On the security defects of an image encryption scheme

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Abstract

This paper studies the security of a recently-proposed chaos-based image encryption scheme, and points out the following problems: 1) there exist a number of invalid keys and weak keys, and some keys are partially equivalent for encryption/decryption; 2) given one chosen plain-image, a subkey K_{10} can be guessed with a smaller computational complexity than that of the simple brute-force attack; 3) given at most 128 chosen plain-images, a chosen-plaintext attack can possibly break the following part of the secret key: $\{K_i \bmod 128\}_{i=4}^{10}$, which works very well when K_{10} is not too large; 4) when K_{10} is relatively small, a known-plaintext attack can be carried out with only one known plain-image to recover some visual information of any other plain-images encrypted by the same key.

Key words: cryptanalysis, image encryption, chaos, known-plaintext attack, chosen-plaintext attack

1 Introduction

Spurred by the rapid development of multimedia and network technologies, multimedia data are being transmitted over networks more and more fre-

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quently. As a result, content protection of multimedia data is urgently needed in many applications, including both public and private services such as military information systems and multimedia messaging systems (MMS). Although any traditional data ciphers (such as DES and AES) can be used to meet this increasing demand of information security, they cannot provide satisfactory solutions to some special properties and requirements in many multimedia-related applications. For example, one requirement is perceptual encryption [1], meaning that the encrypted multimedia data can still be decoded by any standard-compliant codec and displayed, with a relatively low quality, which cannot be realized by simply employing a traditional cipher. As a response to this concern, a large number of specially-designed multimedia encryption schemes have been proposed [2–8]. Meanwhile, security analysis on the proposed schemes have also been developed, and some of these schemes have been found insecure to a certain extent [9–13]. For more discussions about multimedia data encryption techniques, readers are referred to some recent surveys [14–18].

Since 2003, Pareek et al. [19–21] have proposed three different encryption schemes based on one or more one-dimensional chaotic maps, among which the one proposed in [21] was designed for image encryption. Recent cryptanalysis results [22, 23] have shown that the two schemes proposed in [19, 20] are not secure. The present paper focuses on the security analysis of the image encryption scheme proposed in [21], and reports the following findings:

(1) There are several types of security problems with the secret key, and each subkey is involved in at least one problem.
(2) One subkey K_{10} can be separately searched with a relatively small computational complexity, even when only one chosen plain-image is given.
(3) The scheme is insecure against chosen-plaintext attack in the sense that using 128 chosen plain-images may be enough to break part of the key. The attack is especially feasible when K_{10} is not too large.
(4) When K_{10} is relatively small and one plain-image is known, a known-plaintext attack can be used to reveal some visual information of any other plain-images encrypted with the same secret key.

The rest of the paper is organized as follows. The next section gives a brief introduction to the image encryption scheme under study. Section 3 is the main body of the paper, focusing on a comprehensive cryptanalysis, with both theoretical and experimental results. In the last section, some concluding remarks and conclusions are given.

2 The image encryption scheme under study

In this scheme, the plaintext is a color image with separate RGB channels. The plain-image is scanned in the raster order, and then divided into 16-pixel blocks. The encryption and decryption procedures are performed blockwise on the plain-image. Without loss of generality, assume that the size of the plain-image is $M \times N$, and that MN can be exactly divided by 16. Then, the plain-image $\boldsymbol{I}$ can be represented as a 1-D signal $\{I(i)\}_{i=0}^{MN-1}$ with $N_b = MN/16$ blocks, namely, $\boldsymbol{I} = \{I^{(16)}(k)\}_{k=0}^{N_b-1}$, where $I^{(16)}(k) = \{I(16k+i)\}_{i=0}^{15}$. Similarly, the cipher-image is denoted by $\boldsymbol{I}^* = \{I^{*(16)}(k)\}_{k=0}^{N_b-1}$, where $I^{*(16)}(k) = \{I^*(16k+i)\}_{i=0}^{15}$.

The secret key of the encryption scheme under study is an 80-bit integer and can be represented as $K = K_1 \cdots K_{10}$, where each subkey $K_i \in \{0, \ldots, 255\}$. Two chaotic systems are involved in the encryption scheme, and both are realized by iterating the Logistic map

$$f(x) = \mu x(1-x), \tag{1}$$

where μ is the control parameter and fixed to be 3.9999. One chaotic map runs globally throughout the whole encryption process, while another one runs locally for the encryption of each 16-pixel block. The initial condition of the global chaotic map is determined by the six subkeys $K_4 \sim K_9$ as follows:

$$X_0 = \left(\frac{\sum_{i=4}^{6} K_i \cdot 2^{8(i-4)}}{2^{24}} + \frac{\sum_{j=7}^{9}((K_j \bmod 16) + \lfloor K_j/16 \rfloor)}{96} \right) \bmod 1, \tag{2}$$

and the local chaotic map corresponding to each block is initialized according to selected chaotic states of the global map. For the k-th block $I^{(16)}(k)$, the encryption process can be described by the following steps.

- *Step 1: Determining the initial condition of the local chaotic map.* Iterate the global chaotic map until 24 chaotic states within the interval $[0.1, 0.9)$ are obtained. Denoting these chaotic states by $\{\hat{X}_j\}_{j=1}^{24}$, generate 24 integers $\{P_j\}_{j=1}^{24}$, where $P_j = \lfloor 24(\hat{X}_j - 0.1)/0.8 \rfloor + 1$. [1] Then, calculate $B_2 = \sum_{i=1}^{3} K_i \cdot 2^{8(i-1)}$ and set the initial condition of the local chaotic map as

$$Y_0 = \left(\frac{B_2 + \sum_{j=1}^{24} B_2[P_j] \cdot 2^{j-1}}{2^{24}} \right) \bmod 1, \tag{3}$$

[1] In Sec. 2 of [21], the interval is $[0.1, 0.9]$ and $P_j = \lfloor 23(\hat{X}_j - 0.1)/0.8 \rfloor + 1$. However, following this process, $P_j = 24$ when and only when $\hat{X}_j = 0.9$, which becomes a rare event and conflicts with the requirement that P_j has a roughly uniform distribution over $\{1, \ldots, 24\}$. Therefore, in this paper we changed the original process in [21] to a more reasonable one. Note that such a change does not affect the performance of the encryption scheme.

where $B_2[P_j]$ denotes the P_j-th bit of B_2.

- *Step 2: Encrypting the k-th block* $I^{(16)}(k)$. For each pixel in the block, iterate the local chaotic map to obtain K_{10} consecutive chaotic states $\{\hat{Y}_j\}_{j=1}^{K_{10}}$ which fall into the interval [0.1,0.9), and then encrypt the RGB values of the current pixel according to the following formulas:

$$R^* = E_1(R) = g_{K_4,K_5,K_7,K_8,\hat{Y}_{K_{10}}} \circ \cdots \circ g_{K_4,K_5,K_7,K_8,\hat{Y}_1}(R), \quad (4)$$
$$G^* = E_2(G) = g_{K_5,K_6,K_8,K_9,\hat{Y}_{K_{10}}} \circ \cdots \circ g_{K_5,K_6,K_8,K_9,\hat{Y}_1}(G), \quad (5)$$
$$B^* = E_3(B) = g_{K_6,K_4,K_9,K_7,\hat{Y}_{K_{10}}} \circ \cdots \circ g_{K_6,K_4,K_9,K_7,\hat{Y}_1}(B), \quad (6)$$

where $\circ$ denotes the composition of two functions and $g_{a_0,b_0,a_1,b_1,Y}(x)$ is a function under the control of Y as shown in Table 1.

- *Step 3: Updating subkeys* $K_1, \ldots, K_9$. Perform the following updating operation for $i = 1 \sim 9$:

$$K_i = (K_i + K_{10}) \bmod 256. \quad (7)$$

Table 1
The definition of $g_{a_0,b_0,a_1,b_1,Y}(x)$, where $\overline{x}$ denotes the bitwise complement of x, and $\oplus$ denotes the bitwise XOR operation.

$Y \in$	$g_{a_0,b_0,a_1,b_1,Y}(x)=$	$g^{-1}_{a_0,b_0,a_1,b_1,Y}(x)=$
$[0.10, 0.13) \cup [0.34, 0.37) \cup [0.58, 0.62)$	$\overline{x} = x \oplus 255$	
$[0.13, 0.16) \cup [0.37, 0.40) \cup [0.62, 0.66)$	$x \oplus a_0$	
$[0.16, 0.19) \cup [0.40, 0.43) \cup [0.66, 0.70)$	$(x + a_0 + b_0) \bmod 256$	$(x - a_0 - b_0) \bmod 256$
$[0.19, 0.22) \cup [0.43, 0.46) \cup [0.70, 0.74)$	$\overline{x \oplus a_0} = x \oplus (a_0 \oplus 255) = x \oplus \overline{a_0}$	
$[0.22, 0.25) \cup [0.46, 0.49) \cup [0.74, 0.78)$	$x \oplus a_1$	
$[0.25, 0.28) \cup [0.49, 0.52) \cup [0.78, 0.82)$	$(x + a_1 + b_1) \bmod 256$	$(x - a_1 - b_1) \bmod 256$
$[0.28, 0.31) \cup [0.52, 0.55) \cup [0.82, 0.86)$	$\overline{x \oplus a_1} = x \oplus (a_1 \oplus 255) = x \oplus \overline{a_1}$	
$[0.31, 0.34) \cup [0.55, 0.58) \cup [0.86, 0.90]$	$x = x \oplus 0$	

The decryption procedure is similar to the above encryption procedure, except that Eqs. (4)∼(6) in Step 2 are replaced by the following ones:

$$R = E_1^{-1}(R^*) = g^{-1}_{K_4,K_5,K_7,K_8,\hat{Y}_1} \circ \cdots \circ g^{-1}_{K_4,K_5,K_7,K_8,\hat{Y}_{K_{10}}}(R^*), \quad (8)$$
$$G = E_2^{-1}(G^*) = g^{-1}_{K_5,K_6,K_8,K_9,\hat{Y}_1} \circ \cdots \circ g^{-1}_{K_5,K_6,K_8,K_9,\hat{Y}_{K_{10}}}(G^*), \quad (9)$$
$$B = E_3^{-1}(B^*) = g^{-1}_{K_6,K_4,K_9,K_7,\hat{Y}_1} \circ \cdots \circ g^{-1}_{K_6,K_4,K_9,K_7,\hat{Y}_{K_{10}}}(B^*), \quad (10)$$

where $g^{-1}_{a_0,b_0,a_1,b_1,Y}(x)$ is the inverse function of $g_{a_0,b_0,a_1,b_1,Y}(x)$ with respect to x as shown in Table 1.

