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ON THE SYNTHESIS OF REALISTIC SEA STATES IN A LABORATORY FLUME

E.R. Funke and E.P.D. Mansard

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LIST OF SYMBOLS

$A(f)$	Fourier coefficient or periodogram
C_R	Reflection coefficient
$D(s)$	Wave slope distribution
$E(t)$	Smoothed instantaneous wave energy history (SIWEH)
$\bar{E}$	Average value of E(t) = Variance of the wave record
f	Frequency in Hertz
f_p or FP	Peak frequency of the wave spectrum
F_e	Heave exciting force
GF	Groupiness factor
H_s	Significant wave height
H_{max}	Maximum wave height
m_0	Zeroth moment of the variance spectral density of the water surface elevation
$m_{\varepsilon 0}$	Zeroth moment of the variance spectral density of the SIWEH function
Q_k	Smoothing or window function
$S_I(f)$	Variance spectral density of the incident waves
$S_R(f)$	Variance spectral density of the reflected waves
$S_\eta(f)$	Variance spectral density of the water surface elevation
s_{max}	Maximum wave slope
T_{max}	Period of the maximum wave
T_n	Length of the finite wave record
T_p	Peak period of the spectrum

LIST OF SYMBOLS (Cont'd)

$\varepsilon(f)$	Variance spectral density of the SIWEH function
BF	Spectral width parameter
ζ	Damping factor
$F(\ )$	Fourier transform
$F^{-1}(\ )$	Inverse Fourier transform
Δf	Frequency increment
ΔT	Time increment
η	Water surface elevation
λ	Normalized frequency
$\phi(f)$	Phase spectrum
τ	Time shift
ω	Frequency in radians per second

On the Synthesis of Realistic Sea States in a Laboratory Flume

1.0 INTRODUCTION

The testing of models of coastal structures in laboratory wave flumes and basins requires a realistic attack by waves in order to establish the range of forces or the extent of damage on the basis of which values for design parameters may be determined. Techniques and machinery for the generation of such test waves have evolved from the early electric motor driven waveboards for the generation of regular waves to the modern double articulated and hydraulically actuated 'random' wave makers operating under direct computer control. In between these two extremes one may find pneumatic plenums, rotating eccentric cylinders, horizontal or vertical oscillating wedges, submerged vertical pistons and nodding, duck-shaped devices.

The term 'random' waves is used mostly rather loosely and it is not generally intended to suggest by its use that the waves which are being generated are indeed truly random, but rather that the simulated sea state approximates, in some fashion, the wave conditions which are known or expected to prevail in nature. Nearly all presently used methods for the generation of 'random' waves are fundamentally devices or procedures that create a repetitive signal with repetition periods varying from a fraction of a minute to several hours. The only methods which, to the authors' knowledge, do not fall into this category are based on either a 'true' random noise source utilizing cosmic radiation or thermal noise as a primary input, or are based on those 'pseudo-random' noise generators with very long repetition periods so that, for all intents and purposes, the experiment is over before the series repeats itself.

All other devices, be they repetitive reproducers of prototype wave records, pseudo-random wave signal generators with repetition periods equal to the test period, or lastly, the harmonic synthesizers, generate signals which should more properly be called 'repetitive irregular' rather than 'random' waves. Although these various methods differ from each other in implementation, they are all, with the exception of the 'true' random noise, mathematically identical in as much as their outputs can be reproduced by inverse Fourier transform techniques. This statement may be accepted if one considers that any repetitive time series may be expressed as a Fourier series expansion.

There is one special case of non-repetitive 'random' signal generators. This is a discrete frequency summer in which the various frequency components are not harmonically related. As a result, the time series never repeats itself, but the resultant spectrum has of course distinctly recognizable peaks. This device is not common in coastal engineering research laboratories.

It is not the purpose of this report to compare and assess in detail the various techniques used for the generation of 'random' waves, but rather to suggest that in the presence of the considerable diversity in testing procedures, a need for more comprehensive measurements exists in order to characterize the simulated sea state. It is now known that the matching of amplitude spectra between the prototype and the simulated sea state is no longer sufficient (Ref. 1) and the particular sequencing of high and low waves is now also suspected to be a relevant factor (Ref. 2). Therefore attention should now be given to such features as the grouping of waves and perhaps the wave slopes near the structure under test. It is perhaps appropriate to reflect on the relationship between a particular technical implementation and its place in the history of technical evolution. So much of the ingenuity in developing the various devices for random wave generation wentinto the struggle against the many technical and economical limitations which existed at the time. Although many researchers and engineers were seeking for better methods of generating a more realistic sea state, this simply was not practical.

Now, however, with the availability of extremely fast, versatile and moderately priced digital computers connected on-line to powerful hydraulic or electric servo actuator systems, the engineer is no longer hampered to the same degree with technical implementation problems, and the researcher may now direct his attention more specifically to the question: what constitutes a realistic sea state and how can it be synthesized? Improved instrumentation which is more stable and reliable and the convenience of computer aided calibration procedures now permits the simultaneous use of many wave probes. The measured wave data can be conveniently analyzed by any known technique of digital signal processing in the frequency, the time and the space domain and incident and reflected spectra can be easily separated. Perhaps the time has now come that some existing older research equipment be replaced with more modern facilities, that the on-line computer is accepted as a general purpose laboratory instrument and that certain standards in the synthesis and the analysis of simulated sea states be evolved. It is towards this latter objective that this report attempts to make a minor contribution.

Evidently, in order to simulate a sea state one must know something about the prototype sea state which is to be modelled. Although much has been done to describe a sea state in terms of wave height distributions and variance spectral densities, not enough is known about the grouping phenomenon and the distributions of wave slopes. The question must naturally arise: is there any point in attempting synthesis before it is known what is to be synthesized? The authors claim, however, that it is not uncommon to learn something about a prototype situation by way of simulation. In addition, it is believed that the analysis of prototype wave data should, at least to some extent, be concerned with the needs of the control engineer who ultimately has to use the sea state characterizations to conjure up a storm in the laboratory flume. By attempting synthesis at this time, it may be possible to define these relevant parameters and it is therefore worthwhile to get on with the development of the tools of synthesis. The 'Smoothed Instantaneous Wave Energy History' which is being introduced in this report, was born out of the need to synthesize a suitable phase spectrum. Having achieved this objective, the authors now hope that this function may receive a wider acceptance and may, in fact be used in the analysis of prototype wave data.

In order to avoid confusion, it should be pointed out that most of the wave trains which will be illustrated later in this report are, what might be called, "paper waves" as they are purely mathematical/numerical fabrications. Their realization in a wave flume or basin is a different subject which is not covered here. There is little doubt in the authors' mind that the great majority of these wave trains are in fact realizable and therefore this issue should not distract attention from the problems of synthesizing grouped wave trains.

2.0 TESTING WITH 'RANDOM' WAVES - SOME PHILOSOPHICAL QUESTIONS

The natural sea state is known to be a non-stationary random process. It is therefore completely probabilistic in nature, that is to say, that nothing about its characteristics is known with certainty, but only within certain limits of probability. For example, its variance spectral density function and significant wave height will exhibit substantial variations between successive observations of finite duration (Ref. 3). There is a recognizable change in these descriptions as a storm grows, peaks and then fades away. It seems reasonable that other characteristic sea state properties, such as wave groupings, will also undergo a transition during this interval, (Ref. 4). Furthermore, every storm will be different in intensity, in duration and in direction and any attempt to recreate a simulation of the natural sea state will invite questions on what exactly is being simulated.

Without going into the details of the many 'random' signal generation techniques which are now in common use throughout the world, one may recognize two fundamentally different approaches which have a bearing on the preferred implementation technique. These may be described as the 'deterministic' and the 'probabilistic' approach and are summarized in Fig. 1.

The characteristic feature of the 'probabilistic' approach is the creation of a 'random' sea state of long duration, perhaps of several hours or days in model time, before the wave sequence repeats (if ever). The assumption is then made that the long duration simulation represents all the possible wave heights, wave groups and wave slopes which may in fact occur in nature during a long period of time. Consequently, by measuring or observing the response of a test structure in this simulated sea state, one may obtain all the possible outcomes of the structural response and therefore a direct statistical analysis of the response data yields the desired statistical description.

The 'deterministic' approach, on the other hand, does not pretend to simulate the wave climate in its entirety but rather to select those short duration features which are considered to be of specific interest. These are usually the more severe wave conditions. Having measured the response of the test structure to these inputs, one may then build up a statistical description of the response by hypothesizing how the particular short duration simulation represents the prototype sea state statistics.

