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ON THE STRUCTURE OF THE MOON

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Summary

Eckert's result on the Moon's moments of inertia leads, on a straightforward interpretation, to the result that there is a thick layer near the surface with a density greater than that of lead.

W. J. Eckert (1965), after an extensive revision of the theoretical secular motions of the Moon's node and perigee, finds that the observed values, taken with the results from the librations, imply a distribution of density similar to that of a tennis ball. The most striking result is given as an entry in a table without special comment, and seems worth elaboration.

For a spherically symmetrical body with a Wiechert structure, I take radius a, core radius $a\alpha$, density ρ for $a\alpha < r < 1$, ρ_1 for $0 < r < a\alpha$. Then the mean density

$$\bar{\rho} = \rho(1-\alpha^3) + \rho_1\alpha^3$$

and the ratio

$$\frac{C}{Ma^2} = \frac{2}{5}\,\frac{\rho(1-\alpha^5)+\rho_1\alpha^5}{\rho(1-\alpha^3)+\rho_1\alpha^2}$$

$$= \frac{2}{5}\left\{\alpha^2 + \frac{\rho}{\bar{\rho}}(1-\alpha^2)\right\}.$$

The central density ρ_1 cannot be negative; then

$$\bar{\rho} \geqslant \rho(1-\alpha^3)$$

$$\frac{C}{Ma^2} \leqslant \frac{2}{5}\,\frac{1-\alpha^5}{1-\alpha^3}$$

with equality when $\rho_1 = 0$. Some numerical values are given in Table I.

TABLE I

α	C/Ma^2 max	$\rho/\bar{\rho}$ min
0·2	0·4031	1·008
0·4	0·4230	1·068
0·6	0·4705	1·276
0·8	0·5511	2·049
0·85	0·5752	2·585
0·9	0·6044	3·690
0·95	0·6345	7·012
1·00	0·6667	∞

Eckert's g' is $\frac{3}{2}C/Ma^2(3C'/2Mb'^2$ in his notation) and is given as 0·965 with no stated uncertainty. The residuals in the node are given as $-1{\cdot}9''$, $+1{\cdot}0''$ per century for $g' = 0{\cdot}9$ and $1{\cdot}0$. The observed value adopted is that of Spencer Jones from occultations; the standard error may be taken as $2''$, and we may take

$$g' = 0{\cdot}965 \pm 0{\cdot}07.$$

g' cannot exceed 1; the lower value near $0{\cdot}9$ would make

$$C/Ma^2 = 0{\cdot}60,$$

whence

$$\alpha \geqslant 0{\cdot}9, \qquad \rho \geqslant 12{\cdot}5 \text{ g/cm}^3,$$

with $\bar{\rho} = 3{\cdot}39$ g/cm^3.

With a higher value of ρ_1, α and ρ would increase, subject to no change of $\bar{\rho}$ and g'; α will reach 1 and ρ become infinite when $\rho_1 = \bar{\rho}$. But even with this extreme hypothesis there is a probability of about $\frac{5}{6}$ that $\rho > 12{\cdot}5$ g/cm^3.

This result suggests possible explanations.

1. The density in the outer parts of the Moon may be well over that of lead.

2. There may be an error in Eckert's calculations:

3. The values of the constants of the libration, used especially in estimation of $(C-A)/B$ for the Moon from the inclination of its axis, may be wrong by 30%.

4. The value of the motion of the node may have a systematic error, either of observation or of the method of reduction.

I think that the first three are out of the question.

The motion of the perigee gives reasonable agreement between theory and observation; it is not very sensitive to g'.

Reference

Eckert, W. J., 1965. *Astr. J., N.Y.*, **70**, 787–792.