3 Cryptanalysis

In this section, we report our cryptanalysis results about the image encryption scheme under study. These include a comprehensive analysis on invalid keys, weak keys and partially equivalent keys, a chosen-plaintext attack to break K_{10}, a chosen-plaintext attack to break $\{K_i \bmod 128\}_{i=4}^{10}$, a known-plaintext attack, and some other minor security problems.

3.1 Two properties of the scheme

To facilitate the description of the discussion below, we first point out two properties of the scheme under study in this subsection. One is about the subkey updating mechanism, and the other is about the essential equivalent presentation form of the encryption function.

To improve the security of the scheme, an updating mechanism is introduced for subkeys in Eq. (7) of [21]. Because the updating process is performed in a finite-state field, the sequence of each updated subkey produced by such a mechanism is always periodic (see Fact 1 below). As a result, the sequence of the dynamic keys is also periodic. Assuming that the period is T, the N_b plain pixel-blocks $\{I^{(16)}(k)\}_{k=0}^{N_b-1}$ can be divided into T separate sets according to the values of these dynamically updated subkeys: $\left\{\mathbb{I}_j = \bigcup_{k=0}^{N_T-1} I^{(16)}(T\cdot k+j)\right\}_{j=0}^{T-1}$, where $N_T = \lceil N_b/T \rceil$. For blocks in the same set $\mathbb{I}_j$, all the updated subkeys are identical. In other words, for each set $\mathbb{I}_j$ ($1/T$ of the whole plain-image) one can consider that the secret key is fixed. Since $1/T$ of a plain-image may be enough to reveal essential visual information, one can turn to break any set $\mathbb{I}_j$ without considering the updating mechanism.

Fact 1 *For $x, a \in \{0, \ldots, 255\}$, the integer sequence $\{y(i) = (x + ai) \bmod 256\}_{i=0}^{\infty}$, has period $T = 256/\gcd(a, 256)$.*

With respect to the encryption function, one can see from Table 1 that each encryption subfunction is represented in one of the following two formats:

(1) $g_{a_0,b_0,a_1,b_1,Y}(x) = x \oplus \alpha$, where $\alpha \in \{0, 255, a_0, a_1, \overline{a_0}, \overline{a_1}\}$;
(2) $g_{a_0,b_0,a_1,b_1,Y}(x) = x \dotplus \beta$, where $x \dotplus \beta$ denotes $(x + \beta) \bmod 256$ (the same hereinafter), and $\beta \in \{a_0 \dotplus b_0, a_1 \dotplus b_1\} \subset \{0, \cdots, 255\}$.

Because $(x \oplus \alpha_1) \oplus \alpha_2 = x \oplus (\alpha_1 \oplus \alpha_2)$ and $(x \dotplus \beta_1) \dotplus \beta_2 = x \dotplus (\beta_1 \dotplus \beta_2)$, consecutive encryption subfunctions of the same kind can be combined together, and those with $\alpha = 0$ or $\beta = 0$ can be simply ignored. As a result, each encryption function $E_i(x)$ is a composition of $len \le K_{10}$ subfunctions: $\{G_j(x)\}_{j=1}^{len}$, where

$G_j(x) = x \oplus \alpha_{\lceil j/2 \rceil}$ or $x \dotplus \beta_{\lceil j/2 \rceil}$, and $G_j(x)$, $G_{j+1}(x)$ are encryption subfunctions of different kinds. According to the types of $G_1(x)$ and $G_{len}(x)$, $E_i(x)$ has four different formats:

(1) $E_i(x) = ((\cdots((x \dotplus \beta_1) \oplus \alpha_1) \cdots) \oplus \alpha_{\lceil (len-1)/2 \rceil}) \dotplus \beta_{\lceil len/2 \rceil}$;
(2) $E_i(x) = ((\cdots((x \dotplus \beta_1) \oplus \alpha_1) \cdots) \dotplus \beta_{\lceil (len-1)/2 \rceil}) \oplus \alpha_{\lceil len/2 \rceil}$;
(3) $E_i(x) = ((\cdots((x \oplus \alpha_1) \dotplus \beta_1) \cdots) \oplus \alpha_{\lceil (len-1)/2 \rceil}) \dotplus \beta_{\lceil len/2 \rceil}$;
(4) $E_i(x) = ((\cdots((x \oplus \alpha_1) \dotplus \beta_1) \cdots) \dotplus \beta_{\lceil (len-1)/2 \rceil}) \oplus \alpha_{\lceil len/2 \rceil}$.

Note that len is generally less than K_{10}. Assuming that $\{Y_i\}$ distributes uniformly over the interval [0.1,0.9], we can get the following inequality:

$$Prob[len = K_{10}] \leq \begin{cases} 2 \cdot (\frac{5}{8} \cdot \frac{1}{4})^{\frac{K_{10}}{2}}, & \text{when } K_{10} \text{ is even,} \\ (\frac{5}{8} \cdot \frac{1}{4})^{\lfloor \frac{K_{10}}{2} \rfloor} (\frac{5}{8} + \frac{1}{4}), & \text{when } K_{10} \text{ is odd.} \end{cases} \tag{11}$$

From the above equation, we can see that the probability decreases exponentially as K_{10} increases. Because it is difficult to exactly estimate the probability that len is equal to a given value less than K_{10}, we performed a number of random experiments for a 512×512 plain-image to investigate the possibilities. Figure 1 shows a result of 100 random keys when $K_{10} = 66$.

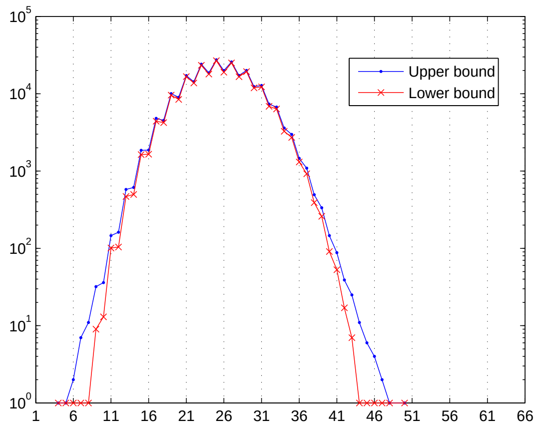


Fig. 1. The number of subfunctions composed of len subfunctions, when $K_{10} = 66$ and other subkeys were generated randomly for 100 times.

Since $G_j(x)$ is a composition of multiple functions $g_{a_0,b_0,a_1,b_1,Y}(x)$ of the same kind, and since that $\overline{a_0} \oplus a_1 = a_0 \oplus \overline{a_1} = a_0 \oplus a_1 \oplus 255$ and $\overline{a_0} \oplus \overline{a_1} = a_0 \oplus a_1$, one can easily deduce that

$$\alpha_i \in \mathbb{A} = \{255, a_0, a_1, a_0 \oplus 255, a_1 \oplus 255, a_0 \oplus a_1, a_0 \oplus a_1 \oplus 255\} \tag{12}$$

and

$$\beta_i \in \mathbb{B} = \{z_1(a_0 \dotplus b_0) \dotplus z_2(a_1 \dotplus b_1) | z_1, z_2 \in \{0, \cdots, K_{10}\} \text{ and } z_1 + z_2 \leq K_{10}\}.$$

Note that $\mathbb{A}$ has an interesting property: $\forall x_1, x_2 \in \mathbb{A} \cup \{0\}$, $x_1 \oplus x_2 \in \mathbb{A} \cup \{0\}$. This property concludes that $\bigoplus_i \alpha_i \in \mathbb{A} \cup \{0\}$, which will be used later in Sec. 3.5 for chosen-plaintext attack.

3.2 Analysis of the key space

In this subsection, we report some *invalid keys*, *weak keys* and *partially equivalent keys* existing in the encryption scheme under study. Here, an *invalid key* means a key that cannot ensure the successful working of the encryption scheme, a *weak key* is a key that corresponds to one or more security defects, and *partially equivalent keys* generate the same encryption result for a certain part of the plain-image. When estimating the key space, invalid keys and weak keys should be excluded, and all keys that are partially equivalent to each other should be counted as one single key [24, Sec. 3.2].

3.2.1 Invalid keys with respect to $K_4 \sim K_9$

When $X_0 = 0$, the global chaotic map will fall into the fixed point 0, which disables the encryption process due to the lack of chaotic states lying in $[0.1, 0.9]$. Now, let us see when $X_0 = 0$ can happen.

Observing Eq. (2), one can see that $X_0 = 0$ is equivalent to

$$\frac{\sum_{i=4}^{6} K_i \cdot 2^{8(i-4)}}{2^{24}} \equiv -\mathrm{FP}\left(\frac{\sum_{j=7}^{9}((K_j \bmod 16) + \lfloor K_j/16 \rfloor)}{96}\right) \pmod 1,$$

where $\mathrm{FP}(x)$ denotes the floating-point value of x. Because $0 \leq \sum_{i=4}^{6} K_i \cdot 2^{8(i-4)} < 2^{24}$ and $0 \leq \sum_{j=7}^{9}((K_j \bmod 16) + \lfloor K_j/16 \rfloor) \leq 15 \cdot 6 = 90 < 96$, one can further simplify the above equation as follows:

$$\frac{\sum_{i=4}^{6} K_i \cdot 2^{8(i-4)}}{2^{24}} = 1 - \frac{\mathrm{FP}\left(\sum_{j=7}^{9}((K_j \bmod 16) + \lfloor K_j/16 \rfloor)\right)}{96}. \tag{13}$$

By the fact that $\frac{\sum_{i=4}^{6} K_i \cdot 2^{8(i-4)}}{2^{24}} \bmod 2^{-24} = 0$, the following equality also holds:

$$\frac{\mathrm{FP}\left(\sum_{j=7}^{9}((K_j \bmod 16) + \lfloor K_j/16 \rfloor)\right)}{96} \bmod 2^{-24} = 0.$$

By checking all the 90 possible values of $\sum_{j=7}^{9}((K_j \bmod 16) + \lfloor K_j/16 \rfloor)$, one can easily get the following result:

$$\sum_{j=7}^{9}((K_j \bmod 16) + \lfloor K_j/16 \rfloor) = 3C, \tag{14}$$

where $C \in [0, 30]$. In this case,

$$1 - \text{FP}\left(\frac{\sum_{j=7}^{9}((K_j \bmod 16) + \lfloor K_j/16 \rfloor)}{96}\right) = 1 - \frac{C}{32}.$$

Substituting the above equation into Eq. (13), one has

$$\sum_{i=4}^{6} K_i \cdot 2^{8(i-4)} = 2^{19}(32 - C). \tag{15}$$

As a result, any key that satisfies Eqs. (14) and (15) simultaneously can lead to $X_0 = 0$. The number of such invalid subkeys $(K_4, \cdots, K_9)$ can be calculated to be $5592406 = 2^{22.415}$, where $5592406 = \lceil 16^6/3 \rceil$ is the number of distinct values of (K_7, K_8, K_9) satisfying Eq. (14), calculated according to the following Proposition 1.