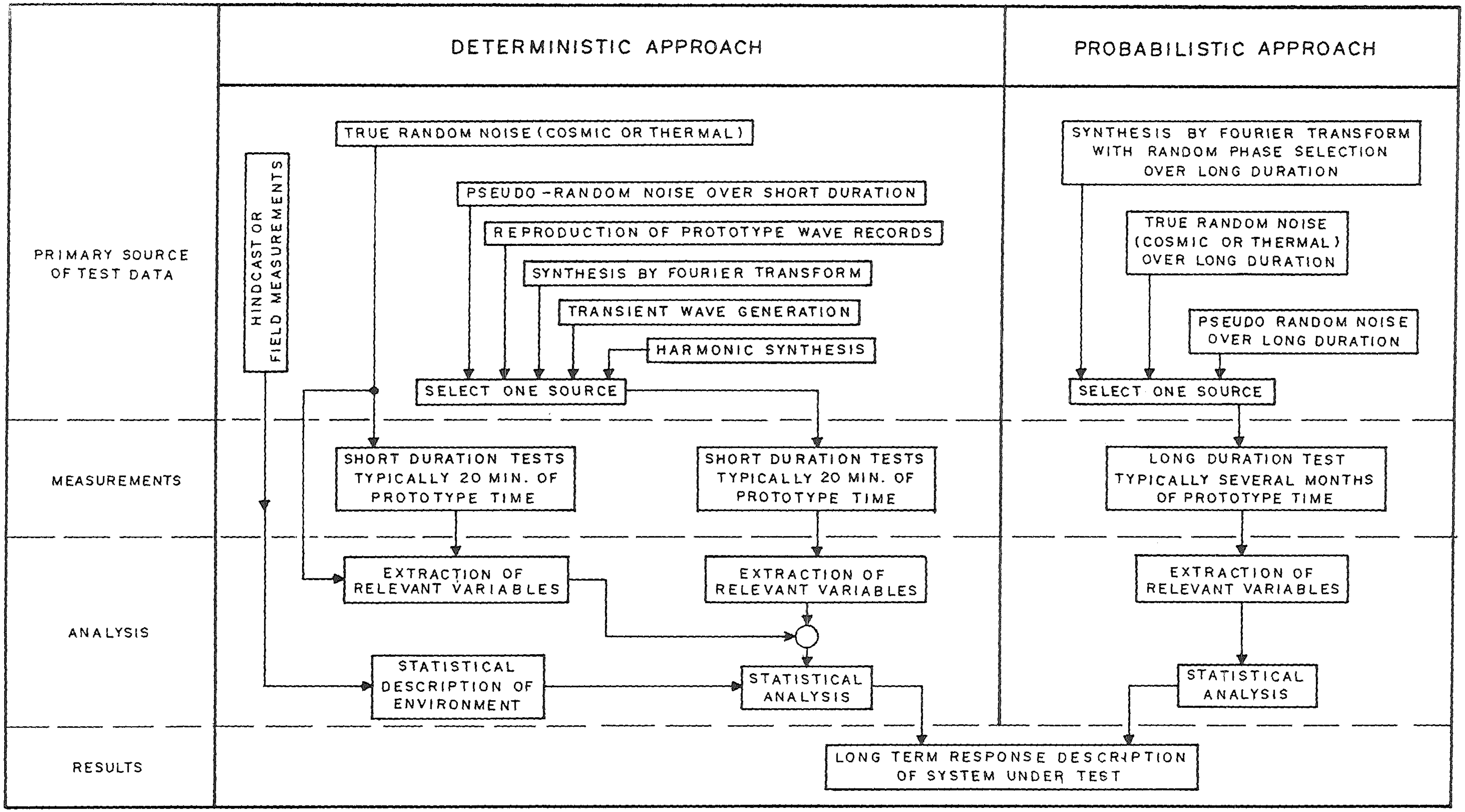


FIG. I

TECHNIQUES FOR TESTING WITH 'RANDOM' WAVES

In the past, researchers have used mono-chromatic waves to represent these short duration extreme value features. However, these mono-chromatic waves cannot create the extreme wave heights or slopes as can be found in nature. Wave transients or even longer wave trains, which have been designed to possess specific wave heights, wave groups, wave slopes and spectral characteristics must also be considered deterministic.

It appears that most laboratories prefer to use the deterministic approach, probably because it is more cost effective as no time is being lost on testing low wave height activity.

In either case, an important distinction must be made between the use of the 'truly random' noise source on the one hand and all other methods, including the 'pseudo random' noise source, on the other. While the latter provide a known and repeatable input condition, the 'truly random' noise source, as mentioned before, is usually of cosmic, radiation or thermal noise origin, and is therefore non-repeatable and never restartable. Individual tests are then also non-repeatable and it is therefore essential to monitor the input conditions for each test and relate the measurements of structural response to their respective inputs. This is indicated by Fig. 1 where the "extraction of relevant variables" for the 'truly random' noise source requires both the input to as well as the output from the test. This approach has been used successfully for the measurement of linear transfer functions where the spectral description of system input and output are sufficient. However, for nearly all coastal or marine dynamic problems, the linear assumption is rarely valid and other wave properties such as slope or grouping may have to be taken into account.

The ability to repeat an experiment or the ability to compare the performance of two different structures under identical test conditions must surely be one of the most important requirements for the simulation of sea states under laboratory conditions. Since the 'truly random' noise source cannot meet this requirement, at least not without intermediate recording and subsequent playback, it cannot be considered entirely satisfactory as a technique for the generation of 'random' waves. The same reservation should apply if a random sample of length T_n of a 'pseudo-random' noise source of repetition period T_r is used and where $T_n = k.T_r$, with k being a positive integer.

While the 'truly random' noise source has a continuous spectrum, all other synthesis techniques including the 'pseudo-random' and the repetitive reproductions of prototype wave trains, have discrete amplitude spectra where the separation of the discrete frequency components equals the inverse of the repetition period of the 'random' wave train, i.e.

$$\Delta f = 1/T_n$$

Suppose that the repetition period is 500 seconds, then the discretization interval in the frequency domain is only 0.002 Hz and more than 1000 discrete frequencies may participate in the synthesis. For all intents and purposes this may be considered continuous especially since non-linear interactions between frequencies in the course of wave propagation may, very likely, smear out the discrete frequency components into a smooth, continuous spectrum.

On the other hand, the harmonic synthesizers do not generally utilize more than 8 or 16 discrete, harmonically related frequency components. The resultant amplitude spectrum will always be discrete and, as a consequence, serious errors can be made in the interpretation of experimental measurements (Ref. 5). This may be explained as follows: If the structure under test responds in a linear fashion to a force proportional to the wave height, then one may measure correctly the structure's response to each of the constituent discrete frequencies of the input. However, if the force is proportional to some non-linear function of the wave height, various super- and sub-harmonic components are effectively being generated which have frequencies at sums and differences of the constituent frequencies. Therefore, the spectrum as seen by the structure under test is quite different than the one that is being generated and it makes a lot of difference exactly how this energy transformation takes place relative to the response characteristic of the structure under test. In view of present day knowledge of the interaction between waves and structures, it appears unwise to continue the use of harmonic synthesizers for the synthesis of realistic sea states unless either the number of participating harmonic components can be significantly increased or the energy of each component can be spread out over a finite band.

Some laboratories now include the reproduction of a prototype wave train (Ref. 6 and 7) as a useful tool for sea state synthesis, while Ref. 8 prefers this technique. This preference is based primarily on the assumption that in the absence of complete understanding of the relevant characteristics of a natural sea state, one must surely be on the safe side by reproducing a sea state which has actually been observed in nature. Of course, this argument has many enthusiastic antagonists who claim that this one sample of a natural sea state has been taken at a time and at a place which may not be relevant to the structure under test. There is also a question about the validity of repeating the finite length record because the spacing between the last and the first wave group do not correspond to an observed "length of run". Last, but not least, the technique severely limits the researcher's ability to test the structure under diverse conditions for which prototype records do not exist. As a result he will be tempted to fiddle the playback through speed and amplitude scaling and then one must surely wonder how closely nature's purity has been preserved.

Instead, the authors feel that one must come to grips with the essential parameters of a natural sea state and determine how these parameters change as a function of time and location and how they are related to the structural response. It is then a matter of imposing these essential parameters on a synthesized sea state in order to achieve realism in the simulation. Because it is then necessary to exercize full control over all aspects of the wave synthesis, whether it is the control of the amplitude spectrum, the wave slope or the wave grouping, it is inevitable to give preference to Fourier transform techniques rather than to any other known method.

3.0 THE SMOOTHED INSTANTANEOUS WAVE ENERGY HISTORY

For the purpose of isolating wave groups in a wave record, the half-wave or the full-wave rectified envelope function has been used (Ref. 9). This function is constructed by interconnecting adjacent wave peaks by straight lines or by first folding up the wave troughs about the record's mean and then interconnecting all adjacent peaks and troughs.

These techniques were tried by the authors. The envelope functions appear to be generally quite rough and they do not readily identify the existence of a group. Instead it was felt that a physically more meaningful description of group activity could be obtained by computing the distribution of wave energy along the time axis where wave energy would be defined as the square of the water surface elevation averaged over a period which is a function of the peak frequency. Fig. 2 illustrates a wave record, the half-wave and the full-wave envelope functions as well as the <u>smoothed instantaneous wave energy history</u> using both rectangular as well as Bartlett smoothing.

The rather lengthy name of the function was chosen to avoid confusion with other wave energy functions which are functions of frequency. It is believed that the term 'history' always implies a function of time. The word 'instantaneous' is to describe that the energy is given at any instance of time along the time axis, and the word 'smoothed' must be included to describe the fact that a smoothing operation by means of a digital low pass filter is performed. For convenience this function will be referred to as the SIWEH.

By comparing the two envelope functions in Fig. 2 with the two SIWEHs, it may be seen that the latter has superior qualities for the identification and isolation of groups.

The smoothed instantaneous wave energy history for rectangular smoothing was defined initially as:

$$E(t) = \frac{1}{T_p} \int_{\tau=-T_p/2}^{T_p/2} \eta^2(t+\tau)\cdot d\tau \qquad 3.1$$

where η is the water surface displacement and $T_p = 1/f_p$ with f_p given as the frequency which corresponds to the peak of the power spectral density of the wave record $\eta(t)$.

