

## ON THE STABILITY OF VISCOUS FLOW OVER A STRETCHING SHEET

By

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**Abstract.** The linear stability of two-dimensional boundary layer flow of an incompressible viscous fluid over a flat deformable sheet is investigated when the sheet is stretched in its own plane with an outward velocity proportional to the distance from a point on it. Using Galerkin's method the stability equations are solved for three-dimensional disturbances periodic in a direction normal to the plane of the basic flow and it is shown that the flow is stable.

**1. Introduction.** Flow in the boundary layer over moving solid surfaces was investigated by Sakiadis [1]. Due to the entrainment of the ambient fluid, this boundary layer is different from that in Blasius flow over a flat plate. Erickson, Fan and Fox [2] extended this problem to the case when the transverse velocity at the moving surface is non-zero and is such that similarity solutions exist. These studies have bearing on the problem of a polymer sheet extruded continuously from a die and are based on the tacit assumption that the moving sheet is inextensible. But situations may also arise in the polymer industry where one has to deal with flow over stretching plastic sheet. McCormack and Crane [3] gave a similarity solution in closed analytical form for steady two-dimensional boundary layer flow over such a sheet which is stretched with a velocity proportional to  $x$  ( $x$  being the distance along the sheet). The corresponding nonsimilar solution for the same problem in the presence of a uniform free stream velocity was obtained by Danberg and Fansler [4].

However the stability of the flow over a stretching sheet does not seem to have received any attention despite its importance in the polymer industry. The purpose of this paper is to study the linear stability of the flow examined in [3] for three-dimensional disturbances.

**2. Stability analysis.** Consider the flow of an incompressible viscous fluid past a sheet coinciding with the plane  $y = 0$ . Two equal and opposite forces are introduced along the  $x$ -axis so that the sheet is stretched keeping the origin fixed. In a polymer processing

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application, the above situation is similar to the case of a flat sheet issuing from a thin slit at the origin ( $x = 0, y = 0$ ) and subsequently being stretched. Assuming boundary layer approximations, the equations of continuity and momentum in the usual notations are

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0, \quad (1)$$

$$u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} = \nu \frac{\partial^2 u_0}{\partial y^2}, \quad (2)$$

respectively. The boundary conditions are

$$u_0 = cx, v_0 = 0 \quad \text{at } y = 0; \quad u_0 \rightarrow 0 \quad \text{as } y \rightarrow \infty, \quad (3)$$

$c$  being a positive constant.

The similarity solution of the above system was given in [3] and [4] as

$$u_0 = cx F'(\eta), \quad v_0 = -(c\nu)^{1/2} F(\eta), \quad (4a)$$

where

$$F(\eta) = 1 - e^{-\eta}; \quad \eta = (c/\nu)^{1/2} y. \quad (4b)$$

The justification for assuming a linear velocity for the sheet as given by (3) is as follows. As pointed out by Vleggaar [5], in a polymer processing application involving spinning of filaments without blowing, laminar boundary layer occurs over a relatively small length of the spinning zone: 0.0–0.5m from the die which may be taken as the origin of Fig. 1. This is in fact the zone over which the major part of the stretching (and also heat transfer) takes place. In such a process the initial velocity is low (about 0.3m/s), and a good approximation of the velocity of the filament or sheet is  $u_0 = cx$  (at any rate for the first 10–60cm of the spinning zone), where  $c$  is the constant velocity gradient. However, it should be noted that in actual practice, the stretching filament or sheet may not always conform to the linear speed assumed here.