Proposition 1 *Given an n-dimensional vector* $\mathbf{A} = (a_1, \cdots, a_n) \in \{0, \cdots, 15\}^n$, *the number of distinct values of* $\mathbf{A}$ *that satisfy* $(a_1 + \cdots + a_n) \bmod 3 = 0$, 1 *and* 2 *are* $\lceil 16^n/3 \rceil$, $\lfloor 16^n/3 \rfloor$ *and* $\lfloor 16^n/3 \rfloor$, *respectively.*

Proof: This proposition can be proved by mathematical induction.

When $n = 1$, one can easily verify that the number of distinct values of $\mathbf{A}$ that satisfy $a_1 \bmod 3 = 0, 1, 2$, are $6, 5, 5$, respectively. Since $6 = \lceil 16/3 \rceil$ and $5 = \lfloor 16/3 \rfloor$, the proposition is true.

Assuming that the position is true for $1 \leq n \leq k$, we prove the case for $n = k+1$. First, rewrite $a_1 + \cdots + a_{k+1}$ as $A_k + a_{k+1}$, where $A_k = a_1 + \cdots + a_k$. Then, observe that $(A_k + a_{k+1}) \bmod 3 = 0$ is equivalent to $A_k \equiv -a_{k+1} \pmod 3$. Thus, the number of distinct values of $\mathbf{A}$ that satisfying $A_k + a_{k+1} \bmod 3 = 0$ is the following sum:

$$\begin{aligned} N[(A_k + a_{k+1}) \bmod 3 = 0] &= \lceil 16^k/3 \rceil \cdot \lceil 16/3 \rceil + 2\lfloor 16^k/3 \rfloor \cdot \lfloor 16/3 \rfloor \\ &= (\lfloor 16^k/3 \rfloor + 1) \cdot \lceil 16/3 \rceil + 2\lfloor 16^k/3 \rfloor \cdot \lfloor 16/3 \rfloor \\ &= 16 \cdot \lfloor 16^k/3 \rfloor + 6. \end{aligned}$$

Assume $16^k = (15+1)^k = 3C + 1$. Then, $16^{k+1} = 48C + 16$ and $\lceil 16^{k+1}/3 \rceil = 16C + \lceil 16/3 \rceil = 16C + 6$. Then $16 \cdot \lfloor 16^k/3 \rfloor + 6 = 16C + 6 = \lceil 16^{k+1}/3 \rceil$. Going through a similar process, one can easily get $N[(A_k + a_{k+1}) \bmod 3 = 1] = N[(A_k + a_{k+1}) \bmod 3 = 2] = \lfloor 16^{k+1}/3 \rfloor$. This completes the mathematical induction, hence finishes the proof of the proposition. ∎

3.2.2 Invalid keys with respect to $K_1 \sim K_3$

For a given block $I^{(16)}(k)$, if $Y_0 = 0$, the local chaotic map will fall into the fixed point 0, which will also disable the encryption process of the corresponding block. According to Eq. (3), $Y_0 = 0$ when the following equality holds:

$$\left(B_2 + \sum_{j=1}^{24} B_2[P_j] \cdot 2^{j-1} \right) \bmod 2^{24} = 0,$$

Since $0 \le B_2 = \sum_{i=1}^{3} K_i \cdot 2^{8(i-1)} < 2^{24}$ and $0 \le \sum_{j=1}^{24} B_2[P_j] \cdot 2^{j-1} < 2^{24}$, the above equality can be simplified as follows:

$$\sum_{j=1}^{24} B_2[P_j] \cdot 2^{j-1} = 2^{24} - B_2. \tag{16}$$

Assuming that P_j distributes uniformly in $\{1, \cdots, 24\}$, B_2 and $(2^{24} - B_2)$ have m and n 0-bits, respectively, the probability for Eq. (16) to hold is

$$p_s = \left(\frac{m}{24}\right)^n \cdot \left(\frac{24-m}{24}\right)^{24-n} = \frac{m^n (24-m)^{24-n}}{24^{24}}.$$

The relationship between the values of p_s and $(25m + n)$ is shown in Fig. 2, from which one can see that the probability is not negligible for some values of (m, n). In fact, because $p_s > 0$ holds for any value of (m, n), we can say that any key is invalid from the strictest point of view. To resolve this problem, the original encryption scheme must be amended. One simple way to do so is setting Y_0 to be a pre-defined value once $Y_0 = 0$ occurs. In the following discussions of this paper and all experiments involved, we set $Y_0 = 1/2^{24}$ when such an event occurs.

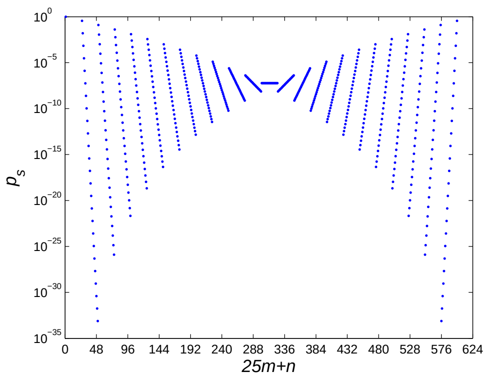


Fig. 2. The value of p_s with respect to the value of $(25m + n)$, where $m, n \in \{0, \cdots, 24\}$.

3.2.3 Weak keys with respect to K_{10}

In the encryption scheme under study, the update process of subkeys $K_1 \sim K_9$ and the number of subfunctions $g_{a_0,b_0,a_1,b_1,Y}(x)$ in each encryption function are both controlled by the subkey K_{10}. In the following, we discuss two weak-key problems with respect to K_{10}, which correspond to the above two processes controlled by K_{10}, respectively.

From Fact 1, one can see that the update of subkeys $K_1 \sim K_9$ has an inherent weakness, i.e., the possible values for the period of the sequence of the updated subkeys is 2^i, $i = 1 \sim 8$. For some values of K_{10}, this period can be very small, which weakens the updating mechanism considerably. The worst situation occurs when $K_{10} = 128$, which corresponds to period two. From the most conservative point of view, T should take the maximal value 256, which means that K_{10} should be an odd number.

The other problem deals with the number of subfunctions $g_{a_0,b_0,a_1,b_1,Y}(x)$ in each encryption function. When $K_{10} = 1$, the probability for a pixel to remain unchanged is 1/8 (under the assumption that Y_i distributes uniformly in the chaotic interval). Though the probability seems quite large, our experiments have shown that very little visual information leaks in the cipher-image. When $K_{10} \geq 2$, experiments have shown that it is almost impossible to distinguish any visual pattern from the cipher-image. As a result, in this case there exists only one major weak key: $K_{10} = 1$. To avoid other potential security defects, $K_{10} \geq 8$ is suggested.

3.2.4 Weak keys with respect to $K_4 \sim K_9$

Observing Table 1, one can see that the encryption subfunction $g_{a_0,a_1,b_0,b_1,y}$ $(x) = x$ or $\bar{x}$ when the following requirements are satisfied:

$$a_0, a_1 \in \{0, 255\} \text{ and } a_0 + b_0 \equiv a_1 + b_1 \equiv 0 \pmod{256}. \tag{17}$$

For the sub-image $\mathbb{I}_j$, if the subkeys corresponding to one encryption function $E_i(x)$ satisfy the above requirements, $E_i(x)$ will also be x or $\bar{x}$. Assuming that the chaotic trajectory of the local chaotic map has a uniform distribution in the interval $[0.1, 0.9]$, the probability of $g_{a_0,a_1,b_0,b_1,y}(x) = \bar{x}$ is $p = 3/8$. Then, according to Proposition 2 given below (note that $\bar{x} = x \oplus 255$), $\forall i = 1 \sim 3$, the probabilities of $E_i(x) = \bar{x}$ and $E_i(x) = x$ are $(1 - (1/4)^{K_{10}})/2$ and $(1 + (1/4)^{K_{10}})/2$, respectively. This means that about half of all plain-pixels in $\mathbb{I}_j$ are not encrypted at all, which may reveal some visual information about the plain-image. As an example, when K = "$3C1DE8FF0151FF012840$" (which corresponds to $T = 4$), one of our experiments showed that 49.9% of all the pixels in $\mathbb{I}_0$ were not encrypted (see Fig. 3 for the encryption result).

a) b)

Fig. 3. The encryption result when K = "$3C1DE8FF0151FF012840$" (represented in hexadecimal format, the same hereinafter): a) the red channel of the plain-image "Lenna"; b) the red channel of the cipher-image. For the other two color channels we have obtained similar results.