Equation 3.1 may be rewritten thus

$$E(t) = \frac{1}{T_p} \int_{\tau=-\infty}^{\infty} \eta^2(t+\tau)\cdot Q_k(\tau)\cdot d\tau \qquad 3.2$$

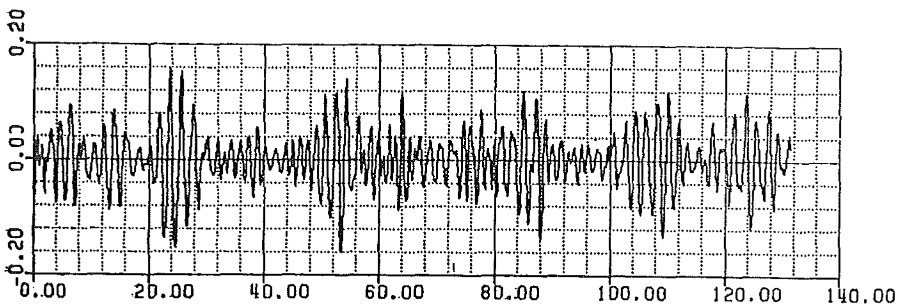
where $Q_k(\tau)$ is a general smoothing or window function which, for the case of equation 3.1, is:

$$Q_0 = 1 \text{ for } -T_p/2 \le \tau \le T_p/2 \qquad 3.3$$

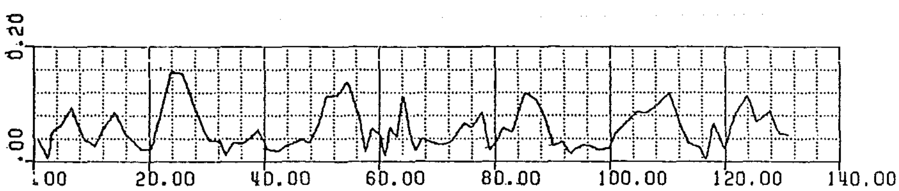
and $\qquad = 0$ everywhere else.

Q_0 is referred to as a rectangular smoothing function and equation 3.2 will be recognized as a convolution operation.

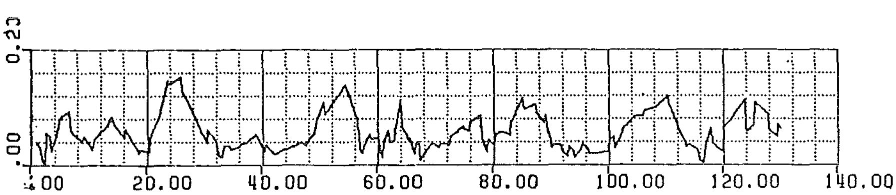
A superior smoothing function is the Bartlett window (Ref. 10), which is given as:



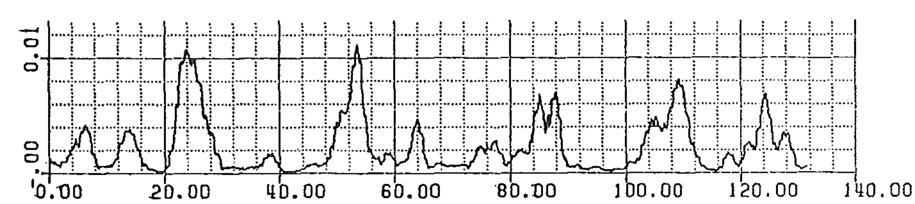
A WAVE RECORD



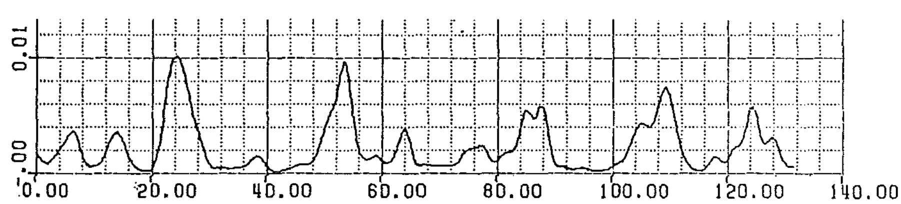
THE HALF-WAVE RECTIFIED ENVELOPE



THE FULL-WAVE RECTIFIED ENVELOPE



THE SMOOTHED INSTANTANEOUS WAVE ENERGY HISTORY
USING RECTANGULAR SMOOTHING



THE SMOOTHED INSTANTANEOUS WAVE ENERGY HISTORY
USING BARTLETT SMOOTHING

FIG.2

A COMPARISON BETWEEN FUNCTIONS FOR WAVE GROUP IDENTIFICATION

$$Q_1(\tau) = 1 - |\tau|/T_p \quad \text{for } -T_p \leq \tau < T_p \qquad 3.4$$

$$= 0 \text{ everywhere else.}$$

Fig. 2 illustrates the smoothed instantaneous wave energy histories for both these smoothing functions. Since the Bartlett function is less sensitive to an exact matching of the dominant wave period, the latter is being adopted.

When processing wave records of finite length, special consideration must be given to the beginning and the end of the record. Also, since Q_1 (the Bartlett window) is known to be limited to the range from $-T_p$ to T_p, the SIWEH is now defined as:

$$E(t) = \frac{1}{T_p} \int_{\tau=-T_p}^{T_p} \eta^2(t+\tau)\cdot Q_1(\tau)\cdot d\tau \qquad 3.5$$

$$\text{for } T_p \leq t \leq T_n - T_p$$

where T_n is the length of the finite wave record. For the beginning and end pieces we have:

$$E(t) = \frac{2}{(T_p+t)} \int_{\tau=-t}^{T_p} \eta^2(t+\tau)\cdot Q_1(\tau)\cdot d\tau \qquad 3.6$$

$$\text{for } 0 \leq t \leq T_p$$

and

$$E(t) = \frac{2}{T_p + (T_n-t)} \int_{\tau=-T_p}^{T_n-t} \eta^2(t+\tau)\cdot Q_1(\tau)\cdot d\tau \qquad 3.7$$

$$\text{for } T_n - T_p \leq t \leq T_n$$

$$\text{with } Q_1(\tau) = 1 - |\tau|/T_p \qquad \text{for } -T_p \leq \tau \leq T_p \qquad 3.8$$
$$= 0 \text{ everywhere else.}$$

4.0 A PHYSICAL INTERPRETATION OF THE SIWEH

When considering the response of a moored offshore structure, the heave-exciting force may be assumed to be

$$F_e = \beta_1 \cdot \eta(t) + \beta_2 \cdot \eta^2(t) \qquad 4.1$$

where β_1 and β_2 are coefficients and η is the water surface elevation (Ref. 5).

By evaluating the η^2 term one may identify four distinct components:

- a constant off-set component,
- a component involving frequencies which are twice the constituent frequencies of $\eta(t)$,
- a component involving frequencies which are the sum of frequency pairs, and
- a component involving frequencies which are the difference of frequency pairs (Ref. 5).

It is the latter term which transports energy into the lower frequency range and, in this way, tends to excite the structural resonance. It is therefore of interest to isolate this fourth term and to assess its spectrum.

By giving consideration to the smoothing operation discussed in Chapter 3.0 in terms of the equivalent low pass filtering effect in the frequency domain, one will recognize that the rectangular data window has a frequency characteristic given by (Ref. 10)[1]

$$F\{Q_0(t)\} = \left(\frac{\sin \omega T_p/2}{\omega T_p/2}\right) \qquad 4.2$$

and similarily for the Bartlett window

$$F\{Q_1(t)\} = \left(\frac{\sin \omega T_p/2}{\omega T_p/2}\right)^2 \qquad 4.3$$

These two functions are plotted in Fig. 3. From this figure one may see that the energy contained in the spectrum of $\eta(t)$ is substantially attenuated by the filter functions, while lower frequencies are allowed to pass. In Fig. 4 the wave record $\eta(t)$ is shown with its periodogram together with the square of the wave record, $\eta^2(t)$, and its periodogram. The Fourier transform of the smoothed instantaneous wave energy history is also included. From this one may conclude that the isolation of the first and fourth terms in the expansion given by Ref. 5 is quite effective.

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[1] This is defined here to provide unity value at zero frequency.