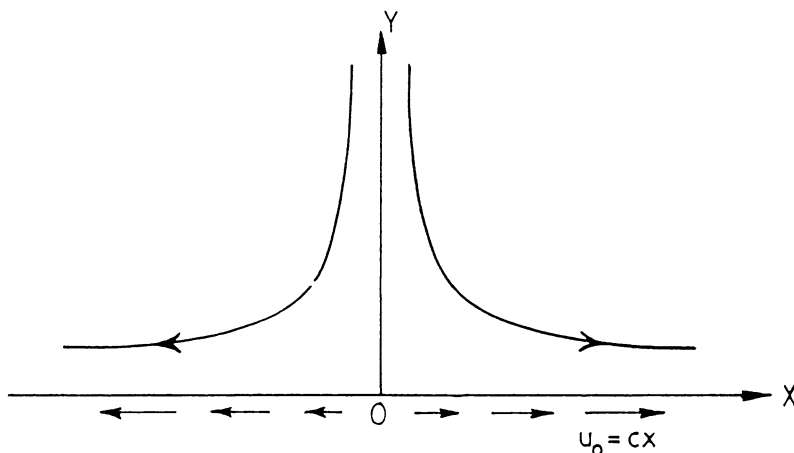


FIG. 1. Sketch of the physical problem.

We now study the stability of the solution given by (4). Since the form of this solution is similar to that for a two-dimensional stagnation-point flow, we examine, following Görtler [6], the stability of (4) with respect to disturbances periodic in a direction normal to the plane of the basic flow. It may be noted that Hämmerlin's [7] detailed study of the disturbance differential equations derived by Görtler suggests that instability can occur in the form of Taylor-Görtler vortices.

We take the perturbed state as

$$u = u_0 + \bar{u} = u_0 + cxf_1(\eta, z, t), \quad (5a)$$

$$v = v_0 + \bar{v} = v_0 - \sqrt{c\nu}f_2(\eta, z, t), \quad (5b)$$

$$w = \bar{w} = \nu f_3(\eta, z, t), \quad (5c)$$

$$p = p_0 + \bar{p} = p_0 + \rho\nu cf_4(\eta, z, t), \quad (5d)$$

where  $u_0$  and  $v_0$  are given by (4),  $p_0$  denotes the basic pressure distribution and  $w$  denotes the perturbation velocity component normal to the  $xy$ -plane.

Following Görtler, we assume that the perturbations are periodic in  $z$  and have an exponential time-dependence as follows:

$$\bar{u} = cxu_1(\eta)(\cos \alpha z)e^{\beta t}, \quad \bar{v} = -(c\nu)^{1/2}v_1(\eta)(\cos \alpha z)e^{\beta t}, \quad (6a)$$

$$\bar{w} = \nu\alpha w_1(\eta)(\sin \alpha z)e^{\beta t}, \quad \bar{p} = \rho\nu cp_1(\eta)(\cos \alpha z)e^{\beta t}. \quad (6b)$$

The linearized three-dimensional perturbation equations are now

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0, \quad (7)$$

$$\frac{\partial \bar{u}}{\partial t} + u_0 \frac{\partial \bar{u}}{\partial x} + \bar{u} \frac{\partial u_0}{\partial x} + v_0 \frac{\partial \bar{u}}{\partial y} + \bar{v} \frac{\partial u_0}{\partial y} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \nu \nabla^2 \bar{u}, \quad (8)$$

$$\frac{\partial \bar{v}}{\partial t} + u_0 \frac{\partial \bar{v}}{\partial x} + \bar{u} \frac{\partial v_0}{\partial x} + v_0 \frac{\partial \bar{v}}{\partial y} + \bar{v} \frac{\partial v_0}{\partial y} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} + \nu \nabla^2 \bar{v}, \quad (9)$$

$$\frac{\partial \bar{w}}{\partial t} + u_0 \frac{\partial \bar{w}}{\partial x} + v_0 \frac{\partial \bar{w}}{\partial y} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} + \nu \nabla^2 \bar{w}, \quad (10)$$

where  $\nabla^2$  is the three-dimensional Laplace operator. Substitution of (6) in (7)–(10) gives upon using (4),

$$u_1 - v_1' + \bar{\alpha}^2 w_1 = 0, \quad (11)$$

$$u_1'' + Fu_1' - (\bar{\beta} + \bar{\alpha}^2 + 2F')u_1 = -F''v_1, \quad (12)$$