Proposition 2 *Given $n > 1$ functions, $f_1(x), \ldots, f_n(x)$, assume that each function is $x \oplus a$ with probability p and is x with probability $1-p$, where $a \in \mathbb{Z}$. Then, the probability of the composition function $F(x) = f_1 \circ \cdots \circ f_n(x) = x \oplus a$ is $P = (1-(1-2p)^n)/2$.*

Proof: Assume that $k = \lceil n/2 \rceil$. Then, $n = 2k$ if it is an even integer and $n = 2k-1$ when it is odd. To ensure $F(x) = f_1 \circ \cdots \circ f_n(x) = x \oplus a$, the number of subfunctions that are equal to $x \oplus a$ should be an odd integer. So,

$$\begin{aligned} P =& \sum_{i=1}^{k} \binom{n}{2i-1} p^{2i-1}(1-p)^{n-(2i-1)} \\ =& (1-p)^n \cdot \sum_{i=1}^{k} \binom{n}{2i-1} (p/(1-p))^{2i-1} \\ =& (1-p)^n \cdot \frac{(1+p/(1-p))^n - (1-p/(1-p))^n}{2} \\ =& (1-(1-2p)^n)/2. \end{aligned}$$

This completes the proof of the proposition. ■

By letting Eq. (17) hold for the three encryption functions $E_1(x)$, $E_2(x)$ and $E_3(x)$, we found a list of weak keys of this kind, as shown in Table 2.

3.2.5 Partially equivalent keys with respect to $K_7 \sim K_9$: Class 1

Observing Eq. (2), one can see that the value of X_0 remains unchanged if the following segments of K_7, K_8, K_9 exchange their values: K_7 mod 16, $\lfloor K_7/16 \rfloor$, K_8 mod 16, $\lfloor K_8/16 \rfloor$, K_9 mod 16, $\lfloor K_9/16 \rfloor$. Now let us find out what will happen if we exchange K_9 mod 16 and $\lfloor K_9/16 \rfloor$, i.e., exchange the upper half and the lower half of K_9. In this case, since the encryption of the red value

Table 2
Some weak keys that cause leaking of visual information.

Weak keys	Visual information leaked from
$(K_4, K_5), (K_7, K_8) \in \{(0,0), (255,1)\}$	Channel R
$(K_5, K_6), (K_8, K_9) \in \{(0,0), (255,1)\}$	Channel G
$(K_6, K_4), (K_9, K_7) \in \{(0,0), (255,1)\}$	Channel B
$(K_4, K_5, K_6, K_7, K_8, K_9) = (0,0,0,0,0,0)$	the whole plain-image

of each pixel is independent of K_9, the red channel of the cipher-image will remain unchanged. Similar results also exist for K_7 and K_8, which correspond to unchanged blue and green channels of the plain-image, respectively. This problem reduces the subkey-space of (K_7, K_8, K_9) from 256^3 to $(16 + (256 - 16)/2)^3 = 136^3$.

3.2.6 Partially equivalent keys with respect to $K_7 \sim K_9$: Class 2

As remarked in Sec. 3.1, each encryption subfunction $g_{a_0,a_1,b_0,b_1,Y}(x)$ can be represented in one of the following two formats: $x \oplus \alpha$ and $x \dot{+} \beta$. The following two facts about $\oplus$ and $\dot{+}$ will lead us to construct another class of partially equivalent keys.

Fact 2 $\forall\, a \in \{0, \dots, 255\}$, $a \oplus 128 = a \dot{+} 128$.

Fact 3 $\forall\, a, b \in \mathbb{Z}$, $(a \oplus 128) \dot{+} b = (a \dot{+} b) \oplus 128$.

Fact 3 means that a change in the MSB (most significant bit) of x, a_0, a_1, b_0, b_1 of any encryption subfunction $g_{a_0,a_1,b_0,b_1,Y}(x)$ is equivalent to XORing 128 on the output of the composition function $E_i(x)$.

Next, Fact 3 is used to figure out the second class of partially equivalent keys about $K_7 \sim K_9$. First, choose any two subkeys from $K_7 \sim K_9$. Without loss of generality, let us take K_7 and K_8. Then, given a secret key K that satisfies $K_7 < 128$ and $K_8 \geq 128$ (or, $K_7 \geq 128$ and $K_8 < 128$), let us change it into another key $\widetilde{K}$ by setting $\widetilde{K}_7 = K_7 \oplus 128$ and $\widetilde{K}_8 = K_8 \oplus 128$. From Eq. (2), it is easy to see that X_0 remains the same for the two keys. This means that both the global and the local chaotic maps have the same dynamics throughout the encryption procedure for the two keys, and that the difference on ciphertexts is determined only by the MSB-changes of K_7 and K_8. In the following, to analyze the influence of the MSB-changes on the ciphertexts, we consider the three color channels separately.

First, consider the encryption process of the green channel of the plain-image, in which K_7 is not involved at all. Assuming that the chaotic trajectory $\{Y_i\}$

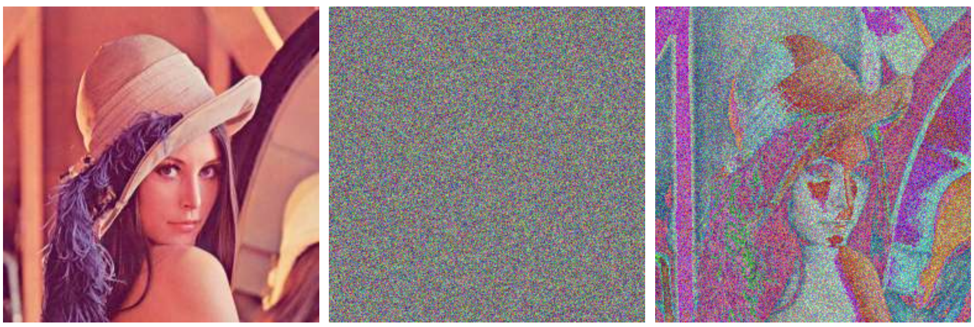
distributes uniformly within the interval $[0.1, 0.9]$, the probability that K_8 has an effect on each encryption subfunction is $p = 3/8$. If K_8 appears for an even number of times in the total K_{10} encryption subfunctions, then the value of $E_2(G)$ will remain the same for the two keys K and $\widetilde{K}$; otherwise, $E_2(G)$ changes its MSB. Thus, using the same deduction as given in the proof of Proposition 2, the probability that $E_2(G)$ remains unchanged can be calculated to be $P_2 = (1 + (1 - 2p)^{K_{10}})/2 = (1 + 4^{-K_{10}})/2$. This means that more than half of all green pixel values in the ciphertexts are identical in probability for the two keys K and $\widetilde{K}$.

For the blue channel, K_8 is not involved in the encryption process. So, following a similar deduction, the probability that $E_3(B)$ remains unchanged is $P_3 = (1 + 4^{-K_{10}})/2 = P_2$.

For the red channel, both K_7 and K_8 are involved, but their differences are neutralized for the encryption subfunction $x \dotplus (K_7 + K_8)$. So, the probability that the differences in K_7 and K_8 have an effect on the ciphertext is reduced to be $p = 2/8 = 1/4$. Thus, the probability that $E_1(R)$ remains unchanged becomes $P_1 = (1 + 2^{-K_{10}})/2 > P_2 = P_3$.

Combining all the above analyses together, it is expected that more than half of all pixel values in the cipher-images will be identical for the two keys K and $\widetilde{K}$. In addition, for other different pixel values, the XOR difference is always equal to 128. By enumerating all possibilities about this security problem, one can conclude that the subkey-space of (K_7, K_8, K_9) is reduced from 256^3 to $4 \cdot 128^3 = 256^3/2$.

To verify the above theoretical results, we have carried out some experiments for a plain-image of size 512×512. One result is shown in Fig. 4, in which the number of identical pixel values in red, green and blue channels are 131241 (50.06%), 130864 (49.92%) and 131383 (50.12%), respectively.



a) b) c)

Fig. 4. The decryption result with partially equivalent keys of Class 2: a) the plain-image "Lenna"; b) the cipher-image corresponding to $K =$ "$1A93DF25CF78\underline{DC44}E160$"; c) the decryption result of subfigure b with a different key $\widetilde{K} =$ "$1A93DF25CF78\underline{5CC4}E160$".

Finally, it is worth mentioning that there exists an internal relationship between the sub-images $\mathbb{I}_j$ and $\mathbb{I}_{j+T/2}$, where $j \in \{0, \cdots, T/2-1\}$, which can be easily deduced from the following fact about the updating process of the subkeys: $K_i + K_{10} \cdot T/2 = K_i + 128 \cdot K_{10}/\gcd(K_{10}, 256) \equiv K_i + 128 = K_i \oplus 128 \pmod{256}$.

3.2.7 Reduction of the key space

Based on the above analyses, we now summarize the influence of invalid, weak and equivalent keys on the key space in Table 3. According to the table, one can roughly estimate that the size of key space is reduced to 2^{75}, which is somewhat smaller than 2^{80} (the one claimed in [21, Sec. 3.3]).

Table 3
Reduction of the key space due to the existence of invalid keys, weak keys and partially equivalent keys.

Subkeys	Size of reduced subkey-space	Reason
$K_1 \sim K_3$	-	$Y_0 = 0$
$K_4 \sim K_9$	$2^{48} - 5592406 \approx 2^{48}$	$X_0 = 0$
$K_7 \sim K_9$	$136^3/2 = 2^{20.2624}$	Equivalent key of Classes 1 and 2
K_{10}	$< (255 - 128 - 1) = 126$	Weak keys about K_{10}

3.3 Guessing K_{10} and $\{K_i\}_{i=1}^{9}$ separately

The encryption process of the first block $I^{(16)}(0)$ depends only on the secret values Y_0 and K_{10}. In other words, for the first block one can consider (Y_0, K_{10}) as an equivalent to the original key K. Then, by guessing the value of (Y_0, K_{10}) one can get the value of K_{10} with complexity $O(2^{32})$. Thus, the other subkeys can be separately guessed with complexity $O(2^{72})$. The total complexity of such an enhanced brute-force attack is $O(2^{32} + 2^{72}) = O(2^{72})$, which is smaller than $O(2^{80})$, the expected complexity of a simple brute-force attack.

3.4 Guessing K_{10} with a chosen plain-image

As remarked in Sec. 3.1, all 16-pixel blocks in $\mathbb{I}_j = \bigcup_{k=0}^{N_T-1} I^{(16)}(T \cdot k + j)$ are encrypted with the same subkeys. If these blocks also correspond to the same values of Y_0, then all the three encryption functions for the R, G, B channels will become identical. Precisely, given two identical blocks, $I^{(16)}(k_0)$ and $I^{(16)}(k_1)$, one can see that the corresponding cipher-blocks will also become identical, if the following two requirements are satisfied:

(A) the distance of the two blocks is a multiple of T, i.e., $(k_0 - k_1) \mid T$;
(B) $Y_0^{(k_0)} = Y_0^{(k_1)}$, where $Y_0^{(k_0)}$ and $Y_0^{(k_1)}$ denote the values of Y_0 corresponding to the two 16-pixel blocks.

