

# On the Solution of NP-hard Linear Complementarity Problems

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## Abstract

In this paper two enumerative algorithms for the Linear Complementarity Problems (*LCP*) are discussed. These procedures exploit the equivalence of the *LCP* into a nonconvex quadratic and a bilinear programs. It is shown that these algorithms are efficient for processing NP-hard *LCPs* associated with reformulations of the Knapsack problem and should be recommended to solve difficult *LCPs*.

**Key Words:** mathematical programming, complementarity, global optimization, enumerative algorithms.

**AMS subject classification:** 90C33, 65K10.

## 1 Introduction

The Linear Complementarity Problem (*LCP*) consists of finding vectors  $z \in \mathbb{R}^n$  and  $w \in \mathbb{R}^n$  such that

$$\begin{aligned}w &= q + Mz \\z &\geq 0, \quad w \geq 0 \\z^T w &= 0\end{aligned}$$

for a given matrix  $M \in \mathbb{R}^{n \times n}$  and a vector  $q \in \mathbb{R}^n$ . This problem has originally appeared in the sixties for the solution of bimatrix games and convex quadratic programs. Since then, it has received an increasing interest,

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