$$v_1'' + Fv_1' - (\bar{\beta} + \bar{\alpha}^2 - F')v_1 = -p_1'. \quad (13)$$

$$w_1'' + Fw_1' - (\bar{\beta} + \bar{\alpha}^2)w_1 = -p_1, \quad (14)$$

where a prime denotes derivative with respect to  $\eta$  and

$$\bar{\alpha}^2 = \nu\alpha^2/c, \quad \bar{\beta} = \beta/c. \quad (15)$$

The no-slip boundary conditions are

$$u_1 = v_1 = w_1 = 0 \quad \text{at } \eta = 0 \quad (16)$$

and since the perturbations vanish at infinity, we must have

$$u_1 = v_1 = w_1 = 0 \quad \text{as } \eta \rightarrow \infty. \quad (17)$$

Using (16) and (17) in (11) gives  $v_1' = 0$  at  $\eta = 0$  and  $\eta = \infty$ . Thus the boundary conditions become

$$u_1 = v_1 = v_1' = 0 \quad \text{at } \eta = 0 \text{ and } \eta = \infty. \quad (18)$$

Elimination of  $w_1$  and  $p_1$  from (11), (13) and (14) gives

$$\begin{aligned} v_1^{iv} + Fv_1''' + (F' - \bar{\beta} - 2\bar{\alpha}^2)v_1'' - \bar{\alpha}^2 Fv_1' + \bar{\alpha}^2(\bar{\beta} + \bar{\alpha}^2 - F')v_1 \\ = u_1''' + Fu_1'' + (F' - \bar{\beta} - \bar{\alpha}^2)u_1'. \end{aligned} \quad (19)$$

Differentiating (12) with respect to  $\eta$  and combining with (19), we get

$$\begin{aligned} v_1^{iv} + Fv_1''' + (F' - \bar{\beta} - 2\bar{\alpha}^2)v_1'' + (F'' - \bar{\alpha}^2 F)v_1' \\ + [\bar{\alpha}^2(\bar{\beta} + \bar{\alpha}^2 - F') + F''']v_1 = 2F'u_1' + 2F''u_1. \end{aligned} \quad (20)$$

Introducing

$$T = e^{-\eta}, \quad L \equiv -T \frac{d}{dT}, \quad (21)$$

Eqs. (12) and (20) become

$$L^2 u_1 + (1 - T)Lu_1 - (\bar{\beta} + \bar{\alpha}^2 + 2T)u_1 = Tv_1, \quad (22)$$

$$\begin{aligned} L^4 v_1 + (1 - T)L^3 v_1 + (T - \bar{\beta} - 2\bar{\alpha}^2)L^2 v_1 - [T + \bar{\alpha}^2(1 - T)]Lv_1 \\ + [\bar{\alpha}^2(\bar{\beta} + \bar{\alpha}^2 - T) + T]v_1 = 2T(Lu_1 - u_1). \end{aligned} \quad (23)$$

Further the boundary conditions (18) reduce to

$$u_1 = v_1 = Lv_1 = 0 \quad \text{at } T = 0 \text{ and at } T = 1. \quad (24)$$

Equations (22) and (23) subject to the boundary conditions (24) constitute an eigenvalue problem for stability.

To solve the above system we use Galerkin's method, a succinct account of which is given by Duncan [8]. We expand  $u_1$  in a set of trial functions

$$T(1 - T), T^2(1 - T), T^3(1 - T), \dots \quad (25)$$

each of which satisfies (24). Similarly  $v_1$  is expanded in the following set

$$T(1 - T)^2, T^2(1 - T)^2, T^3(1 - T)^2, \dots \quad (26)$$

each of which also satisfies (24). We begin with one-term approximation

$$u_1 = AT(1 - T), \quad v_1 = BT(1 - T)^2, \quad (27)$$

where  $A$  and  $B$  are constants and introduce the following notation

$$f_{m,n} = T^m(1 - T)^n. \quad (28)$$

Then it can be readily shown that

$$Lf_{m,n} = nf_{m+1,n-1} - mf_{m,n} \quad (m, n \geq 1), \quad (29a)$$

$$Lf_{m,0} = -mf_{m,0}. \quad (29b)$$

Substitution of (27) in (22) gives on using (28) and (29) the following residual

$$R_1 = A[-3f_{2,0} + (1 - \bar{\beta} - \bar{\alpha}^2)f_{1,1} - f_{2,1} - f_{1,2}] - Bf_{2,2}. \quad (30)$$

