Prog. Theor. Phys. Vol. 46 (1971), No. 1

On the Stability of α-Cluster Structures in ^{8}Be and ^{12}C Nuclei

Yasuhisa ABE, Jun HIURA
and Hajime TANAKA
Hokkaido University, Sapporo

May 6, 1971

Recently many investigations have been made concerning cluster structures of light nuclei both experimentally and theoretically.[1] None of them, however, have ever treated the stability or the polarization of constituent clusters. This problem is just challenged by introducing a new model,[2] i.e., a molecular orbital (M. O.) model of the atomic nuclei.

The model has its basis on the Hartree-Fock (H. F.) approximation[3] for nuclear intrinsic states, where the basis vectors are constructed from many-centered, cluster orbitals. Neglecting the spin-orbit coupling, we have four-fold degeneracy in each H. F. orbital. Its space part $|\lambda\rangle$ is expanded by the orthonormal basis vectors $|\alpha\rangle$:

$$|\lambda\rangle=\sum_{\alpha} C_{\lambda\alpha}|\alpha\rangle\,,$$

where the coefficients $C_{\lambda\alpha}$ are determined by solving the usual H. F. equation. The set $|\alpha\rangle$ is

$$|\alpha\rangle=\sum_{m,\,A} D_{\alpha mA}|m;A\rangle\,,$$

where $|m;A\rangle$ are "cluster" or "local" orbitals, taken as the harmonic oscillator wave functions (m) at each center (A), and the coefficients $D_{\alpha mA}$ are determined by a classification using an irreducible representation of an assumed point group symmetry and by an orthogonalization among orbitals belonging to the same representation. Then our treatment may be called "LCCO" (linear combination of cluster orbitals) method in analogy to the LCAO-MO-SCF one[4] in molecular physics.

In the case that the $|m;A\rangle$ are truncated only to the $(1s)$ shell $(N=0)$, no iteration is needed, because the number of the occupied orbitals $|\lambda\rangle$ is the same as that of $|\alpha\rangle$ available. The total nuclear wave function constructed with these $|\lambda\rangle$ is exactly the same as that of Brink's α-particle model.[5] Then the M. O. model offers the framework by which it is possible to study the polarization effects, or the stability of α-clusters in nuclei by including the higher

orbitals at each center. In the present investigation the set of $|m;A\rangle$ is taken up to $N=1$ oscillator states at each center. Thus $C_{\lambda\alpha}$ give a measure of polarizations of α-clusters. Furthermore, we have two parameters which characterize clusterization: sizes $b=\sqrt{\hbar/M\omega}$ of the clusters and distances d between them. These values should be determined by minimizing the total intrinsic energy.

As the first step, the M. O. model is applied to ^{8}Be and ^{12}C nuclei. The dumbbell and the equilateral triangle geometries of the α-clusters are assumed respectively. Then these systems have the point group symmetries of $C_{\infty h}$ and D_{3h}, with which the basis vectors are classified. The self-consistent symmetry[3] is imposed in the H. F. scheme with respect to the relevant point group operation. The two-body nuclear force used is Volkov's No. 1[6] with the Majorana exchange mixture $M=0.6$. The kinetic energy of the center-of-mass motion is subtracted approximately from the total Hamiltonian. The numerical results of the M. O. model and the α-particle one are compared in Figs. 1 and 2 for the binding energies versus distance d and for the density distributions $\rho(x,y,z)$, with $b=1.31$ fm.