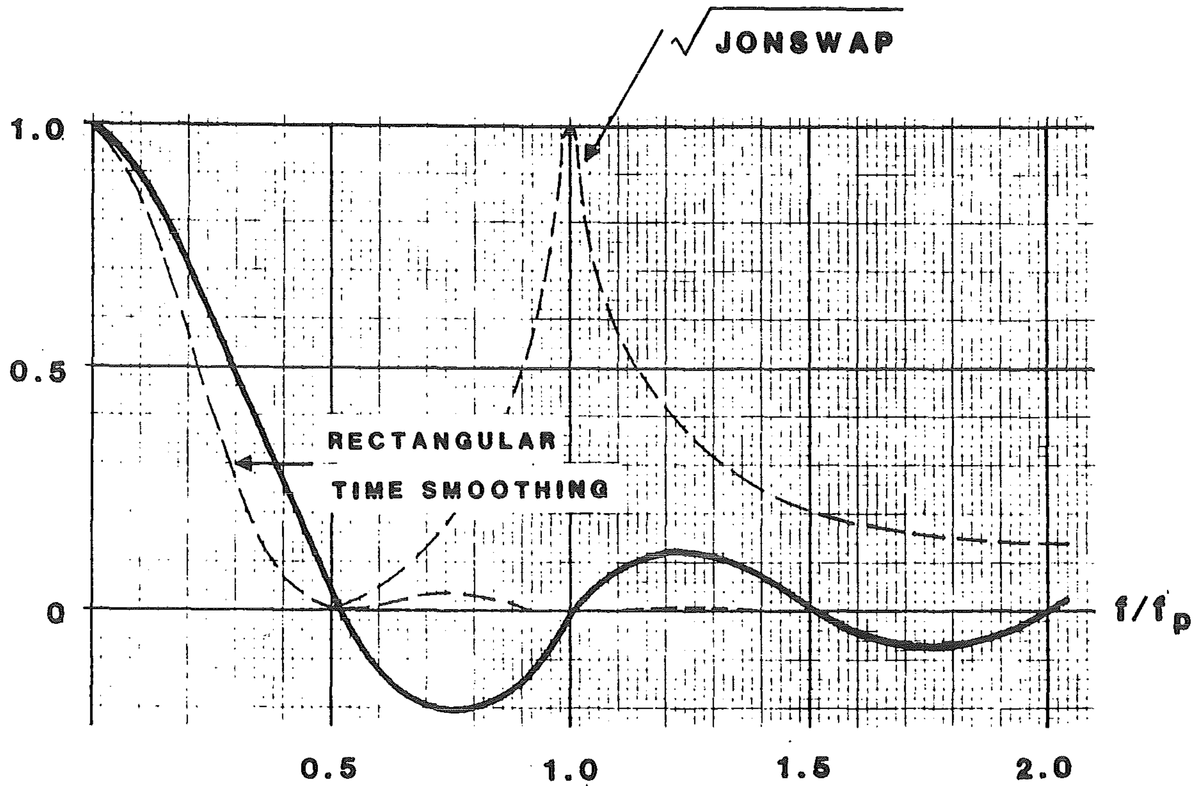


FIG.3 THE FREQUENCY CHARACTERISTICS OF TWO SMOOTHING FUNCTIONS USED FOR THE SMOOTHED INSTANTANEOUS WAVE ENERGY HISTORY

The spectrum of the smoothed instantaneous wave energy history is considered to be an important descriptor which may ultimately be used to assess the response of moored structures and, perhaps, harbour resonances as well. It may also serve for the synthetic creation of grouped or non-grouped wave trains. For reasons of its usefulness to describe the extent of group activity, this function is defined here as a spectral density and has the dimension of meters**4 per Hertz. It shall be referred to as the <u>SIWEH spectral density</u>. For convenience, this definition is given here by the continuous transform

$$\varepsilon(f) = \frac{2}{T_n} \left| \int_0^{T_n} (E(t) - \overline{E}) \cdot e^{-j\omega t} \cdot dt \right|^2 \qquad 4.4$$

where T_n is the length of the finite record and $\overline{E}$ is the average of E(t) which is equal to the variance of the wave record, i.e.

$$\overline{E} = \frac{1}{T_n} \int_0^{T_n} E(t) \cdot dt = \int_0^{\infty} S_\eta(f) \cdot df \qquad 4.5$$

It will be noticed that the SIWEH spectrum is computed without smoothing. It has been found that much of the group periodicity information is lost if a smoothing function is applied to it.

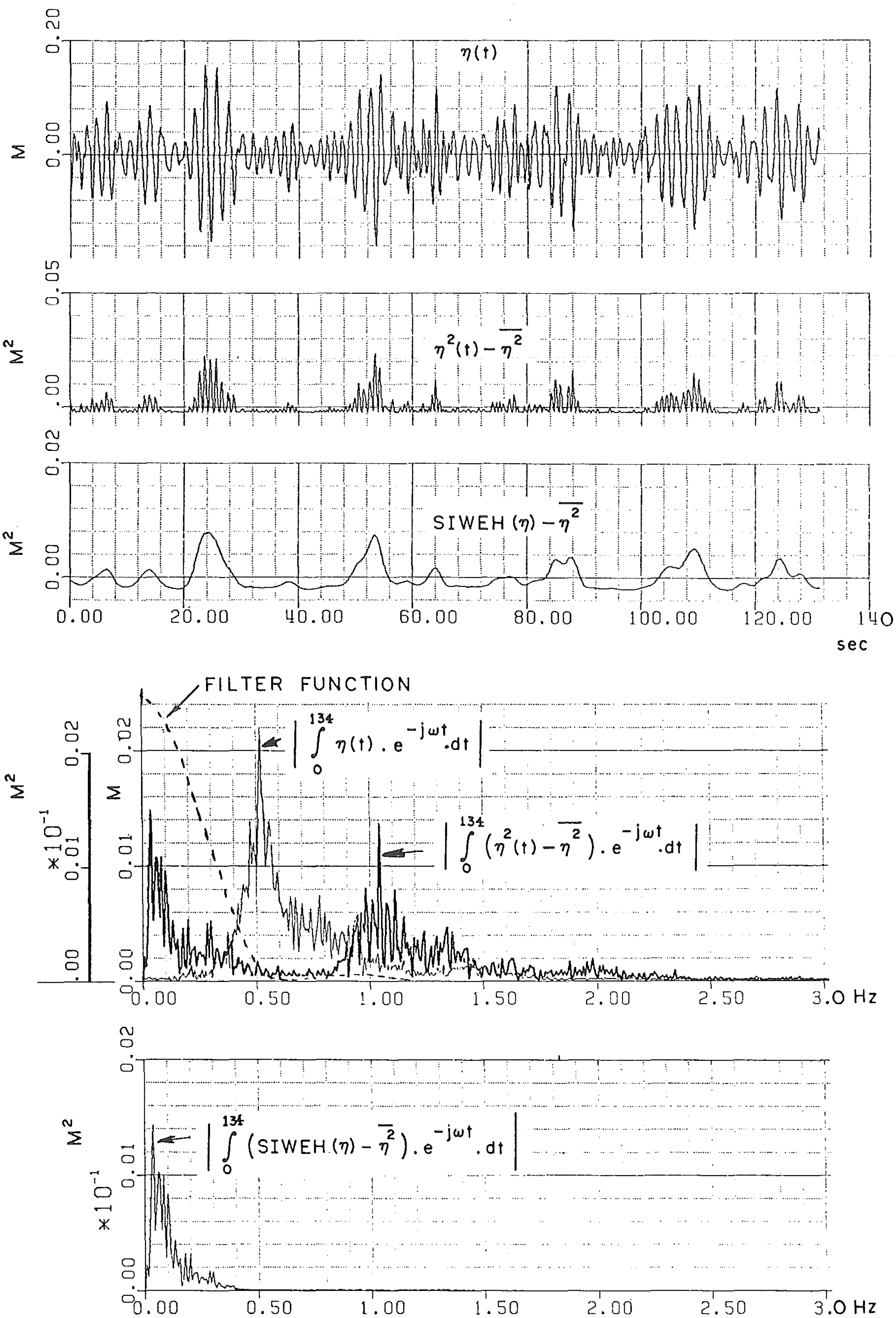


FIG.4

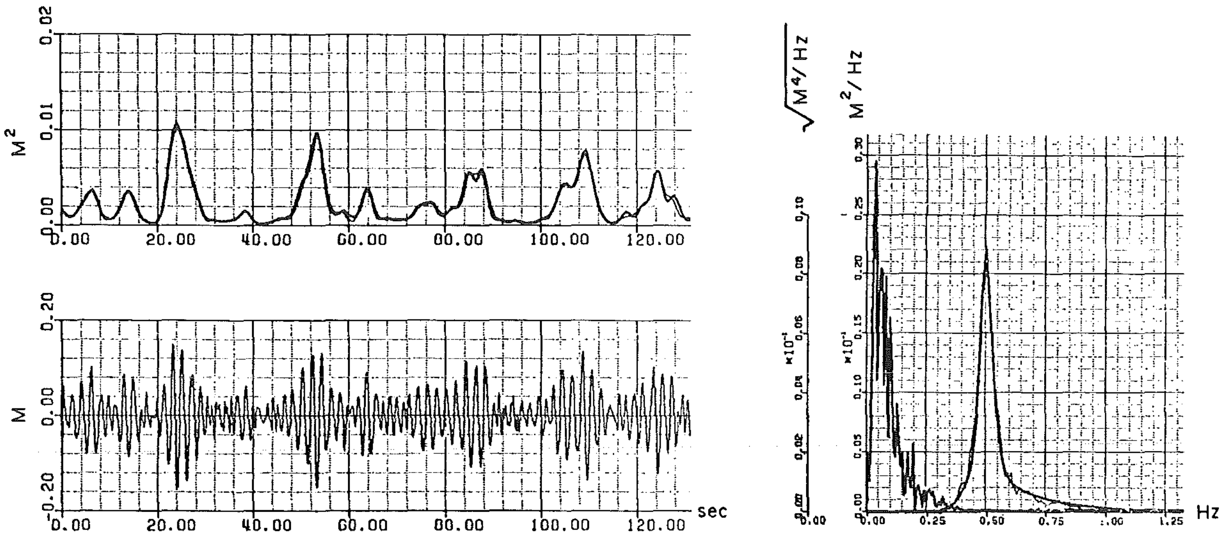
SPECTRUM OF SQUARED WATER SURFACE ELEVATION

At this time it is not clear what the relationship is between $\varepsilon(f)$ and $S_\eta(f)$ the variance spectral density of the water surface displacement. Some investigators (Ref. 9) have suggested a relationship in terms of the auto-correlation of the power spectral density $S_\eta(f)$ of the water surface displacement and the spectral density of the envelope function. However, this does not appear to be satisfactory as may be demonstrated by the following example.