Noting that

$$\int_0^1 f_{m,n} dT = \frac{m!n!}{(m+n+1)!}, \quad (31)$$

the orthogonality condition

$$\int_0^1 T(1-T)R_1 dT = 0$$

reduces to

$$A\left[\frac{3}{2} + \frac{1}{3}(\bar{\beta} + \bar{\alpha}^2)\right] + \frac{B}{14} = 0. \quad (32)$$

Similarly using (27) through (29) in (23), the following residual is obtained:

$$\begin{aligned} R_2 = & [50B - (2\bar{\beta} + 4\bar{\alpha}^2)B - 2A]f_{3,0} + [-30B + 6(\bar{\beta} + 2\bar{\alpha}^2)B + 4A]f_{2,1} \\ & + [1 - \bar{\beta} - 2\bar{\alpha}^2 + \bar{\alpha}^2(\bar{\beta} + \bar{\alpha}^2)]Bf_{1,2} - 20Bf_{3,1} \\ & + [17 - 3\bar{\alpha}^2]Bf_{2,2} + [\bar{\alpha}^2 - 1]Bf_{1,3} + 2Bf_{4,0}. \end{aligned}$$

The orthogonality of  $R_2$  to  $T(1-T)^2$  then gives as before

$$\frac{A}{105} + \left[\frac{4}{15} + \frac{\bar{\beta}}{70} + \frac{\bar{\alpha}^2}{42} + \frac{1}{105}(\bar{\alpha}^2\bar{\beta} + \bar{\alpha}^4)\right]B = 0. \quad (33)$$

Equations (32) and (33) have a non-trivial solution if the determinant of the coefficients of  $A$  and  $B$  vanishes. This gives on simplification

$$\begin{aligned} \left(\frac{1}{210} + \frac{\bar{\alpha}^2}{315}\right)\bar{\beta}^2 + \left(\frac{139}{1260} + \frac{17}{630}\bar{\alpha}^2 + \frac{2\bar{\alpha}^4}{315}\right)\bar{\beta} \\ + \left(\frac{587}{1470} + \frac{157}{1260}\bar{\alpha}^2 + \frac{\bar{\alpha}^4}{45} + \frac{\bar{\alpha}^6}{315}\right) = 0. \end{aligned} \quad (34)$$

Since the coefficients in (34) are all positive for real values of the wave number  $\bar{\alpha}$ , it follows that  $\bar{\beta}$  can neither be positive nor can have positive real part. Hence from (6) and (15) we conclude that the flow is stable.

In order to see to what extent the one-term approximation (27) gives reliable results, we proceed to solve the eigenvalue problem with a two-term approximation for  $u_1$  and  $v_1$  as follows:

$$u_1 = ET(1-T) + RT^2(1-T) = Ef_{1,1} + Rf_{2,1}, \quad (35a)$$

$$v_1 = GT(1-T)^2 + HT^2(1-T)^2 = Gf_{1,2} + Hf_{2,2}, \quad (35b)$$

where  $E$ ,  $R$ ,  $G$  and  $H$  are constants. It is important to note that in (35a),  $f_{1,1}$  and  $f_{2,1}$  are linearly independent. Similarly  $f_{1,2}$  and  $f_{2,2}$  in (35b) are also linearly independent. Such linear independence is necessary for the application of Galerkin's method. We then

calculate the residual by substituting (35a) and (35b) in (22). The orthogonality of this residual to both  $f_{1,1}$  and  $f_{2,1}$  then gives as before

$$\left[ \frac{3}{20} + \frac{\bar{\alpha}^2}{30} + \frac{\bar{\beta}}{30} \right] E + \left[ \frac{13}{105} + \frac{\bar{\alpha}^2}{60} + \frac{\bar{\beta}}{60} \right] R + \frac{G}{140} + \frac{H}{280} = 0, \quad (36)$$

$$\left[ \frac{1}{10} + \frac{\bar{\alpha}^2}{60} + \frac{\bar{\beta}}{60} \right] E + \left[ \frac{79}{840} + \frac{\bar{\alpha}^2}{105} + \frac{\bar{\beta}}{105} \right] R + \frac{G}{280} + \frac{H}{504} = 0. \quad (37)$$