Therefore, if the probability of the two cipher-blocks to be identical is sufficiently large, one may use the distance between them to determine the value of T and narrow down the search space of K_{10}.

It should be noted that the following two cases can both ensure the requirement (B): 1) the sequences $\{P_j\}$ corresponding to the two blocks are identical; 2) the sequences $\{P_j\}$ corresponding to the two blocks are different (which may have $t \in \{0, \cdots, 23\}$ identical elements), but the values of Y_0 are still identical. The second case is tightly related to the ratio of 0-bits and 1-bits in B_2. As an extreme example, when $B_2 = 0$ or $2^{24} - 1$ (all the bits of B_2 are 0 or 1), $B_2[P_j]$ will be fixed to be 0 or 1, respectively. Assuming that the number of 1-bits in B_2 is m, one can easily calculate the probability of $B_2\left[P_j^{(k_0)}\right] = B_2\left[P_j^{(k_1)}\right]$ to be $(m/24)^2 + (1 - m/24)^2$, and then the probability of $Y_0^{(k_0)} = Y_0^{(k_1)}$ be $P_B = ((m/24)^2 + (1 - m/24)^2)^{24}$. We have carried out a large number of experiments to verify this theoretical estimation and the results are shown in Fig. 5. In these experiments, all possible values of B_2 were exhaustively generated to estimate the probability (as the mean value) for $\min(m, 24 - m) \leq 4$, and $\binom{24}{4} = 10,626$ random keys were generated for $\min(m, 24 - m) > 4$.

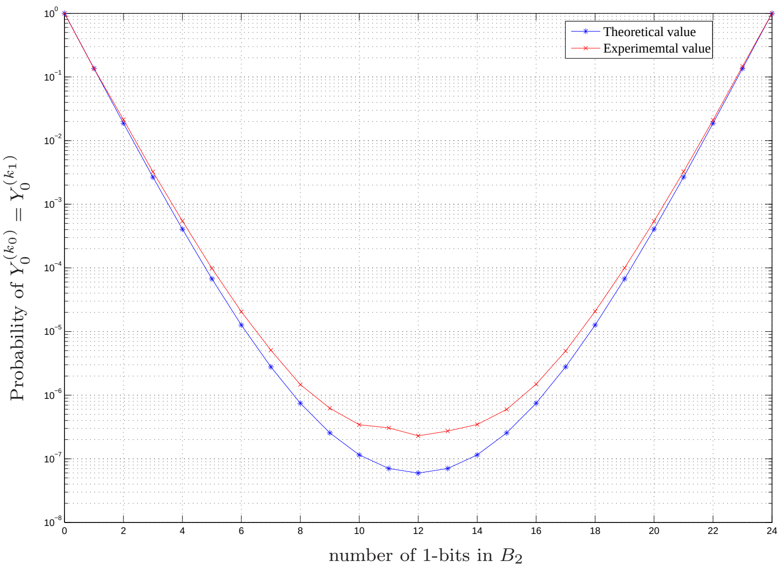


Fig. 5. Probability of $Y_0^{(k_0)} = Y_0^{(k_1)}$ with respect to the number of 1-bits in B_2.

Since $Prob((k_0 - k_1) \mid T)$ is $1/T$, the final probability that both requirements hold is P_B/T. According to Fig. 5, this probability may be large enough for an attacker to find some identical blocks in the same set $\mathbb{I}_j$, especially when $\min(m, 24 - m)$ and T are both relatively small.

To show how the attack works, we chose a 512×512 plain-image in which all blocks are identical but all pixels in each block are different from each other, and performed the attack for a secret key $K =$ "$2A84BCF35D70664E4740$". As a result, we found 9 pairs of identical blocks whose indices are listed in Table 4. Because all these indices should satisfy the requirement $(k_0 - k_1) \mid T$, we can get an upper bound of T by solving their greatest common divisor of the differences of the 9 indices. Thus, one immediately gets

$$\gcd(3161 - 1941, 7083 - 2015, 15255 - 3023, 9163 - 4159, 12113 - 5061, \\ 16355 - 5507, 12454 - 9166, 12259 - 9655, 13102 - 11090) = 4.$$

This means $T \in \{2, 4\}$, thus immediately leading to $\gcd(K_{10}, 256) \in \{128, 64\}$ and $K_{10} \in \{64, 128, 192\}$ according to Fact 1. As can be seen, in this example the size of the subkey space corresponding to K_{10} is reduced from 256 to 3, which is quite significant.

Table 4
The indices of 9 pairs of identical blocks in the cipher-image corresponding to the plain-image of fixed value zero.

k_0	1941	2015	3023	4159	5061	5507	9166	9655	11090
k_1	3161	7083	15255	9163	12113	16355	12454	12259	13102

3.5 Breaking $\{K_i \bmod 128\}_{i=4}^{10}$ with chosen-plaintext attack

This subsection presents one of the most important results of this work, since it shows how to partially break the encryption algorithm using a very cost-effective chosen-plaintext attack, in which 128 or even less plain-images are created. First, in Sec. 3.5.1 some mathematical devices are introduced. Next, in Sec. 3.5.2 the steps used to recover subkeys $\{K_i \bmod 128\}_{i=4}^{10}$ are described in detail. Finally some experimental results are given in Sec. 3.5.3 validate the proposed attacks.

3.5.1 Preliminaries

First, we prove some useful properties related to the composite functions $E_i(x)$. These properties are essential for the attack to be introduced below in this subsection.

Theorem 1 *Let $F(x) = G_{2m+1} \circ \cdots \circ G_1(x)$ be a composite function defined over $\{0, \ldots, 2^n - 1\}$, where $m, n \in \mathbb{Z}^+$, $G_{2i}(x) = x \oplus \alpha_i$ for $i = 1 \sim m$, $G_{2i+1}(x) = (x + \beta_i) \bmod 2^n$ for $i = 0 \sim m$ and $\alpha_i, \beta_i \in \{0, \ldots, 2^n - 1\}$. If $F(x) = x \oplus \gamma$ for some $\gamma \in \{0, \ldots, 2^n - 1\}$, then $\gamma \equiv \bigoplus_{i=1}^{m} \alpha_i \pmod{2^{n-1}}$.*

Proof: Let $x = \sum_{j=0}^{n-1} x_j \cdot 2^j$, $\alpha_i = \sum_{j=0}^{n-1} \alpha_{i,j} \cdot 2^j$, $\beta_i = \sum_{j=0}^{n-1} \beta_{i,j} \cdot 2^j$, and $F(x) = \sum_{j=0}^{n-1} F_j(x) \cdot 2^j$.

The proof is based on the following fact.

If F verifies $F(x) = x \oplus \gamma$ for some $\gamma = \sum_{j=0}^{n-1} \gamma_j \cdot 2^j$, then, for any $j = 0 \sim n-1$, the result of the computation of $F_j(x)$ depends only on the value of the j-th bit of x, that is, x_j. In other words, the value of $F_j(x)$ is independent of F_{j^*} if $j^* \neq j$.

We are going to check the computation of $F(x)$ starting from the least significant bit. To get the value of $F_0(x)$, we only need to calculate $\widetilde{F_0}(x) = (\cdots((x_0 + \beta_{0,0}) \oplus \alpha_{1,0} + \beta_{1,0}) \oplus \cdots \oplus \alpha_{m,0} + \beta_{m,0})$, and then get the least significant bit of $\widetilde{F_0}(x)$.[2] Note that the carry bit generated in each $+$ operation influences only the most significant bits $F_1(x) \sim F_{n-1}(x)$, and for the least significant bit of $\widetilde{F_0}(x)$ the operation $+$ is equivalent to $\oplus$. Therefore, we immediately get $F_0(x) = x_0 \oplus \beta_{0,0} \oplus \alpha_{1,0} \oplus \beta_{1,0} \cdots \oplus \alpha_{m,0} \oplus \beta_{m,0} = x_0 \oplus (\alpha_{1,0} \oplus \cdots \oplus \alpha_{m,0}) \oplus (\beta_{0,0} \oplus \cdots \oplus \beta_{m,0})$.

Then, let us study how the carry bits generated by $+$ operations in the calculation on $\widetilde{F_0}(x)$ affect the value of $F_1(x)$, as an effort to determine the value of $\beta_{0,0} \oplus \cdots \oplus \beta_{m,0}$. Note the following two facts about carry bits:

- when $\beta_{i,0} = 0$, no carry bit appears for any value of x_0;
- when $\beta_{i,0} = 1$, a carry bit appears when $x_0 = 0$ or 1 after the operation $+\beta_{i,0}$, and only for one possible value of x_0 there will be a carry bit[3].

Denoting the number of $\beta_{i,0}$ whose value equals to 1 by N_0, the above facts mean that N_0 can be obtained by counting carry bits when $x_0 = 0$ and when $x_0 = 1$. That is, $N_0 = \sum_{x_0 \in \{0,1\}} N_0(x_0) = N_0(0) + N_0(1)$, where $N_0(x_0)$ denotes the number of carry bits generated in the calculation process on $\widetilde{F_0}(x)$ with respect to x_0.

The independence of $F_1(x)$ of x_0 means that $N_0(0) = N_0(1)$, and as a result $N_0 = N_0(0) + N_0(1) = 2N_0(0)$ is an even number. This immediately leads to the conclusion $\beta_{0,0} \oplus \cdots \oplus \beta_{m,0} = 0$. Thus, $F_0(x) = x_0 \oplus (\alpha_{1,0} \oplus \cdots \oplus \alpha_{m,0})$.

Next, consider $F_1(x)$. In this case, $\widetilde{F_1}(x) = (\cdots((x_1 + \beta_{0,1} + CB_0(x_0)) \oplus \alpha_{1,1} + \beta_{1,1} + CB_1(x_0)) \oplus \cdots \oplus \alpha_{m,1} + \beta_{m,1} + CB_m(x_0))$, where $CB_i(x_0)$ denotes the bit carrying from $\widetilde{F_0}(x)$ during the i-th $+$ operation (which is equal to 0 when a carry bit does not exist). Then, due to the same reason as in the case of $F_0(x)$,

[2] Here, $+ \bmod 2^n$ is replaced by $+$ in the calculation process, because $\bmod 2^n$ does not affect any bit of $F(x)$.