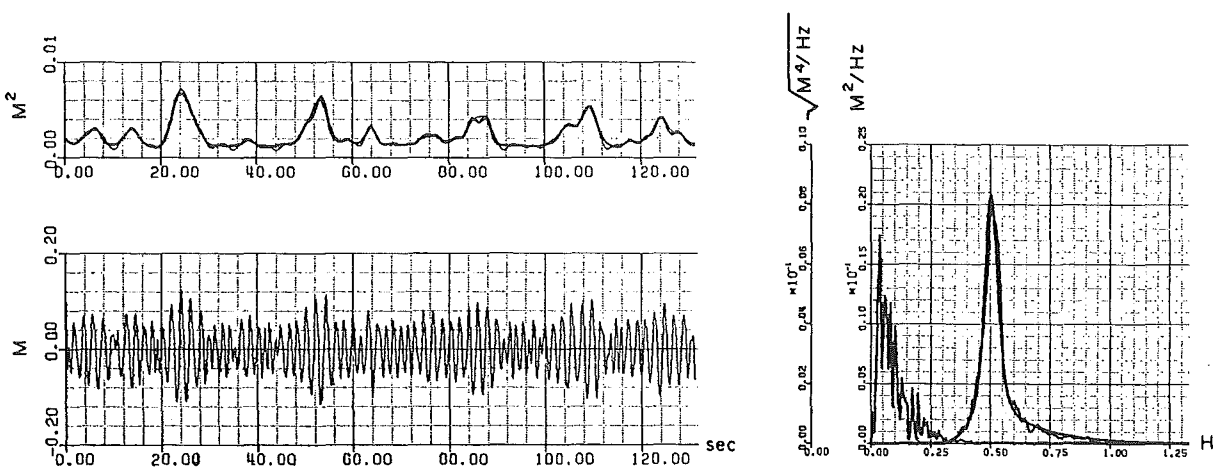
Fig. 5 gives three wave records which are all synthetic creations based on a common variance spectral density of $\eta(t)$. They differ in as much as their smoothed instantaneous wave energy histories have been modulated to give three different degrees of groupiness. All three have the same variance spectral density, however, their SIWEH-spectral densities are different in magnitude although not in shape. As will be shown in Chapter 8.0, also the shape of the SIWEH spectral density may vary substantially and still permit a matching to a given variance spectral density.

Furthermore, as shown in Appendix A1, one may obtain an expression for the SIWEH spectral density in terms of the square of the Fourier series expansion of $\eta(t)$ i.e.

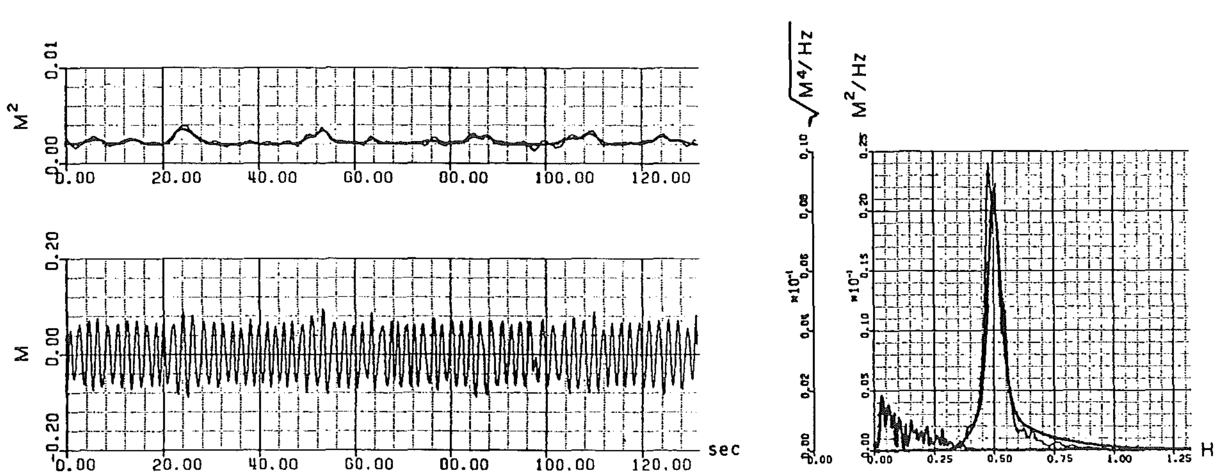
$$\varepsilon(k\cdot\Delta f) = \frac{1}{2\cdot\Delta f}\left(\left(\sum_{i=1}^{N-k} \Pi_{i,k}\cdot\cos\,\theta_{i,k}\right)^2+\left(\sum_{i=1}^{N-k} \Pi_{i,k}\cdot\sin\,\theta_{i,k}\right)^2\right) \quad 4.6$$



GROUPINESS FACTOR , GF = 0.95



GF = 0.57



GF = 0.20

FIG.5

THREE WAVE TRAINS WITH COMMON VARIANCE BUT DIFFERENT SIWEH SPECTRAL DENSITIES where $\Pi_{i,k}$ and $\theta_{i,k}$ are defined in Appendix A1.

It may therefore be seen that the summation of 4.6 depends not only on the terms $\Pi_{i,k}$, which are derived from the Fourier coefficients of $\eta(t)$, but also on the phase angles, $\theta_{i,k}$, which are obtained by differencing the phase spectrum of $\eta(t)$, a fact which is not accounted for if the variance spectral density $S_\eta(f)$ is auto-correlated. While the auto-correlation of any function always leads to maximum values of zero shift, the SIWEH spectral density has not exhibited this characteristic for any of the cases which have been investigated so far. This must indicate that the values of $\theta_{i,k}$ for small values of k are probably uniformly scattered between $-\pi$ to π and as a result the sum of sin $\theta_{i,k}$ or cos $\theta_{i,k}$ for all i approaches zero.

For the reasons and examples given above, it must therefore be incorrect to assume either a unique relationship between the variance spectral density on the one hand and the SIWEH spectral density on the other or that the broadness of the variance spectral density $S_\eta(f)$ is necessarily an indication of wave groupiness.

5.0 A DESCRIPTION OF WAVE GROUP ACTIVITY BY THE SIWEH

In order to describe wave group activity with sufficient detail, it is postulated that four parameters are required.

The first parameter must indicate if there are wave groups present and to what degree. This must necessarily be an indication on how uniform or how lumped the distribution of wave energy is along the time axis. The SIWEH provides an excellent basis for the establishment of such a GROUPINESS FACTOR which is proposed to be defined as follows:

$$GF = \sqrt{\frac{1}{T_n} \int_0^{T_n} (E(t) - \bar{E})^2 \cdot dt} \Big/ \bar{E} \qquad 5.1$$

$$= \sqrt{m_{\varepsilon 0}} / m_0 \qquad 5.2$$

where $m_{\varepsilon 0}$ and m_0 are the zeroth moment of the SIWEH spectral density and the variance spectral density of $\eta(t)$ respectively.[1] In words, the groupiness factor is the standard deviation of the SIWEH about its mean and normalized with respect to this mean. The GF has been evaluated for the three examples in Figs. 5 and 6 and it may be noticed that it describes well the extent of wave grouping.

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[1] This definition of GF is based on a verbal communication by A. Tørum who suggested the normalization with respect to $\bar{E}$.

The other three parameters related to wave grouping must describe what the average group length and group repetition period is, and provide at least some indication on the variation of the group repetition period.

From the point of view of the control engineer, who is to synthesize a grouped wave in a laboratory flume, such descriptions should allow him to match up the specifications of both the water surface displacement spectrum and the group activity and, from this, create a wave train which has the desired properties. From this point of view it appears easiest to describe the grouping properties simply in terms of the SIWEH spectral density.

Since there are no suitable models at this time which could describe the SIWEH spectral density in terms of the above mentioned wave group parameters for group length and repetition period, it may be necessary for the time being to provide a graphical description of the SIWEH spectral density as a specification for wave group activity. A description of a wave record may be derived in terms of its variance spectral density $S_\eta(f)$ and its SIWEH spectral density $\varepsilon(f)$. This should be a fairly comprehensive characterization of the wave activity and the wave groupiness factor as defined by equation 5.2. The only reservation to this approach is the fact that the SIWEH spectral density $\varepsilon(f)$ alone, i.e. without its phase spectrum, will not account for the group width parameter.

