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On the stability of Laplace resonance for Galilean moons (Io, Europa, Ganymede)

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Abstract: This paper presents the application of recent ansatz for estimation of stability of the Laplace resonance for Galilean moons (Io, Europa, Ganymede). We estimate over time the eccentricity + semi–major axis in a binary system experiencing the net tidal friction, including the additional tidal heating which comes from the transformation of net transfer of angular momentum between the Galilean moons of Jupiter (due to dynamical features of the Laplace resonance). Presumably, there should be a net transfer of angular momentum between Io and Europa (for the reason that tidal heating on Ganymede seems to be negligible with respect to Io and Europa). We established the fact that Laplace resonance should be valid and stable on a timescale of centuries in the future, but there might be chaotic perturbations less than 0.1% for the accuracy of such phenomenon. Moreover, the presented ansatz can be used to predict a scheme for optimizing the maneuvers of spacecrafts in the vicinity of Ganymede (due to absence of net transfer of angular momentum between Ganymede and other Galilean moons). The main conclusion stems from previously suggested approach (Ershkov 2017a) is that there is a net transfer of angular momentum between Io and Europa, which results in the tidal heating. The main results are also relevant in view of the future mission for the icy moons of Jupiter.

Key words: Tidal dissipation, galilean satellites of Jupiter, Laplace resonance, net transfer of angular momentum.

1 - INTRODUCTION

The Laplace resonance is known to be one of the old famous problems in celestial mechanics, besides we should especially note that a lot of great scientists have been reporting their researches related to this phenomenon during the last 200 years.

In accordance with (Peale et al. 1979) the Galilean satellites are numbered in the conventional manner with 1 to 4 corresponding respectively to Io, Europa, Ganymede, and Callisto (the largest moons of Jupiter). Let n_i denotes the mean orbital motion, the relation

$$n_1 - 3n_2 + 2n_3 = 0, \tag{1}$$

is satisfied exactly within infinitesimal observational error. This is the well–known Laplace relation (here mean orbital motion n_i, according to definition, is equal to the average angular frequency of a body in elliptical orbit, and is defined by $n_i = \pi/T_i$, where T_i is the orbital period).

All the Galilean moons are known to be captured in synchronous rotation by Jupiter (except Europa in the closest past, but currently we can assume – at first approximation – that Europe is being captured in tidal locking by Jupiter too). We should especially note that tidal locking (also called gravitational locking or captured rotation (Barnes 2010)) occurs when the long-term interaction between a pair of co-orbiting astronomical bodies drives the rotation rate of at least one of them into the state where there is no more net transfer of angular momentum between moon and its orbit around the Planet.

This condition of "no net transfer of angular momentum" must be satisfied over the course of one orbit around the Planet; besides, let us outline that there is no interchange of the angular momentum between Jupiter and, separately, each of the Galilean moons.

But, nevertheless, Galilean moons (which are related to each other by the Laplace resonance – Io, Europa, Ganymede) might be interchanging angular momentum between each other, inside the aforementioned triple Laplace relation (1).

For example, let us consider the case of Io for the presenting with respect to the fact that total angular momentum inside system "Jupiter–Io" should be variable insofar.

In accordance with the results, which were reported in (Goldreich & Gold 1963), if we only consider the case where the satellite always presents the same face to the Planet (this is the non-rare case which is observed in Solar system and includes Galilean moons of Jupiter, the greater satellites of Saturn as well as the Moon of Earth) then it is easy to understand why the tide raised on satellite may not influence on its eccentricity. The eccentricity ($0 < e < 1$) is in this case, as reported in (Goldreich & Gold 1963):

$$e = \sqrt{1 + \frac{2EL^2}{m^3 M^2 G}} \tag{2}$$

where E is the energy of the orbit ($E = -(GmM)/2a$, where a is semi–major axis), L is the angular momentum, m is the mass of satellite, M is the mass of Jupiter, G is gravitational constant in the Newton's law of universal gravitation.

If the satellite is not spinning (in its relative rotation to the Planet) (Goldreich & Gold 1963), then the tide, which was raised on it's surface, may only produce a radial perturbation force. This means that L is not changed by the tide.

Since any energy dissipation in the satellite decreases E and since we have $E < 0$, and Laplace resonance (1) helps to maintain Io's orbital eccentricity (e) at a stable and constant value (0.0041, according to the data of astrometric observations (Ershkov 2017a)), we find from (2) that there should be a net transfer of angular momentum between the Galilean moons of Jupiter – due to dynamical features of the Laplace resonance (1). Presumably, it should be a net transfer of angular momentum between Io and Europa (for the reason that tidal heating on Ganymede seems to be negligible with respect to Io and Europa).