[3] To be more precise, if there is a carry bit when $x_0 = 0$, then there will not be a carry bit when $x_0 = 1$ and vice versa.

we have $F_1(x) = x_1 \oplus (\alpha_{1,1} \oplus \cdots \oplus \alpha_{m,1}) \oplus (\beta_{0,1} \oplus CB_0(x_0) \cdots \oplus \beta_{m,1} \oplus CB_m(x_0))$. Observing the expression of $\widetilde{F_1}(x)$, we can easily note the following facts:

- when $\beta_{i,1} = CB_i(x_0) = 0$: no carry bit appears for any value of x_1;
- when $\beta_{i,1} = CB_i(x_0) = 1$: one carry bit always appears for any value of x_1;
- when $\beta_{i,1} = 0$, $CB_i(x_0) = 1$, or when $\beta_{i,1} = 1$, $CB_i(x_0) = 0$: one carry bit appears for only one value of x_1.

As a summary, only one carry bit may be generated from a pair of $\beta_{i,1}$ and $CB_i(x_0)$, which means that one can consider $\beta_{i,1} + CB_i(x_0)$ as a single value $\beta^*_{i,1}(x_0)$.

Denoting the number of $\beta^*_{i,1}$ whose value equals to 1 by $N_1(x_0)$, the above facts imply that $N_1(x_0) = \sum_{x_1 \in \{0,1\}} N_1(x_0, x_1) = N_1(x_0, 0) + N_1(x_0, 1)$, where $N_1(x_0, x_1)$ means the number of carry bits generated in the calculation process on $\widetilde{F_1}(x)$ with respect to x_0 and x_1. Then, because the value of $F_2(x)$ is independent of x_1, we can get $N_1(x_0, 0) = N_1(x_0, 1)$ and $N_1(x_0)$ is even. This means that $\beta_{0,1} \oplus CB_0(x_0) \cdots \oplus \beta_{m,1} \oplus CB_m(x_0) = 0$ and so $F_1(x) = x_1 \oplus (\alpha_{1,1} \oplus \cdots \oplus \alpha_{m,1})$.

The above deduction can be simply applied to other bits $F_2(x) \sim F_{n-1}(x)$. As a result, we get $F_i(x) = x_i \oplus (\alpha_{1,i} \oplus \cdots \oplus \alpha_{m,i})$, $\forall i = 0 \sim n-1$.

Finally, combining all the cases together, we have the result that $F(x) \equiv x \oplus (\alpha_1 \oplus \cdots \oplus \alpha_m) \pmod{2^{n-1}}$. This means that $\gamma \equiv \bigoplus_{i=1}^{m} \alpha_i \pmod{2^{n-1}}$ and the theorem is thus proved. ■

Corollary 1 *For the image encryption scheme under study, if there exists* $\gamma \in \{0, \ldots, 255\}$ *such that* $E_i(x) = x \oplus \gamma$*, then* $\gamma \in \{\bigoplus_i \alpha_i, (\bigoplus_i \alpha_i) \oplus 128\}$.

Proof: Consider the four classes of $E_i(x)$ as shown in Sec. 3.1.

(1) $E_i(x) = ((\cdots((x \dotplus \beta_1) \oplus \alpha_1) \cdots) \oplus \alpha_{\lceil (len-1)/2 \rceil}) \dotplus \beta_{\lceil len/2 \rceil}$: From Theorem 1, one has $\gamma \in \left\{\bigoplus_{i=1}^{\lceil (len-1)/2 \rceil} \alpha_i, \left(\bigoplus_{i=1}^{\lceil (len-1)/2 \rceil} \alpha_i\right) \oplus 128\right\}$.

(2) $E_i(x) = ((\cdots((x \dotplus \beta_1) \oplus \alpha_1) \cdots) \dotplus \beta_{\lceil (len-1)/2 \rceil}) \oplus \alpha_{\lceil len/2 \rceil}$: From Theorem 1, one has $\alpha_{\lceil len/2 \rceil} \oplus \gamma \in \left\{\bigoplus_{i=1}^{\lceil (len-1)/2 \rceil} \alpha_i, \left(\bigoplus_{i=1}^{\lceil (len-1)/2 \rceil} \alpha_i\right) \oplus 128\right\}$, which means $\gamma \in \left\{\bigoplus_{i=1}^{\lceil len/2 \rceil} \alpha_i, \left(\bigoplus_{i=1}^{\lceil len/2 \rceil} \alpha_i\right) \oplus 128\right\}$.

(3) $E_i(x) = ((\cdots((x \oplus \alpha_1) \dotplus \beta_1) \cdots) \oplus \alpha_{\lceil (len-1)/2 \rceil}) \dotplus \beta_{\lceil len/2 \rceil}$: Assuming that $x' = x \oplus \alpha_1$, we have $E_i(x) = x \oplus \gamma = x' \oplus (\alpha_1 \oplus \gamma)$. Then, applying Theorem 1 on x', we can easily get $\alpha_1 \oplus \gamma \in \left\{\bigoplus_{i=2}^{\lceil (len-1)/2 \rceil} \alpha_i, \left(\bigoplus_{i=2}^{\lceil (len-1)/2 \rceil} \alpha_i\right) \oplus 128\right\}$, thus $\gamma \in \left\{\bigoplus_{i=1}^{\lceil (len-1)/2 \rceil} \alpha_i, \left(\bigoplus_{i=1}^{\lceil (len-1)/2 \rceil} \alpha_i\right) \oplus 128\right\}$.

(4) $E_i(x) = ((\cdots((x \oplus \alpha_1) \dotplus \beta_1) \cdots) \dotplus \beta_{\lceil (len-1)/2 \rceil}) \oplus \alpha_{\lceil len/2 \rceil}$: Using a similar process to the above class, one gets $\gamma \in \left\{\bigoplus_{i=1}^{\lceil len/2 \rceil} \alpha_i, \left(\bigoplus_{i=1}^{\lceil len/2 \rceil} \alpha_i\right) \oplus 128\right\}$.

The above four conditions together complete the proof of the corollary. ■

From Corollary 1 and Eq. (12), we get the following result:

$$\gamma \bmod 128 = \bigoplus_i \alpha_i \bmod 128 \in \mathbb{A}^* = \{x \bmod 128 | x \in \mathbb{A} \cup \{0\}\}. \qquad (18)$$

Assuming that $a_0^* = a_0 \bmod 128$ and $a_1^* = a_1 \bmod 128$, we have

$$\mathbb{A}^* = \{0, 127, a_0^*, a_1^*, a_0^* \oplus 127, a_1^* \oplus 127, a_0^* \oplus a_1^*, a_0^* \oplus a_1^* \oplus 127\}. \qquad (19)$$

Observing the above equation, we can easily notice the following facts:

(1) when $a_0^* = a_1^* \in \{0, 127\}$, $\#(\mathbb{A}^*) = 2$;
(2) when $a_0^* \in \{0, 127\}$ and $a_1^* \notin \{0, 127\}$ (or $a_1^* \in \{0, 127\}$ and $a_0^* \notin \{0, 127\}$), $\#(\mathbb{A}^*) = 4$;
(3) when $a_0^*, a_1^* \notin \{0, 127\}$ and $a_0^* \oplus a_1^* \in \{0, 127\}$, $\#(\mathbb{A}^*) = 4$;
(4) when $a_0^*, a_1^* \notin \{0, 127\}$ and $a_0^* \oplus a_1^* \notin \{0, 127\}$, $\#(\mathbb{A}^*) = 8$.

Apparently, if we can get the set $\mathbb{A}^*$, it will be possible to get the values of a_0^* and a_1^*. The complexity of such a process is summarized as follows:

(1) when $\#(\mathbb{A}^*) = 2$, there are only 2 possible values of (a_0^*, a_1^*): (0,127) or (127,0);
(2) when $\#(\mathbb{A}^*) = 4$, assuming that $\mathbb{A}^* = \{0, 127, a, a \oplus 127\}$, there are 8 possible values of (a_0^*, a_1^*): $(0, a)$, $(0, a \oplus 127)$, $(127, a)$, $(127, a \oplus 127)$, (a, a), $(a, a \oplus 127)$, $(a \oplus 127, a)$, $(a \oplus 127, a \oplus 127)$;
(3) when $\#(\mathbb{A}^*) = 8$, there are 24 possible values of (a_0^*, a_1^*): $a_0^* \in \mathbb{A}^*/\{0, 127\}$ and $a_1^* \in \mathbb{A}^*/\{0, 127, a_0^*, a_0^* \oplus 127\}$.

One can see that in any case the complexity is much smaller than $2^7 \times 2^7 = 2^{14}$, the complexity of exhaustively searching all the bits of a_0^* and a_1^*. This idea is the key for the chosen-plaintext attack proposed in this subsection.

Next, let us find out how to distinguish XOR-equivalent encryption functions. According to Proposition 3, one can achieve such a goal by checking the following 255 equalities: $F(x_1) \oplus F(x_1 \oplus i) = i$, where x_1 is an arbitrary integer in $\{0, \ldots, 255\}$ and $i = 1 \sim 255$.

Proposition 3 *Let $F(x)$ be a function defined over $\{0, \ldots, 2^n - 1\}$, where $n \in \mathbb{Z}^+$. Then, $F(x) = x \oplus \gamma$ for any $x \in \{0, \ldots, 2^n - 1\}$ if and only if the following requirement holds: there exists $x_1 \in \{0, \ldots, 2^n - 1\}$ such that $F(x_1) \oplus F(x_1 \oplus i) = i, \forall i \in \{1, \ldots, 2^n - 1\}$.*

Proof: The "only if" part is obvious. Now, let us prove the "if" part. Note that $F(x_1) \oplus F(x_1 \oplus i) = i$ also holds when $i = 0$. So, when $i = x \oplus x_1$, we have $F(x_1 \oplus x \oplus x_1) = F(x) = F(x_1) \oplus x \oplus x_1 = x \oplus (x_1 \oplus F(x_1))$. When

$i = x_1$, we have $F(x_1) \oplus F(x_1 \oplus x_1) = x_1$ and then get $x_1 \oplus F(x_1) = F(0)$. Therefore, $F(x) = x \oplus F(0)$, where $F(0) = \gamma$ is a fixed value. ■

For the encryption functions $E_i(x)$ composed of $\oplus$ and $\dot{+}$, the above result can be further simplified. From Proposition 4, it is enough to check the following 127 equalities: $F(x_1) \oplus F(x_1 \oplus d) = d$, where x_1 is an arbitrary integer in $\{0, \ldots, 255\}$ and $d \in \{1, \cdots, 127\}$.