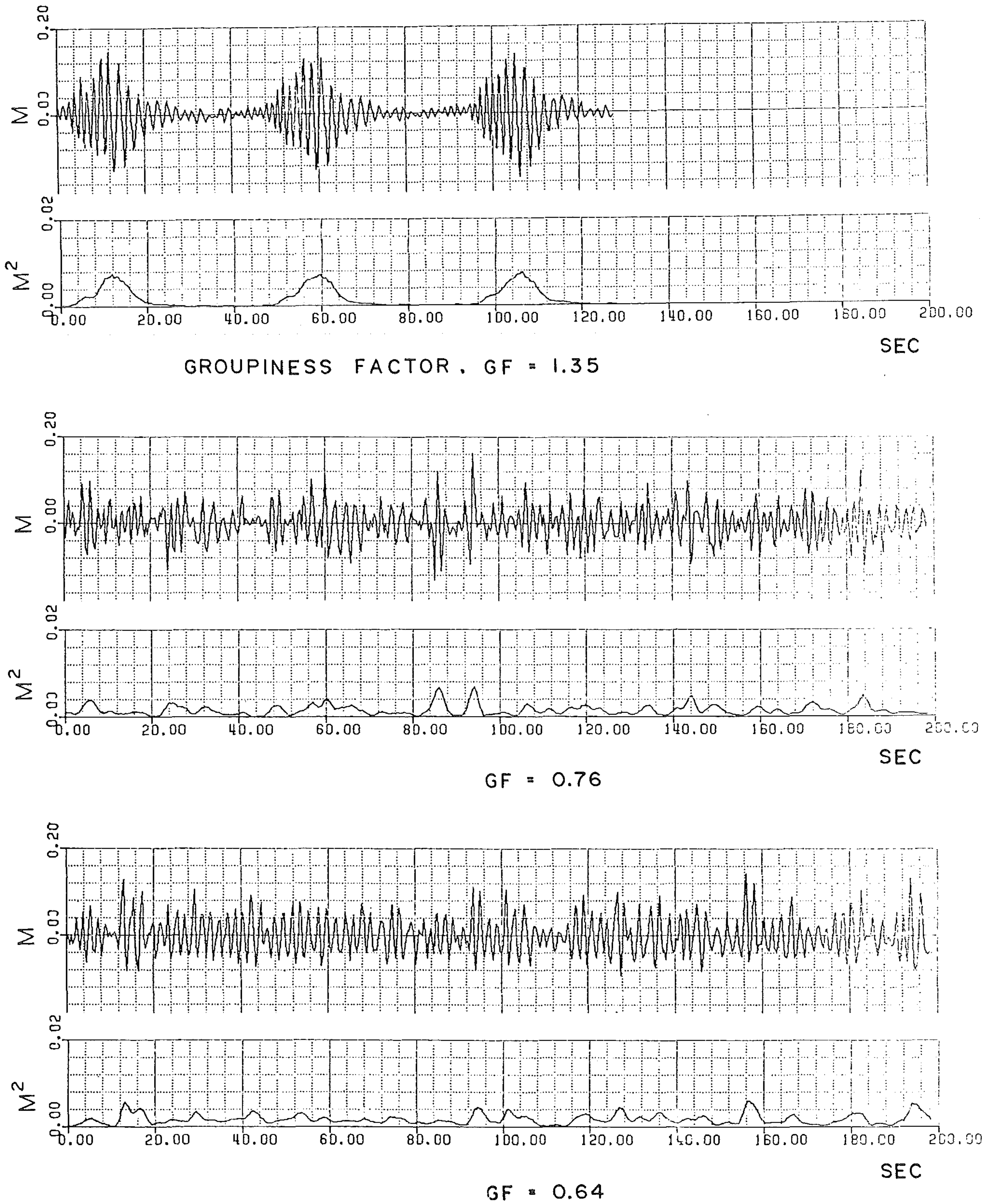


FIG.6

SIWEH EXAMPLES

Fig. 7 gives the SIWEH spectral density for the data presented in Fig. 6 while Fig. 8 explores the use of the auto-correlation function of the SIWEH as a possible but not preferred means of summarizing wave group activity. Since the auto-correlation function is always normalized with respect to its zero-shift value, i.e. the record variance, it is inevitable that the 3 SIWEHs of Fig. 5 have similar auto-correlation functions. In the authors' view the SIWEH spectral density is perhaps more useful in revealing group periodicity while the SIWEH auto-correlation may describe more effectively the average group width by its first zero-downcrossing. To a lesser extent also periodicities can be identified by the existence of peaks and the time at which they occur.

6.0 THE SYNTHESIS OF GROUPED WAVE TRAINS BY MEANS OF THE SIWEH

The process of synthesis of wave data, as described here requires the synthetic creation of one or several wave trains as a function of time on the basis of specifications in the frequency domain. The method which lends itself most readily to synthesis is based on the inverse Fourier transform. For this purpose, it is common practice to use either the Fourier amplitude spectrum or the square root of the

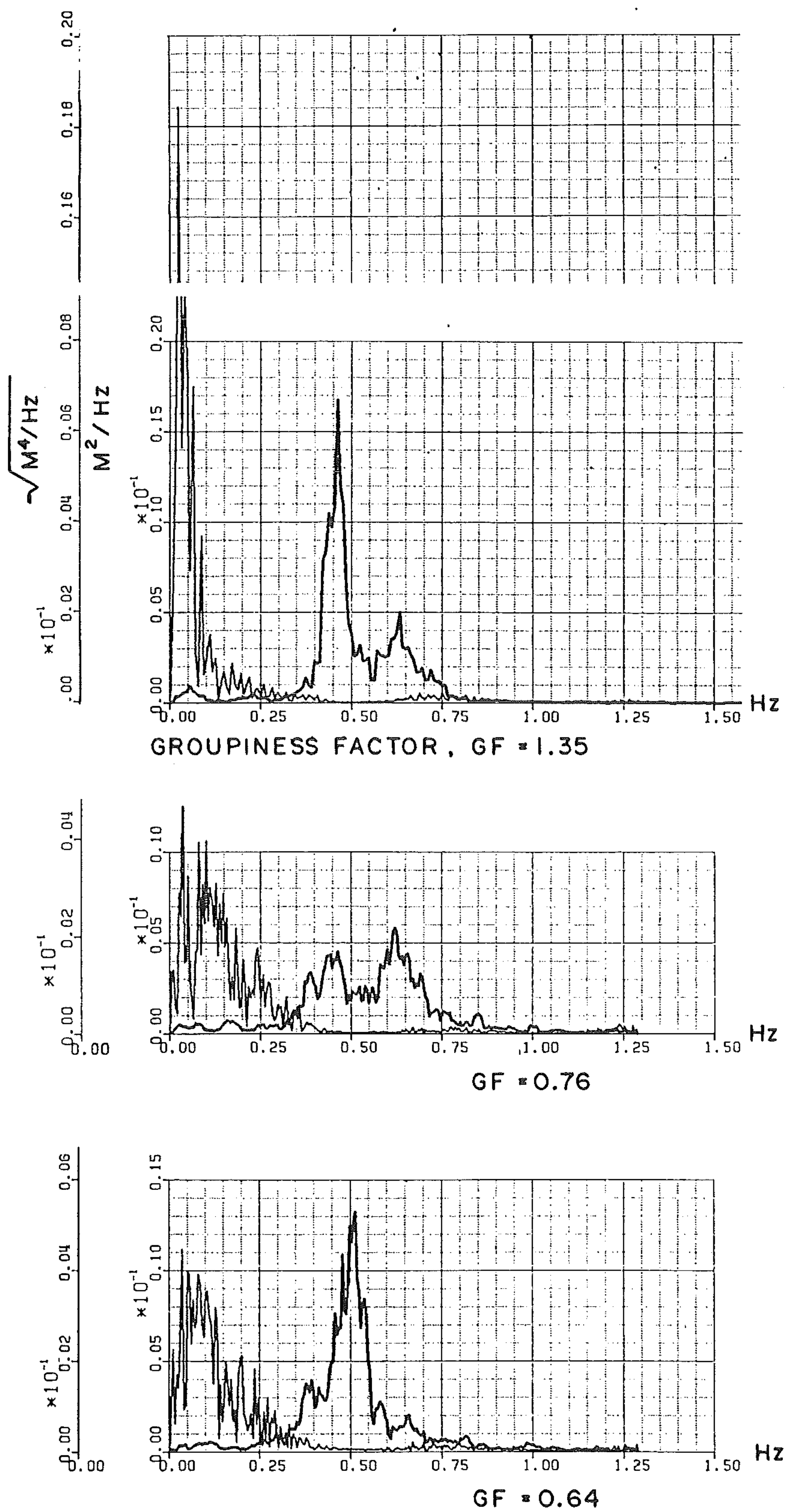


FIG. 7

SIWEH & VARIANCE SPECTRAL DENSITIES
FOR DATA FROM FIG. 6

variance spectral density of the desired water surface displacement and then assume some arbitrary phase spectrum. This phase spectrum is usually created by means of a random number generation algorithm which can supply randomly selected phases which are uniformly distributed between $-\pi < \phi < \pi$. The amplitude and the phase spectra are then paired and inverse Fourier transformed. This procedure of phase spectrum generation is based on the general belief that the phases of prototype waves are random and uniformly distributed, an assumption which is now being challenged (Ref. 11).

In order to create a phase spectrum which will indeed generate a wave train with the desired grouping phenomenon, one may use the SIWEH as follows.

Given:

E1(i) for i = 1,N with repetition period T_n = N.ΔT and

N = 2**k, k a positive integer and

S(ℓ) for ℓ = 1,N/2+1 at frequencies (ℓ-1)*ΔF with

ΔF = 1/T_n, and assuming that

MEAN[E1(i)] = SUM[S(ℓ).ΔF]

proceed as follows:

1. Obtain the peak frequency FP such that

   S(FP) = Maximum value of S((ℓ-1)*ΔF) for all ℓ

2. Obtain $\overline{E}$ = MEAN [E1(i)]

3. Compute the desired periodogram A(ℓ) = $\sqrt{2*S(\ell)*\Delta F}$

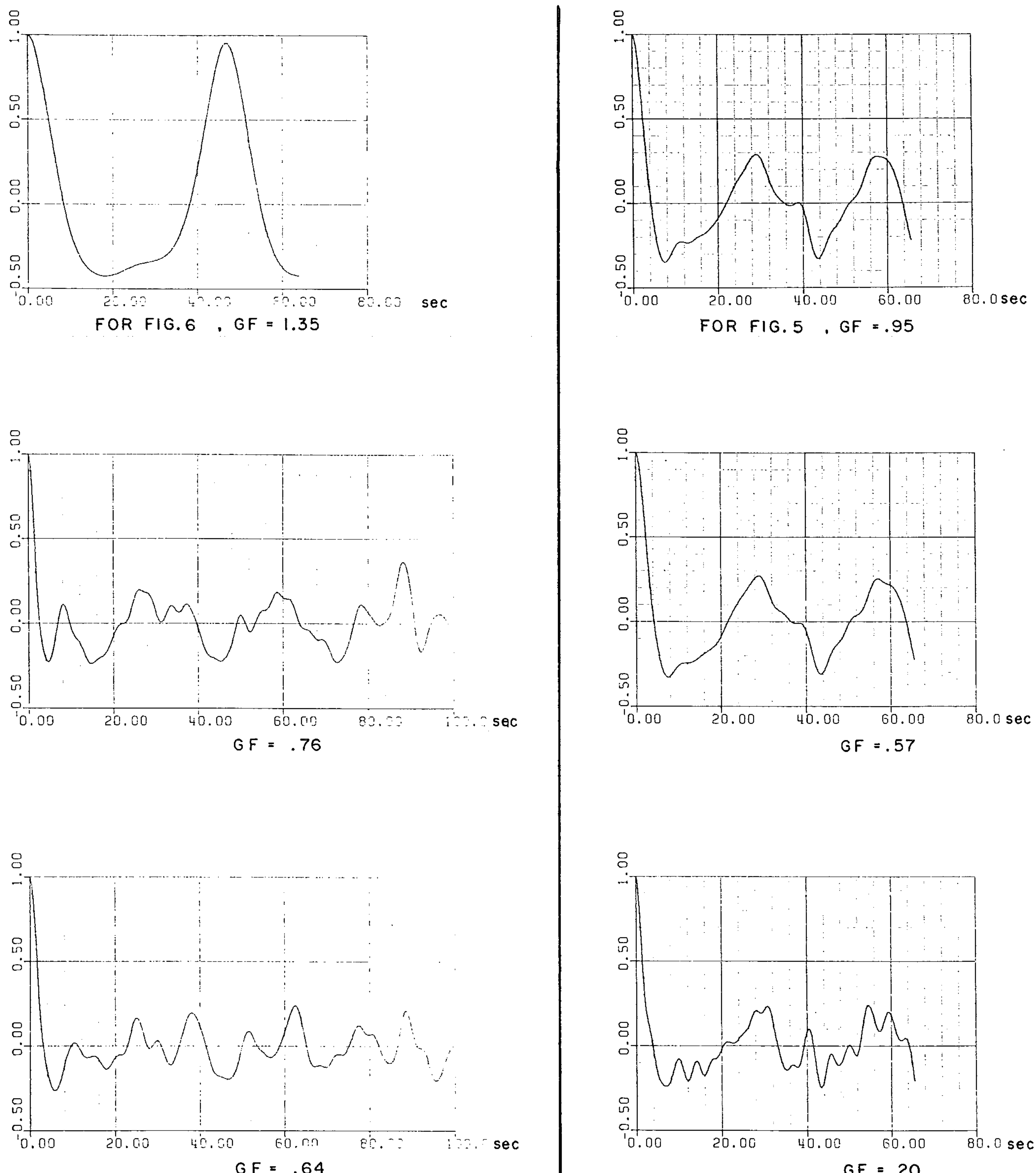


FIG. 8

AUTO - CORRELATIONS OF SIWEHs
FROM FIG. 6 AND FIG. 5

4. Compute a phase modulated sinusoid which is designed so that the frequency within a wave group is approximately FP Hertz but of a higher frequency between groups, i.e.

$$X(i) = SIN(2\pi * FP * (i-1) * \Delta T + THET(i))$$

where THET(i) is obtained from E1(i) by the following procedure:

- If $E1(i) \geq \overline{E}/5$, $B = E1(i)$
  else $B = \overline{E}/5$
  and set $U(i) = (E/B)^2$
- Find UMIN = Minimum value of U(i), i=1, N
- Compute $FI(i) = RL*(U(i)-UMIN)^2$, i=1,N
  (with RL typically = 0.02)
- Limit $FI(i) \leq 5*FP$
- Set THET(1) = 0 and then

$$THET(i) = THET(i-1) + 2\pi * \Delta T * FI(i)$$

for i = 2,N

5. Amplitude modulate X(i), i.e.

$$y1(i) = \sqrt{E1(i)} * X(i)$$

6. Fourier transform y1(i) to obtain an amplitude and a phase spectrum, i.e

$$\mathcal{F}\{y1(i)\} = Y1(\ell.\Delta F) * \exp(-j.\phi 1(\ell.\Delta F))$$

7. Discard $Y1(\ell.\Delta F)$ and replace it with $A(\ell.\Delta F)$ then inverse Fourier transform

$$\mathcal{F}^{-1}\{A(\ell.\Delta F).\exp(-j.\phi 1.(\ell.\Delta F))\} = y2(i)$$

8. Compute the SIWEH for y2(i)

$$E2(i) = SIWEH(y2(i))$$

9. Correct the wave train by iteration, i.e.

$$y3(i) = y2(i) * \sqrt{E1(i)} / \sqrt{E2(i)}$$

10. Substitute y1(i) = y3(i) and go back to step 6. Repeat 2 or 3 times and branch out.

Fig. 9 illustrates the operation, giving intermediate results. Although the iteration phase in Fig. 9 does not show much of an improvement, there are some cases where this computational step is well worthwhile.

7.0 THE SYNTHESIS OF A SIWEH FROM A SIWEH SPECTRAL DENSITY

As a next step one must wonder where the SIWEH, as used in the above example came from and if it is possible to generate other grouped wave trains which may have similar wave group properties. Given a SIWEH-spectral density, one may indeed generate a family of different SIWEHs all of which have the same SIWEH spectral density. Fig. 10 illustrates this possibility. Here, the original SIWEH spectral density is paired with a number of different phase spectra of more or less randomly selected phases. Each of these SIWEHs is then used to compute wave trains in the manner described above. These are also shown in Fig. 10.

The actual process of creating a phase spectrum for the SIWEH spectral density is not quite as simple as the random

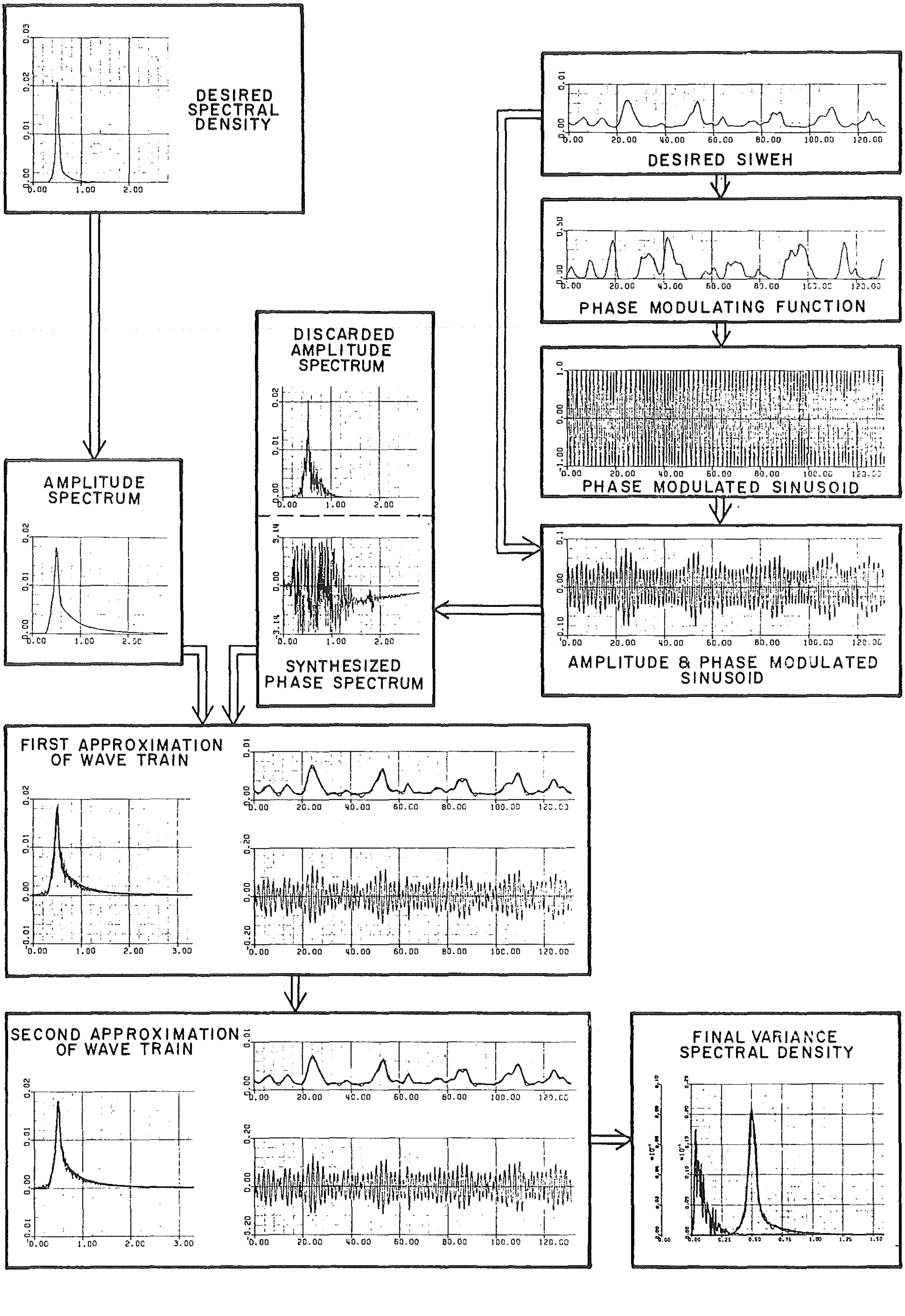


FIG.9

THE SYNTHESIS OF A GROUPED WAVE TRAIN

selection of phases for different frequencies. This latter procedure always leads to a Gaussian distribution of amplitudes in the time function which results from inverse Fourier transformation. Inspection of any SIWEH will reveal that its amplitude distribution is not Gaussian and that there must be a considerable amount of harmonic content in the frequency domain for the SIWEH to exhibit the extent of distortion in the time domain. The implication of this is that the second and higher harmonics must be, more or less, in phase with their fundamental.

The method adopted here for the creation of a suitable phase spectrum is again based on a process of iteration. This method may not be the most efficient procedure but it seems to work quite well and since Fourier transformations by array processors are executed with great speed, the proposed method does not appear to be unpracticable for those who have access to such digital devices. This method may be defined as follows:

Given:
- a SIWEH spectral density, EPS
- the repetition period T_n of the desired SIWEH (in seconds)
- the groupiness factor, GF, of the desired SIWEH
- the variance m_0 of desired water surface displacement. This is to be imposed as the mean of the desired SIWEH.