As for the satellites with a large amount of fluids layers inside (or on their surface as Ocean), there obviously exists a physical mechanism of transformation of the incoming net transfer of angular momentum into the additional tidal heating via interacting of fluid tides or Rossby waves (Tyler 2011, 2014) with the solid structure of the borders of coastline in the Ocean, for example.

Because Ganymede's orbital eccentricity is relatively low - on average 0.0015 – we conclude that tidal heating is negligible now (that's why the net transfer of angular momentum into the system "Jupiter-Ganymede" from Io and Europa is also negligible).

If we consider the orbital dynamics of Galilean moons, we should take into consideration the net tidal heating (including tidal heating which comes to the heat budget from transformation of the net transfer of angular momentum via crushing of fluid tides on the solid boundaries of fluids layers in these moons).

The aforementioned assumption is very important for the understanding of natural mechanism at the explanation of basic governing features in Laplace resonance (1): indeed, it is the net tidal heating that governs the orbital dynamics of Galilean moons in Laplace relation.

2 - MATHEMATICAL MODEL OF ORBITAL DYNAMICS FOR GALILEAN MOONS, CAPTURED IN LAPLACE RESONANCE (SEMI-ANALYTICAL ANSATZ)

Having stated the aforementioned physical mechanism, let us consider mathematical model of orbital dynamics of Galilean moons. According to comprehensive research (Goldreich & Gold 1963), in this article it was stated that the tides on the Galilean moons are probably more important than tides on the Jupiter (insofar as their effects on eccentricity were exhibited).

Basing on the aforementioned assumption, let us recall that in (Ershkov 2017a) the systems of equations (3a), (3b) were presented which have been stated for mutual evolution of the eccentricity e along with the semi–major axis a of the moons of Planet (in (Ershkov 2017a) such system was denoted as (A2)). Here and below we note that the tidal effects are introduced by means of the Love number k_2, which is describing the response of the potential of the distorted body in regard to the experiencing tides, as well as by the quality factor Q, which is inversely proportional to the amount of energy dissipated essentially as heat by tidal friction; so, tidal effects are introduced in the combination k_2/Q for Planet and satellite.

In particular, we recall that we have (as a first approximation) for the tides raised in the $1:1$ spin-orbit satellite (Peale & Cassen 1978):

$$\frac{da}{dt} = -\frac{21 k_2^s M n R_s^5}{Q^s m a^4} e^2 \tag{3a}$$

$$\frac{de}{dt} = -\frac{21 k_2^s M n}{2 Q^s m} (R_s/a)^5 e \tag{3b}$$

here the sign s denotes the case of satellite; m is the mass of satellite, M is the mass of Jupiter, n is the osculating mean motion, R_s is equatorial radius of satellite.

Besides, we should note that according to the Kepler's law of orbital motion the square of mean motion is:

$$n^2 = \frac{G(M+m)}{a^3} \tag{4}$$

Let us denote just for simplicity

$$B = \pm \frac{21 k_2^s M \sqrt{G(M+m)} (R_s)^5}{Q^s m} \tag{5}$$

The mathematical procedure for reduction of the system of Eqs. (3) was reported in (Ershkov 2017a), with only the resulting formulae (5–7) left in the main text of the current research.

For the case of eccentricity $e \longrightarrow 0$, we could obtain from Eqs. (3) (here below $\triangle t$ should be considered as a long time-period scale; $\{a_0, e_0\} = \{a(0), e(0)\} = const$):

$$e \cong e_0 \exp\left[-\frac{B}{2}\frac{\exp\left(13\, e_0^2/2\right)}{a_0^{13/2}} \triangle t\right], \tag{6}$$

as well as we could obtain the appropriate expression for the semi–major axis

$$a \cong a_0 \exp\left(e_0 \exp\left[-\frac{B}{2}\frac{\exp\left(13\, e_0^2/2\right)}{a_0^{13/2}} \triangle t\right]\right) \tag{7}$$

where the scale-factor a_0 should be given by the initial conditions.

3 - DISCUSSION

The main motivation of this research is the ensueing and elegant application of the recent ansatz (Ershkov 2017a) for estimation of stability of the Laplace resonance for Galilean moons (Io, Europa, Ganymede). As for scientific originality of the current research (in comparison with the results of work (Ershkov 2017a)), the aforementioned analysis let us conclude that there is a net transfer of angular momentum between Io and Europa, which results in the tidal heating (this main conclusion portends a flow of constructive research novelty). The main results are also relevant in view of the future mission for the icy moons of Jupiter.