Proceeding exactly in the same manner, the two orthogonality relations deduced from (23) are

$$\begin{aligned} \frac{E}{105} + \frac{R}{105} + \left[ \frac{4}{15} + \frac{\bar{\alpha}^2}{42} + \frac{\bar{\alpha}^4}{105} + \left( \frac{1}{70} + \frac{\bar{\alpha}^2}{105} \right) \bar{\beta} \right] G \\ + \left[ \frac{317}{1260} + \frac{23}{1260} \bar{\alpha}^2 + \frac{\bar{\alpha}^4}{280} + \left( \frac{1}{105} + \frac{\bar{\alpha}^2}{280} \right) \bar{\beta} \right] H = 0, \end{aligned} \quad (38)$$

$$\begin{aligned} \frac{E}{420} + \frac{R}{252} + \left[ \frac{59}{315} + \frac{23\bar{\alpha}^2}{2520} + \frac{\bar{\alpha}^4}{280} + \left( \frac{1}{168} + \frac{\bar{\alpha}^2}{280} \right) \bar{\beta} \right] G \\ + \left[ \frac{64}{315} + \frac{13\bar{\alpha}^2}{1260} + \frac{\bar{\alpha}^4}{630} + \left( \frac{1}{180} + \frac{\bar{\alpha}^2}{630} \right) \bar{\beta} \right] H = 0. \end{aligned} \quad (39)$$

For the existence of a non-trivial solution the determinant of the coefficients of  $E$ ,  $R$ ,  $G$  and  $H$  in (36) through (39) must vanish. After a lengthy algebra, this condition can be written as

$$c_4 \bar{\beta}^4 + c_3 \bar{\beta}^3 + (c'_2 + c''_2) \bar{\beta}^2 + (c'_1 + c''_1) \bar{\beta} + (c'_0 + c''_0) = 0. \quad (40)$$

where,

$$c_4 = 36288 + 32508\bar{\alpha}^2 + 3780\bar{\alpha}^4, \quad (41a)$$

$$c_3 = 2526552 + 2155356\bar{\alpha}^2 + 238140\bar{\alpha}^4 + 15120\bar{\alpha}^6 \quad (41b)$$

$$\begin{aligned} c'_2 = 49926744 + 38105802\bar{\alpha}^2 + 7044282\bar{\alpha}^4 \\ + 519372\bar{\alpha}^6 + 22680\bar{\alpha}^8, \end{aligned} \quad (41c)$$

$$c''_2 = -9504 - 2844\bar{\alpha}^2, \quad (41d)$$

$$\begin{aligned} c'_1 = 313922196 + 183510936\bar{\alpha}^2 + 70474320\bar{\alpha}^4 + 7767648\bar{\alpha}^6 \\ + 454356\bar{\alpha}^8 + 15120\bar{\alpha}^{10}, \end{aligned} \quad (41e)$$

$$c''_1 = -309384 - 33264\bar{\alpha}^2 - 5688\bar{\alpha}^4, \quad (41f)$$

$$\begin{aligned} c'_0 = 491272992 + 367592148\bar{\alpha}^2 + 134811936\bar{\alpha}^4 + 34895070\bar{\alpha}^6 \\ + 2842434\bar{\alpha}^8 + 140616\bar{\alpha}^{10} + 3780\bar{\alpha}^{12}, \end{aligned} \quad (41g)$$

$$c''_0 = -3184992 - 414072\bar{\alpha}^2 - 23760\bar{\alpha}^4 - 2844\bar{\alpha}^6. \quad (41h)$$

It may be noticed from above that

$$c''_2 \ll c'_2, \quad c''_1 \ll c'_1 \quad \text{and} \quad c''_0 \ll c'_0. \quad (42)$$