It is noticeable that the binding energy curves of the M. O. model are almost straight horizontal line over the relatively wide region of d in both cases. Then it seems at a glance that the localization of α-clusters, separated with each other, disappears and then α-cluster structures are unstable against collapse through an agency of the polarizations of constituent clusters. The fact, however, is not so simple. The density distribution of the M. O. model is hardly affected at $d=d_\alpha$, the minimum distance of the α-particle model, because the polarizations (mixing of the $N=1$ orbitals) are very small there, smallest compared with those at neighbors. Thus it is almost

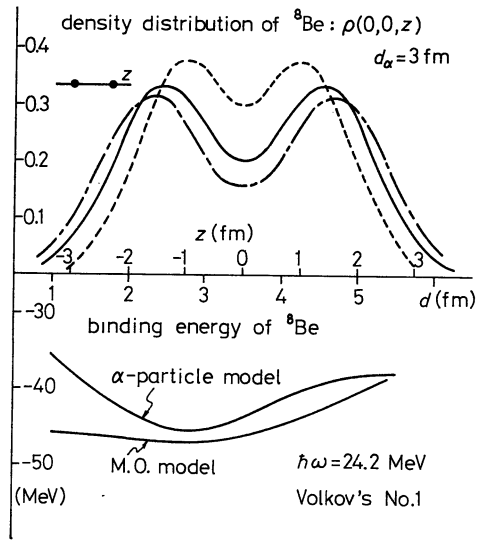

Fig. 1. The density distributions and the binding energy curves for ^{8}Be. For the upper parts, ·····: density distribution of the α-particle model at $d=1.0$ fm, ——: that of the M. O. model at $d=1.0$ fm, — — —: that of the α-particle model at $d=d_\alpha=3.0$ fm.

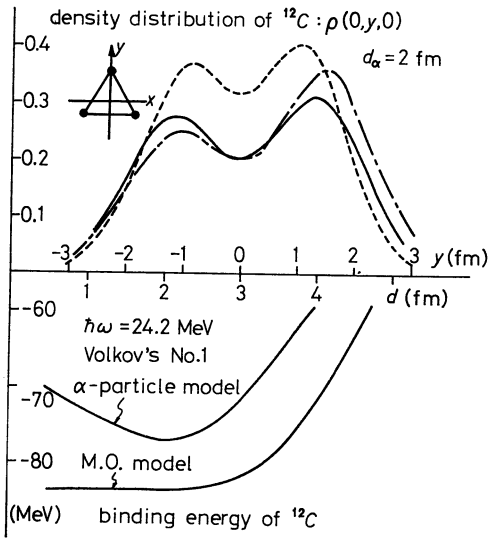

Fig. 2. The curves correspond to those in Fig. 1 but for ^{12}C. The only one exception is that ····· and —— are at $d=0.5$ fm, with — · — at $d=d_\alpha=2.0$ fm.

the same as that of the latter model. On the other hand at smaller distances, for example, the density distributions are much affected by the polarizations of clusters, and surprisingly, the polarization effects result in the simulation of the density to that of the α-particle model.

In conclusion the dumbbell α-cluster structure for ^{8}Be is very stable against the polarizations, and the equilateral triangle α-cluster structure for ^{12}C has the same tendency. The latter case, however, is not so distinct as the former one, because the polarizations are larger for ^{12}C than for ^{8}Be at $d=d_{\alpha}$. Details will be published elsewhere.

The authors are very grateful to Prof. K. Ikeda and Dr. H. Horiuchi for valuable discussions. They would also like to thank Prof. R. Tamagaki and Prof. T. Marumori for their stimulating interest. This work has been done as a part of the "Annual Research Project for α-Like Four-Body Correlation and Molecular Aspect in Nuclei" organized by the Research Institute for Fundamental Physics, Kyoto.

1) *Proc. Int. Conf. on Clustering Phenomena in Nuclei, Bochum, 1969*, ed. E. R. A. Beck (IAEA, Vienna, 1969).
2) Y. Abe, p. 303 of Ref. 1), see also H. Tanaka, p. 137 of Ref. 1).
The original idea was motivated by the work of K. Ikeda (p. 277 of Ref. 1)).
3) G. Ripka, Adv. in Nucl. Phys. I, p. 183.
4) C. C. J. Roothaan, Rev. Mod. Phys. **23** (1951), 69.
5) D. M. Brink, *Scuola Internazionale di Fisica "Enrico Fermi" XXXVI* (1966), p. 247.
6) A. B. Volkov, Nucl. Phys. **74** (1965), 33.