Let us discuss the application of formulae (6-7) for the orbital dynamics of Galilean moons, as well as their contribution into the stability of Laplace resonance (1).

The intriguing fact is that there is no any dynamics (Moore 2003) for Io's orbital eccentricity e: it remains stable at the same constant value (0.0041, according to the data of astrometric observations (Peale et al. 1979)) due to a net transfer of angular momentum between the Galilean moons in Laplace resonance (1). That's why expression for Io's eccentricity, calculated from (6), is simply reduced to the constant (0.0041); therefore we obtain from (7)

$$a_1 = a_1(0) \exp\left(0.0041\right) \cong 1.0041 a_1(0) \tag{8}$$

It means that Io's semi-major axis is assumed to be increased less than circa on 0.5% (by the net tidal heating) since its initial magnitude $a_1(0)$. Indeed, models of its orbit suggest that the amount of tidal heating within Io changes with time (Yoder 1979). All in all, we can come to the conclusion from (4) and (8) that n_1 is proved to be simply the appropriate constant in (1).

Ganymede's orbital eccentricity is relatively (Showman & Malhotra 1997) low - on average 0.0015 - but it is changing quasi–periodically due to solar and planetary gravitational perturbations (including the influence of net transfer of angular momentum from Io and Europa). The range of changes of Ganymede's orbital eccentricity is 0.0009 – 0.0022 on a timescale of centuries (its eccentricity oscillates near the average value of 0.0015).

So, we can come to conclusion from (6) that tidal heating is currently the negligible in case of Ganymede's orbital dynamics and its semi-major axis (7) is assumed to be increased less than circa on 0.2% (by net tidal heating) since its initial magnitude $a_2(0)$.

According to formulae (1) and (7) ($B < 0$ in (5)–(7)), n_1 is proved to be decreased exponentially, but quasi-periodically in the long time–period $\triangle t$ of observations to the appropriate constant value in (1).

As for the dynamics of the Europa's orbital eccentricity (Showman & Malhotra 1997), it quite differs from the aforementioned Galilean moons (Io and Ganymede): despite of its orbital eccentricity is sufficiently low - on average 0.009 - it exceeds the Io's orbital eccentricity more than twice (0.0041) and the Ganymede's orbital eccentricity 6 times (0.0015). It means that tidal heating that governs by the Europa's orbital dynamics should not be considered as negligible process (that's why the net transfer of angular momentum into the system "Jupiter-Europa" from Io and Ganymede is also large enough insofar). In addition to this, quasi-periodical variation in the tidal heating should be taking place since the height of the tide will vary with the oscillation in distance between satellite and Planet (when Europa rotates on its elliptical orbit around Jupiter) (Goldreich & Gold 1963).

So, Europa's orbital eccentricity is being changed quasi-periodically, with the main tendency of its exponentially increasing: indeed, according to the formula (6), Europa's eccentricity oscillates near the average meaning of 0.009 (let us note that we should choose the sign of $B < 0$ in expression (5) for the appropriate dynamics of eccentricity).

Thus, we can come to conclusion from (6) that in case of Europa the tidal heating should be taken into consideration, but nevertheless, Europa's semi–major axis is assumed to be increased less than circa on 0.9% (by the net tidal heating) since its initial magnitude $a_3(0)$. According to (4) and (7), n_3 is proved to be decreasing exponentially, but quasi-periodically in the long time-period $\triangle t$ of observations to the appropriate constant value in (1).

Ending discussion, we can come to a conclusion that in the expression for the Laplace resonance (1): n_1 is proved to be simply a constant, but $\{n_2, n_3\}$ are proved to be decreasing exponentially, but quasi-periodically in the long time–period $\triangle t$ of observations to the appropriate constant values in (1). Thus, Laplace resonance (1) should be valid and stable on a timescale of centuries in the future, but infinitesimal quasi–periodical perturbations are possible there according to (4)–(7) $(0.009)^{(3/2)} = 0.00086$ which is less than 0.1% (namely, 0.086%).

4 - CONCLUSION

Having been presented previously in clear mathematical formula interrelating the mean orbital motions of the largest moons of Jupiter (Io, Europa, Ganymede, and Callisto), the Laplace relation is proved to be stable during a centuries in case these moons are rotating around their common centre of mass with Jupiter on elliptic orbits always maintaining the configuration with mean motion's resonances in so way that Laplace relation is satisfied exactly within infinitesimal observational error (which is less than 0.1%).