Hence neglecting  $c_0''$ ,  $c_1''$  and  $c_2''$  in comparison with  $c_0'$ ,  $c_1'$  and  $c_2'$  respectively. Eq. (40) can be expressed as

$$\begin{aligned} & [252\bar{\beta}^2 + (5292 + 504\bar{\alpha}^2)\bar{\beta} + (10962 + 5292\bar{\alpha}^2 + 252\bar{\alpha}^4)] \\ & \times [(144 + 129\bar{\alpha}^2 + 15\bar{\alpha}^4)\bar{\beta}^2 + (7002 + 5556\bar{\alpha}^2 + 372\bar{\alpha}^4 + 30\bar{\alpha}^6)\bar{\beta} \\ & + (44816 + 11898\bar{\alpha}^2 + 5524\bar{\alpha}^4 + 243\bar{\alpha}^6 + 15\bar{\alpha}^8)] = 0. \end{aligned} \quad (43)$$

Since in both the quadratic factors, the coefficients are all positive for real values of  $\bar{\alpha}$ , it follows that  $\bar{\beta}$  can neither be positive nor can it have positive real part. Hence the flow is stable.

As a specific example, we assume  $\bar{\alpha}^2 = 10$ . In this case Eq. (34) based on one-term approximation gives the following values for  $\bar{\beta}$ .

$$(\bar{\beta}_1)_{1\text{-term}} = -14.5, \quad (\bar{\beta}_2)_{1\text{-term}} = -13.3. \quad (44)$$

On the other hand, Eq. (43) based on two-term approximation gives

$$(\bar{\beta}_1)_{2\text{-term}} = -12.33, \quad (\bar{\beta}_2)_{2\text{-term}} = -11.58, \quad (45)$$

$$(\bar{\beta}_3)_{2\text{-term}} = -28.67, \quad (\bar{\beta}_4)_{2\text{-term}} = -32.64. \quad (46)$$

While the two-term approximation generates two additional values of  $\bar{\beta}$  given by (46), it can be seen on comparing the values of  $\bar{\beta}_1$  and  $\bar{\beta}_2$  in (44) and (45) that the agreement is not bad.

However to make sure that we indeed get good convergence we next proceed to the three-term approximation.

$$u_1 = Jf_{1,1} + Kf_{2,1} + Mf_{3,1}, \quad (47)$$

$$v_1 = Nf_{1,2} + Pf_{2,2} + Qf_{3,2}, \quad (48)$$

where  $J, K, \dots, Q$  are constants.

Omitting the details of calculation and proceeding in exactly the same way as before, we find after a fairly heavy algebra the dispersion relation for the stability problem as

$$UV = 0. \quad (49)$$

Here  $U$  and  $V$  are both cubic polynomials in  $\bar{\beta}$  given by

$$\begin{aligned} U = & 36\bar{\beta}^3 + (2214 + 108\bar{\alpha}^2)\bar{\beta}^2 + (19164 + 4428\bar{\alpha}^2 + 108\bar{\alpha}^4)\bar{\beta} \\ & + (25812 + 19164\bar{\alpha}^2 + 2214\bar{\alpha}^4 + 36\bar{\alpha}^6), \end{aligned} \quad (50)$$

$$\begin{aligned} V = & (43560 + 65098\bar{\alpha}^2 + 12826\bar{\alpha}^4 + 484\bar{\alpha}^6)\bar{\beta}^3 \\ & + (6144864 + 7288072\bar{\alpha}^2 + 1281874\bar{\alpha}^4 + 50578\bar{\alpha}^6 + 1452\bar{\alpha}^8)\bar{\beta}^2 \\ & + [118793400 + 117452148\bar{\alpha}^2 + 20725892\bar{\alpha}^4 + 2424422\bar{\alpha}^6 \\ & + 62678\bar{\alpha}^8 + 1452\bar{\alpha}^{10}]\bar{\beta} \\ & + [436898870 + 205745496\bar{\alpha}^2 + 115904052\bar{\alpha}^4 + 13508308\bar{\alpha}^6 \\ & + 1207646\bar{\alpha}^8 + 24926\bar{\alpha}^{10} + 484\bar{\alpha}^{12}]. \end{aligned} \quad (51)$$