Proposition 4 *Consider any encryption function* $E_i(x)$ $(i = 1 \sim 3)$ *defined in Eqs. (4)∼(6). If there exists* $x_1 \in \{0, \ldots, 255\}$ *such that* $E_i(x_1) \oplus E_i(x_1 \oplus d) = d$, $\forall d \in \{1, \ldots, 127\}$, *then* $E_i(x) = x \oplus E_i(0)$.

Proof: From Fact 3, one has $E_i(x_1) \oplus E_i(x_1 \oplus 128) = 128$ and $E_i(x_1) \oplus E_i(x_1 \oplus j \oplus 128) = j \oplus 128$ for $j = 1 \sim 127$. This means that $E_i(x_1) \oplus E_i(x_1 \oplus j) = j$ holds $\forall j \in \{1, \ldots, 255\}$. Then, from Proposition 3, $E_i(x) = x \oplus E_i(0)$. ■

Next, let us investigate the probability that a given encryption $E_i(x)$ is equivalent to $x \oplus \gamma$. Again, because the theoretical analysis is quite difficult, we carried out a number of random experiments with a 512×512 plain-image for different values of K_{10}, where $K_1 \sim K_9$ were generated at random. Generally speaking, this probability becomes smaller when K_{10} increases, but it fluctuates in a wide range for different values of $K_1 \cdots K_9$. Two typical examples are shown in Fig. 6, in which the XOR-equivalent encryption functions involving the second kind of encryption subfunctions (i.e., functions of the form $x \dot{+} \beta$) and those not involving these encryption subfunctions were counted separately.

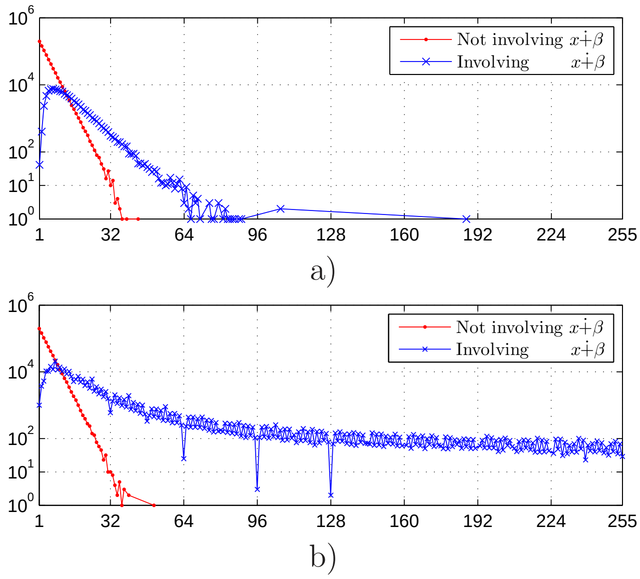


Fig. 6. The number of pixels satisfying $E_1(x) = x \oplus \gamma$ under different values of K_{10}: a) $K_1 \sim K_9$ = "8DB87A1613D75ADF2D"; b) $K_1 \sim K_9$ = "2A84BCF35 D70664347".

3.5.2 Description of the attack

Based on the above discussions, a chosen-plaintext attack can be developed by choosing 128 plain-images $\{I_l\}_{l=0}^{127}$ of size $M \times N$ as follows: $I_l = I_0 \oplus l$,[4] where I_0 can be freely chosen. To facilitate the following description about the attack, denote the encryption function $E_i(x)$ corresponding to the j-th pixel of the k-th block by $E_{i,k,j}(x)$, and the parameters a_0, a_1 corresponding to the k-th block by $a_{0,i,k}$, $a_{1,i,k}$, respectively. Similarly, for each updated subkey K_j, the value corresponding to the k-th block is denoted by $K_{j,k}$. Then, according to the discussion in Sec. 3.2.6, we have the following:

Fact 4 *Given two XOR-equivalent encryption functions $E_{i,k_1,j_1}(x) = x \oplus \gamma_{k_1,j_1}$ and $E_{i,k_2,j_2}(x) = x \oplus \gamma_{k_2,j_2}$, if $k_1 \equiv k_2 \pmod{T/2}$, then $\gamma_{k_1} \equiv \gamma_{k_2} \pmod{128}$.*

The proposed chosen-plaintext attack works in the following steps.

Step 1 – Finding XOR-equivalent encryption functions

For each color channel, scan the 128 plain-images to find encryption functions $E_{i,k,j}$ that are equivalent to $x \oplus \gamma_k$, where $\gamma_k = E_{i,k,j}(0)$ (according to Proposition 4). Record all the XOR-equivalent encryption functions corresponding to each color channel in an $S_i \times 2$ matrix $\mathbf{A}_i$, where S_i denotes the number of blocks containing such encryption functions. The first and the second rows of $\mathbf{A}_i$ contain the block indices and the corresponding values of γ_k, respectively. Here, note that all XOR-equivalent encryptions in the same block are identical, since they share the same parameters $a_{0,i,k}$ and $a_{1,i,k}$.

The output of this step is composed of three matrices $\{\mathbf{A}_i\}_{1 \leq i \leq 3}$, which require $\sum_{i=1}^{3} 2S_i$ memory units.

Step 2 – Estimating $\mathbb{A}_{i,k}^*$ (for each guessed value of K_{10})

Exhaustively search the value of K_{10} and get the period $T = 256/\gcd(K_{10}, 256)$. Then, for each matrix $\mathbf{A}_i$, generate the following $T/2$ sets: $\left\{\widetilde{\mathbb{A}}_{i,k}\right\}_{k=0}^{T/2-1}$, where $\widetilde{\mathbb{A}}_{i,k} = \{\mathbf{A}_i(s,2) \bmod 128 | s \equiv k \pmod{T/2}\}$. Next, expand each $\widetilde{\mathbb{A}}_{i,k}$ to construct $\widetilde{\mathbb{A}}_{i,k}^* = \left\{x_1 \oplus x_2 \oplus x_3 \,\middle|\, x_1, x_2, x_3 \in \widetilde{\mathbb{A}}_{i,k} \cup \{0, 127\}\right\}$, which is an approximation of the following set:

$$\mathbb{A}_{i,k}^* = \{0, 127, a_{0,i,k}^*, a_{1,i,k}^*, a_{0,i,k}^* \oplus 127, a_{1,i,k}^* \oplus 127, a_{0,i,k}^* \oplus a_{1,i,k}^*, a_{0,i,k}^* \oplus a_{1,i,k}^* \oplus 127\},$$

where $a_{0,i,k}^* = (a_{0,i,0} + k \cdot K_{10}) \bmod 128$ and $a_{1,i,k}^* = (a_{1,i,0} + k \cdot K_{10}) \bmod 128$. Note that $a_{0,i,0}$ and $a_{1,i,0}$ are the two subkeys corresponding to the color channel in question.

[4] In this paper, we use $I_l = I_0 \oplus l$ to denote the following facts: $\forall i = 0 \sim MN-1$, $R_l(i) = R_0(i) \oplus l$, $G_l(i) = G_0(i) \oplus l$ and $B_l(i) = B_0(i) \oplus l$.

Then, if there exists $k \in \{0, \cdots, T/2-1\}$ such that $\#\left(\widetilde{\mathbb{A}}^*_{i,k}\right) \notin \{2, 4, 8\}$, one can immediately conclude that the current value of K_{10} is wrong and then remove it from the list of candidate values for K_{10}.

The output of this step includes a list of N candidate values of K_{10} and at most $3T/2$ sets $\{\widetilde{\mathbb{A}}_{i,k}\}_{\substack{1\leq i\leq 3\\ 0\leq k\leq T/2-1}}$ for each candidate value of K_{10}. The total number of memory units required is not greater than $6 \times 3NT/2 = 9NT \leq 12 \times 256 \times 128 = 294912 \approx 2^{18.2}$, which is practical for a PC to store the intermediate data. Here, note that 0 and 127 are always in $\mathbb{A}^*$, so they do not need to be kept.

Step 3 – Determining $\{K_i \bmod 128\}_{i=4}^{10}$

For each color channel, choosing the set $\widetilde{\mathbb{A}}^*_{i,k_0}$ of the greatest size[5], one can exhaustively search all possible values of $(a^*_{0,i,k_0}, a^*_{1,i,k_0})$, i.e., search all possible values of $a^*_{0,i,0} = (a^*_{0,i,k_0} - k_0 \cdot K_{10}) \bmod 128$ and $a^*_{1,i,0} = (a^*_{1,i,k_0} - k_0 \cdot K_{10}) \bmod 128$. Note that $a^*_{0,1,0} = K_4 \bmod 128$ and $a^*_{1,1,0} = K_7 \bmod 128$ (red channel), $a^*_{0,2,0} = K_5 \bmod 128$ and $a^*_{1,2,0} = K_8 \bmod 128$ (green channel), $a^*_{0,3,0} = K_6 \bmod 128$ and $a^*_{1,3,0} = K_9 \bmod 128$ (blue channel).

All the guessed values of $(a^*_{0,i,0}, a^*_{1,i,0})$ are verified by employing the relationship between $\mathbb{A}^*_{i,k_0}$ and other sets $\{\mathbb{A}^*_{i,k}\}_{k\neq k_0}$. If all possible values of $(a^*_{0,i,0}, a^*_{1,i,0})$ are eliminated, the current value of K_{10} can also be eliminated. Note that the other three values of a valid candidate $(a^*_{0,i,0}, a^*_{1,i,0} \oplus 128, K + 10 \bmod 128) = (u, v, w)$ will also pass the verification process due to Fact 5 below: $(u \oplus 127, v \oplus 127, 128 - w)$, (v, u, w), and $(v \oplus 127, u \oplus 127, 128 - w)$.