In a real physical world, there should be a natural mechanism, maintaining the aforementioned balance of mean orbital motions for the celestial bodies which are locked into the resonance. We have come to conclusion it should be the net transfer of angular momentum between moons in the process of their mutual interinfluences for changing by the angular momentum from one to each other (due to dynamical features of the orbital evolution of moons of Jupiter under the influence of the resonance phenomena). Moreover, we should recall that there are only a few known perfect mean-motion resonances in the Solar system, involving planets, dwarf planets or larger satellites:

1. 2:3 Pluto-Neptune;
2. 2:4 Tethys-Mimas (Saturn's moons), not classical case;
3. 1:2 Dione-Enceladus (Saturn's moons);
4. 3:4 Hyperion-Titan (Saturn's moons);
5. 1:2:4 Ganymede-Europa-Io (Jupiter's moons, ratio of orbits).

We have investigated the last one of the aforesaid resonances here. In general case, the changing by the angular momentum between moons of Jupiter occurs chaotically during their mutual motions but there are restrictions in this process which have been outlined in the current research. It is worthnoting that illuminating such restrictions allows us to explore the stability of Laplace relation.

There are a lot of theories of tidal dissipation, but most of them could be associated with two main types: 1) tidal friction for bodies with fluid layers, 2) solid tidal dissipation. Indeed, the rheological law for the actual rheological parameters, obeyed by the material of the bodies and their role in dissipation, differs from one of the aforementioned types to another.

This paper presents the application of ansatz, suggested earlier in (Ershkov 2017a), for estimation of stability of the Laplace resonance (1). Such the mathematical technique (Ershkov 2017a) helps with the analytical integration over time of eccentricity + semi-major axis in a binary system experiencing tidal friction. The described formulation incorporates formulae for the tidal friction that is surely not appropriate for the aforementioned bodies with fluid layers. In this respect we confine ourselves to mention the papers (Tyler 2011, 2014) where all the difficulties concerning the most complicated cases of tidal friction for bodies with fluid layers are remarked.

As noted above, we have found from (2) that in case of Laplace resonance there should be a net transfer of angular momentum between the Galilean moons of Jupiter - due to dynamical features of the Laplace relation (1). Presumably, there should be a net transfer of angular momentum between Io and Europa (for the reason that tidal heating on Ganymede seems to be negligible with respect to Io and Europa).

As for the satellites with a large amount of fluids layers inside (or on their surface as Ocean), there should be a physical mechanism of transformation of incoming net transfer of angular momentum into the additional tidal heating via interacting of fluid tides or Rossby waves with the solid structure of the borders of coastline in Ocean or other boundaries of fluids layers in the satellites.

The last but not least, let us note that appropriate corrections to formulae (5)–(7) should be accomplished with the data of astrometric observations: we should adjust analytical solutions with respect to the actual data of orbits of satellites.

Moreover, the presented ansatz can be used to predict a scheme for optimizing the maneuvers of spacecraft in the vicinity of Ganymede (due to absence of net transfer of angular momentum between Ganymede and other Galilean moons).

Finally, we should especially note that a kind of kinematic ansatz for the application of the recent approach (Ershkov 2017a) with the main aim to estimate the stability of the Laplace resonance for

Galilean moons (Io, Europa, Ganymede) is presented here. The kinematic ansatz (Ershkov & Shamin 2018a,b) means that instead of considering the dynamical causes for the Laplace resonance, we should investigate it as to be exceptionally kinematic phenomenon in regard to the influence on elements of orbit as well as its influence on the net transfer of the angular momentum between Io and Europa (during the process of their mutual orbital evolution). That's why the methodology of the study and the analysis of the results have been presented without a kind of dynamical approach. In this respect we confine ourselves to mention the paper (Paita et al. 2018) in which all the significant details concerning the dynamical approach for the Laplace resonance are remarked.

Also, a remarkable articles should be cited, which concern the problem under consideration (Chernousko et al. 2017) (where fundamental law of angular momentum conservation in application to the dynamics of celestial bodies has been discussed), (Ershkov 2017b) (where tidal phenomena has been discussed relate to rotational regimes of satellite around the primary) and (Pathak et al. 2019a,b) (where resonant phenomena are discussed in the frame of PR3BP). The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Author contributions

In this research, Dr. Sergey Ershkov is responsible for the general ansatz and the solving procedure, simple algebra manipulations, calculations, results of the article in Sections (1–3) and also is responsible for the search of approximate solutions. Dr. Dmytro Leshchenko is responsible for theoretical investigations as well as for the deep survey in literature on the problem under consideration. Both authors agreed with the results and conclusions of each other in Sections (1–4).