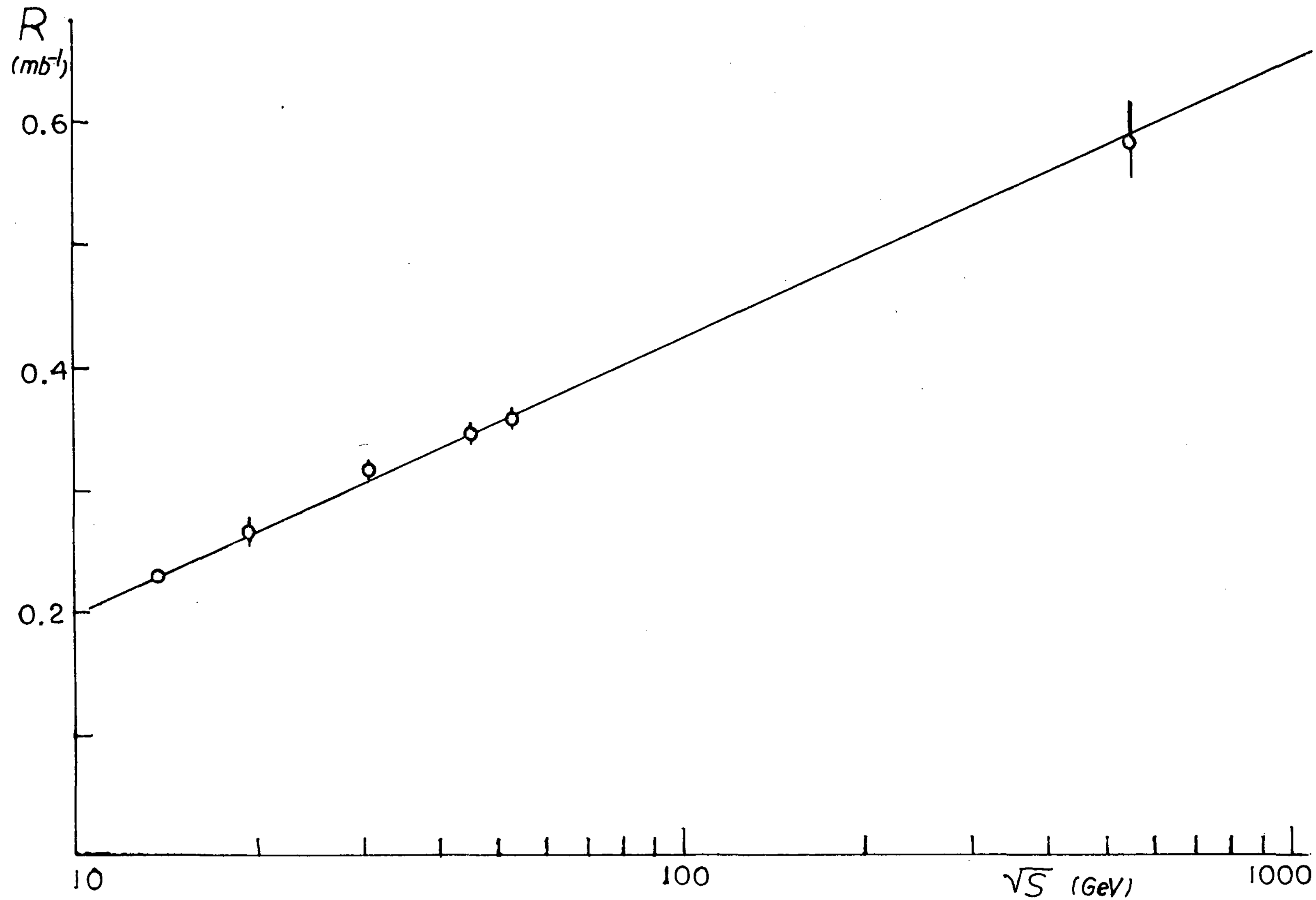

Figure 18

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720