Fact 5 *Given* $x, a, c \in \{0, \cdots, 127\}$*,* $x + ac \equiv (x \oplus 127 + (128 - a)c) \oplus 127 \pmod{128}$*.*

The output of this step is a list of candidate values of

$$K^* = (K_4 \bmod 128, \cdots, K_9 \bmod 128, K_{10} \bmod 128).$$

In the worst case, the number of all possible values is $N \times 24^3 \leq 256 \times 24^3 = 3538944 \approx 2^{21.6}$, which is still much smaller than the number of all possible values of the subkey K^*: $2^{6\times 7+8} = 2^{50}$. In the best case, the number of candidate values is only $2 \times 2^3 = 16$ (according to Fact 5).

[5] The greatest size may be 8, 4 or 2. When it is 4 or 2, $\widetilde{\mathbb{A}}^*_{i,k_0}$ may not be a good estimation of $\mathbb{A}^*_{i,k_0}$ and as a result cannot be used to support the attack. This case often occurs when K_{10} is relatively large, thus leading to a very small occurrence probability of XOR-equivalent encryption functions (see Fig. 6).

3.5.3 Experimental Results

To validate the feasibility of the above attack, we have carried out a real attack with a randomly-generated secret key $K =$ "$2A84BCF25E6A664E4C41$". As a result, we got the following output from Step 2:

$$\begin{aligned}
K_{10} &\in \{1, 3, \cdots, 255\}, \\
\mathbb{A}^*_{0,6} &= \{0, 127, 108, 20, 7, 107, 120, 108\}, \\
\mathbb{A}^*_{0,28} &= \{0, 127, 115, 125, 14, 0, 12, 113\}, \\
\mathbb{A}^*_{0,79} &= \{0, 127, 116, 117, 1, 10, 11, 126\}, \\
\mathbb{A}^*_{1,19} &= \{0, 127, 16, 33, 49, 111, 94, 78\}, \\
\mathbb{A}^*_{1,28} &= \{0, 127, 106, 122, 21, 5, 111, 16\}, \\
\mathbb{A}^*_{2,7} &= \{0, 127, 19, 78, 108, 49, 34, 93\}, \\
\mathbb{A}^*_{2,18} &= \{0, 127, 34, 93, 3, 33, 124, 94\}.
\end{aligned}$$

The final output of the attack (i.e., the output of Step 3) is shown in Table 5.

Finally, note that one may also be able to distinguish some XOR-equivalent encryption functions even with less than 128 chosen plain-images. To investigate such a possibility, we have carried out some experiments by choosing the following $(n+1) < 128$ plain-images instead: $\{I_l\}_{l=0}^{n}$, where $I_l = I_0 \oplus l$ for any $l > 0$. Let $N(n)$ be the number of XOR-equivalent encryption functions detected with the above $n+1$ chosen plain-images. The ratio $r(n) = N(127)/N(n)$ gives an estimation of the probability that a detected XOR-equivalent encryption function is real. For three randomly-generated keys, the values of $r(n)$ with respect to different values of n are shown in Fig. 7, from which one can see that the value of $r(n)$ always increases significantly when n increases from $2^i - 1$ to 2^i $(i = 1 \sim 6)$. We also carried out experiments for other random keys, and found out that this fact holds for most of them. According to this experimental result, we can choose the following 13 plain-images to minimize the number of chosen plaintexts: I_0, $I_1 = I_0 \oplus 1$, $I_2 = I_0 \oplus 2$, $I_3 = I_0 \oplus 3$, $I_4 = I_0 \oplus 4$, $I_5 = I_0 \oplus 7$, $I_6 = I_0 \oplus 8$, $I_7 = I_0 \oplus 15$, $I_8 = I_0 \oplus 16$, $I_9 = I_0 \oplus 31$, $I_{10} = I_0 \oplus 32$, $I_{11} = I_0 \oplus 63$ and $I_{12} = I_0 \oplus 64$. Then, for 1,000 randomly-generated secret keys, our experiments show that the average value of $r^* = N(127)/N^*$ is about 0.825, where N^* denotes the number of detected XOR-equivalent encryption functions with the 13 chosen plain-images. Note that the value of r^* is not accurate when N^* is too small. If only those keys corresponding to $N^* \geq 100$ are considered, the average value of r^* increases to about 0.9234. If only those corresponding to $N(n) \geq 1000$ are counted, the average value of r^* becomes about 0.9826. In practice, one may have to use more than 13 chosen plain-images to run the proposed attack, but it is expected that $O(20)$ chosen plain-images are enough in most cases.

Table 5
The final output of a real attack, where the underlined data form the real values of $\{K_i \bmod 128\}_{i=4}^{10}$.

<table>
<tr><th rowspan="2">$K_{10} \bmod 128$</th><th colspan="6">$\{K_i \bmod 128\}_{i=4}^{9}$</th></tr>
<tr><th>$i=4$</th><th>$i=7$</th><th>$i=5$</th><th>$i=8$</th><th>$i=6$</th><th>$i=9$</th></tr>
<tr><td rowspan="8">63</td><td rowspan="4">25</td><td rowspan="4">13</td><td rowspan="2">33</td><td rowspan="2">49</td><td>51</td><td>21</td></tr>
<tr><td>21</td><td>51</td></tr>
<tr><td rowspan="2">49</td><td rowspan="2">33</td><td>51</td><td>21</td></tr>
<tr><td>21</td><td>51</td></tr>
<tr><td rowspan="4">13</td><td rowspan="4">25</td><td rowspan="2">33</td><td rowspan="2">49</td><td>51</td><td>21</td></tr>
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<tr><td rowspan="2">49</td><td rowspan="2">33</td><td>51</td><td>21</td></tr>
<tr><td>21</td><td>51</td></tr>
<tr><td rowspan="8"><u>65</u></td><td rowspan="4">102</td><td rowspan="4">114</td><td rowspan="2">94</td><td rowspan="2">78</td><td>76</td><td>106</td></tr>
<tr><td>106</td><td>76</td></tr>
<tr><td rowspan="2">78</td><td rowspan="2">94</td><td>76</td><td>106</td></tr>
<tr><td>106</td><td>76</td></tr>
<tr><td rowspan="4"><u>114</u></td><td rowspan="4"><u>102</u></td><td rowspan="2">94</td><td rowspan="2">78</td><td>76</td><td>106</td></tr>
<tr><td>106</td><td>76</td></tr>
<tr><td rowspan="2"><u>78</u></td><td rowspan="2"><u>94</u></td><td>76</td><td>106</td></tr>
<tr><td><u>106</u></td><td><u>76</u></td></tr>
</table>

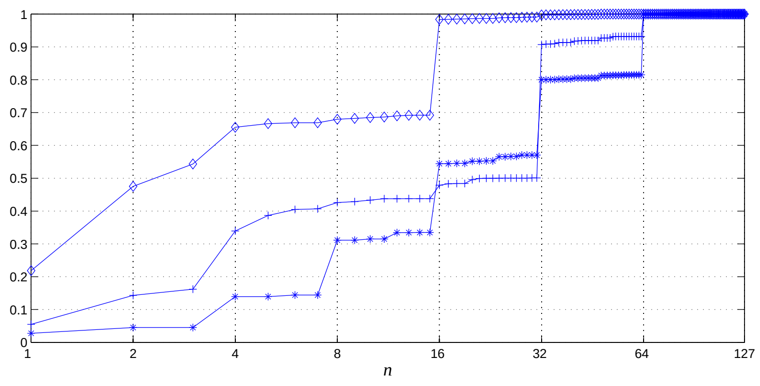
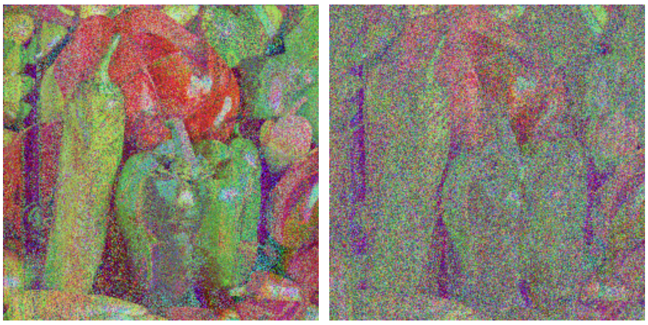


Fig. 7. The values of $r(n)$ with respect to different values of $n = 1 \sim 127$, where the three lines correspond to the results of three randomly-generated keys.

3.6 Known-plaintext attack based on a masking image

According to the results shown in Fig. 6, we know that many encryption functions are equivalent to XOR operations. Therefore, if we consider all the encryption functions as XOR-equivalent ones, then a masking image can be obtained by simply XORing a known plain-image and the corresponding cipher-image pixel by pixel. By using this masking image as an equivalent of the secret key to decrypt other cipher-images, all the pixels encrypted by real XOR-equivalent encryption functions will be correctly recovered. If the number of such correctly-recovered pixels is sufficiently large, some visual information about the plain-images may be obtained. It is expected that this known-plaintext attack can work well when K_{10} is relatively small. Figure 8 shows two examples of this attack when $K_{10} = 6$ and 30, from which one can see that some important visual information about the plain-image is revealed.



a) b)

Fig. 8. The result of breaking a plain-image "Peppers" with a masking image obtained when "Lenna" (Fig. 4a) is the known plain-image: a) $K =$ "8DB87A1613D75ADF2D06"; b) $K =$ "8DB87A1613D75ADF2D1E".

4 Conclusion

In this paper, the security of a recently-proposed image encryption scheme has been analyzed in detail. It is found that there exists a number of invalid keys, weak keys and partially equivalent keys, which reduce the size of the key space. Some attacks to a number of subkeys have also been developed: 1) given a chosen plain-image, a subkey K_{10} can be guessed with a complexity less than 2^8; 2) part of the key may be recovered with a chosen-plaintext attack using at most 128 chosen plain-images. The scheme under study can also be broken with only one known plain-image, when the subkey K_{10} is small. In addition, some other insecure problems about the scheme have been discussed throughout. The cryptanalysis presented in this paper shed some new light on attacking other encryption schemes that are composed of multi-round encryption functions, a relatively difficult but important topic to be further investigated in the near future.

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