Equation (49) implies either  $U = 0$  or  $V = 0$ , and hence for stability, neither of these equations should have any root with a positive real part. To test this we use the Routh-Hurwitz criterion [9] for stability of a third-order system. If  $a_0, a_1, a_2$ , and  $a_3$  are the coefficients of  $\bar{\beta}^3, \bar{\beta}^2, \bar{\beta}$  and  $\bar{\beta}^0$  respectively, in (50), then for non-existence of any root of  $U = 0$  with positive real part, this criterion demands that  $a_1 a_2 - a_0 a_3 > 0$ . This is indeed true since

$$a_1 a_2 - a_0 a_3 = 41499864 + 11183400\bar{\alpha}^2 + 637632\bar{\alpha}^4 + 10368\bar{\alpha}^6. \quad (52)$$

Similarly if  $b_0, b_1, b_2$  and  $b_3$  are the coefficients of  $\bar{\beta}^3, \bar{\beta}^2, \bar{\beta}$  and  $\bar{\beta}^0$  respectively in (51), then

$$\begin{aligned} b_1 b_2 - b_0 b_3 = & 7.1093797 \times 10^{14} + 1.5500988 \times 10^{15}\bar{\alpha}^2 \\ & + 1.1115895 \times 10^{15}\bar{\alpha}^4 + 3.1153282 \times 10^{14}\bar{\alpha}^6 \\ & + 4.8217339 \times 10^{13}\bar{\alpha}^8 + 4.4832861 \times 10^{12}\bar{\alpha}^{10} \\ & + 2.1997299 \times 10^{11}\bar{\alpha}^{12} + 7.6159607 \times 10^9\bar{\alpha}^{14} \\ & + 1.4617574 \times 10^8\bar{\alpha}^{16} + 1874048\bar{\alpha}^{18} \end{aligned} \quad (53)$$

which is clearly positive. Hence it follows from (52) and (53) that neither  $U = 0$  nor  $V = 0$  has any root with positive real part. Thus the system is stable.

For  $\bar{\alpha}^2 = 10$ , the roots of  $U = 0$  and  $V = 0$  with the smallest magnitude are

$$(\bar{\beta}_1)_{3\text{-term}} = -11.65, \quad (\bar{\beta}_2)_{3\text{-term}} = -11.09, \quad (54)$$

respectively, the other roots being  $-18.43$  and  $-61.42$  for  $U = 0$  and  $-19.52$  and  $-80.03$  for  $V = 0$ . On comparing the values of  $\bar{\beta}_1$  and  $\bar{\beta}_2$  from (44), (45) and (54), we find that the convergence of the Galerkin method using 3-term approximation is fairly good and there is hardly any point in continuing this approximation any further, which will only involve cumbersome algebra.

**3. Discussion.** It is worth pointing out that the two dimensional flow whose stability is discussed here is akin to both stagnation-point flow and an asymptotic suction profile. The stability of this flow on a deformable wall is studied with respect to Görtler-type disturbances because the streamlines are curved. It turns out that the flow is stable and this may be explained physically as follows. In such a flow there are certainly regions where the circulation (product of local velocity and the local radius of curvature of the streamline for the basic flow) decreases as the local centre of curvature is approached normal to the curved streamlines. This is due to the fact that the fluid velocity increases as the sheet is approached due to the stretching of the sheet. Thus Rayleigh's criterion [10] suggests stability. In a two-dimensional stagnation-point flow, however, the above circumstances are reversed and one does find centrifugal instability as in the analyses of Görtler and Hämmerlin.

It should be noted, however, that our stability analysis is confined to infinitesimal Görtler-type disturbances, which are non-propagating. It cannot, therefore, be ruled out that the flow may be unstable to other types of disturbances which may be infinitesimal or



of finite amplitude. Experiments are, therefore, needed to confirm our theoretical prediction about stability. Nevertheless we feel that the present study, albeit confined to linear theory, throws some light on the stability of a flow having important bearing on polymer industry.

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