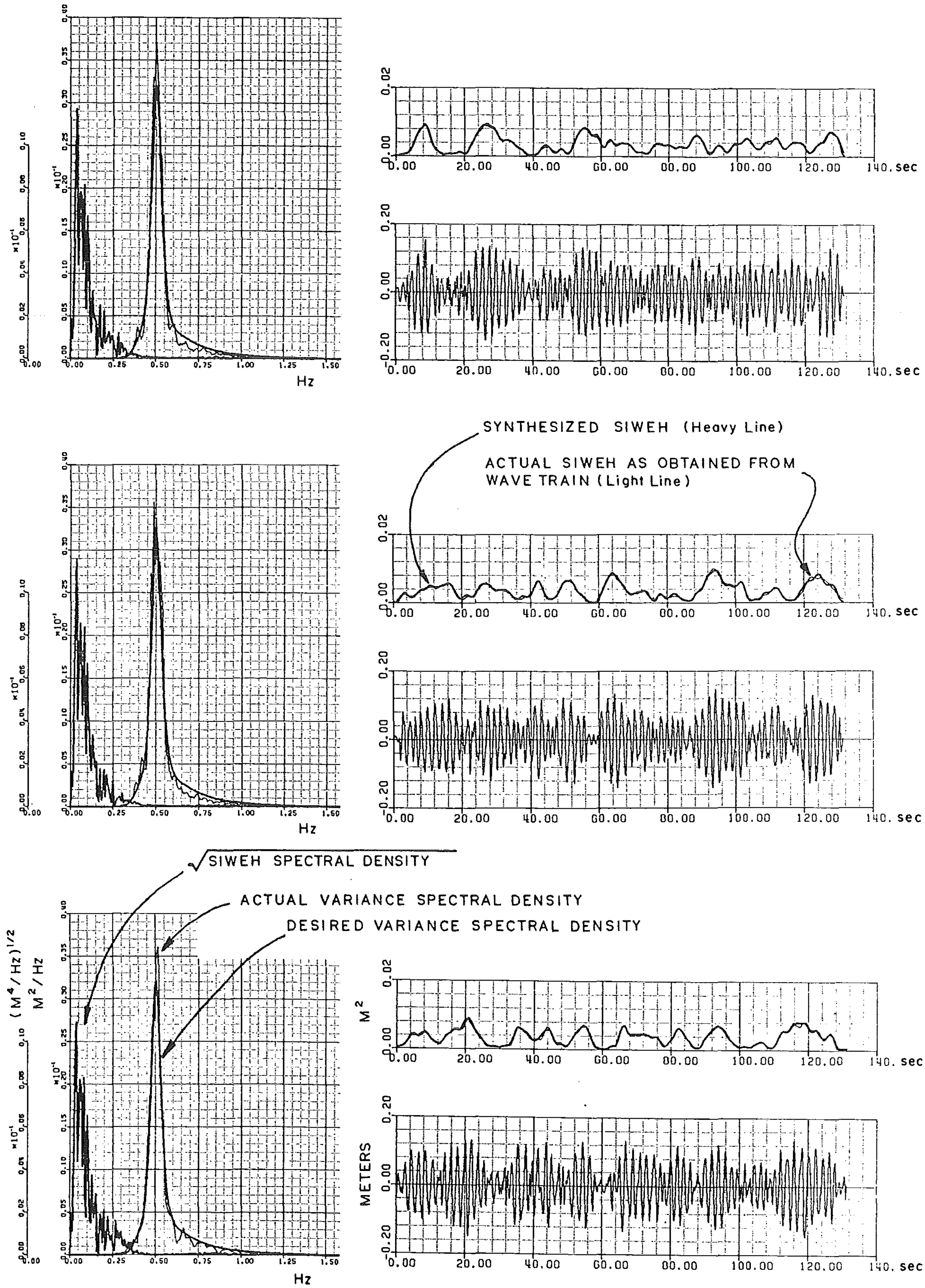


FIG.10

THREE DIFFERENT SIWEHs FROM COMMON SPECTRAL DENSITIES (GF = 0.61)

Proceed as follows:

1. Re-sample EPS such that for EPS((i-1).ΔF), i=1,N/2+1 and $\Delta F = 1/T_n$ and 2**k = N (k a positive integer).

2. Re-scale EPS(i), i=2,N/2+1 to accommodate the groupiness factor specification, i.e

$$S1 = \text{SUM of EPS(i)},\ i=2,N/2+1$$

then compute

$$C = (GF*m_0)**2/S1$$

3. Obtain the periodogram

$$A(\ell) = \sqrt{2*EPS(\ell)*\Delta F*C} \quad \ell=2,N/2+1$$

$$A(1) = 0$$

4. Compute a phase spectrum by randomly selecting phases which are uniformly distributed between $-\pi < \phi 1(\ell) \leq \pi$

5. Inverse Fourier transform to get a first estimate of the SIWEH, i.e.

$$E1(i) = \mathcal{F}^{-1}\{A(\ell).\exp[\phi 1(\ell)]\} \text{ for } i=1,N/2+1$$

6. Clipp E1(i) such that

if $E1(i) \geq -\alpha*m_0$, then $E2(i) = E1(i)$

else $E2(i) = -\alpha*m_0$

7. Count the number of times that E1 was less than $-m_0$. If this count was equal to zero, then branch out to 10.

8. Fourier transform E2(i), i.e.

$$B(\ell)\cdot\exp[\phi 2(\ell)] = \mathcal{F}\{E2(i)\}$$

9. Replace the phase spectrum $\phi 1$ by $\phi 2$, discard $B(\ell)$ and go back to 5.

10. Compute the SIWEH from $E(i) = E2(i) + m_0$.

Experiments have been carried out with different values of α. To quote one example, for and $\alpha = 0.6$, 20 iterations were required before the condition for a branch out of the iteration loop was satisfied. The authors are certain that this can be improved by some variation of the proposed algorithm.

It was particularly this complication in the synthesis of a SIWEH from a SIWEH-spectral density which caused the authors to wonder if the definition of the SIWEH-spectral density should not be based on the square root of the SIWEH rather than the SIWEH itself because the square root of the SIWEH is nearly Gaussian and the random phase selection would have been sufficient. However, it is felt that the advantage of being able to give the groupiness factor GF, in terms of the zeroth moments as stated in equation 5.2 far exceeds other disadvantages.

With the help of the tools described here as well as in chapter 6.0, one may now create a grouped wave train from the specification of a SIWEH-spectral density, a variance spectral density, and the repetition period only. By allowing a re-scaling of the SIWEH-spectral density, the groupiness factor may be a fourth input which may serve to extend the usefulness and the flexibility of the technique.

It is in the nature of the finite Fourier transform that the time series synthesized by inverse transformation is repetitive. The origin of the series may be relocated readily through phase shifting techniques without affecting the general properties of the function in the frequency or the time domain. The implication of this is that the group spacing between the last group of one scan of the time series and the first group of the next scan is consistent with the general description of group activity in terms of the SIWEH spectral density. However, in spite of this argument it may still be preferable, in some applications, to extend the repetition period to much longer intervals which would exceed the capability of most mini-computer systems both in terms of speed as well as storage. As the maximum frequency of the SIWEH spectral density is much lower than the maximum frequency of the variance spectral density, the SIWEH may be sampled at a rate which is correspondingly lower and the SIWEH over this longer repetition period may be described by fewer samples than is required for the wave train itself. As a result SIWEH synthesis for longer repetition periods can be handled by computers with limited capability.

For this reason it is proposed to proceed as follows:

- Create a SIWEH over the required longer repetition period using minimum sampling.

- Break up the SIWEH into subsections where each subsection is of a length equal to or shorter than the maximum period of a wave train which can be synthesized as per Chapter 6.0. It is also important that the subsections be selected so that they start and end in a region of low wave activity, i.e. minimum values of the SIWEH.
- Resample each subsection to be compatible with the sampling rate of the wave train to be synthesized.
- Synthesize wave trains as per Chapter 6.0 for each of the subsections and string these together.
- Resample the time series to be compatible with the regeneration rate of the digital to analog signal generator.

8.0 MODELS FOR SIWEH SPECTRAL DENSITIES

No attempt has been made to date to analyse a large number of prototype wave records in terms of their SIWEH spectral densities and for this reason it is not known what range of groupiness factors and what type of SIWEH spectral densities can be expected.

In order to test the importance of the harmonically related peaks in the process of synthesis in some of the SIWEH spectral densities which have been obtained, a model for a smooth SIWEH spectral density which has no harmonic peaks has been developed. This model is:

$$\varepsilon(\lambda) = \frac{C}{\sqrt{(1-\lambda^2)^2 + 4\zeta^2\lambda^2}} \cdot \frac{\lambda}{\sqrt{(1+\lambda^2)}} \qquad 8.1$$

The function of equation 8.1 is frequently encountered in linear dynamic system analysis and is used to describe the amplitude response of a third order linear system in the frequency domain. λ is a normalized frequency, i.e.

$$\lambda = f/f_0$$

and ζ is a damping factor which controls the width of the spectral peak which occurs in the vicinity of $\lambda=1$. The smaller is ζ, the narrower is the spectral peak. f_0 is the peak frequency for $\zeta = 0$.

Fig. 11 gives these theoretical SIWEH spectral densities for C=1 and ζ=0.1, 0.2, 0.3, 0.5 and 1. Fig. 12 illustrates SIWEHs which are synthesized from this theoretical spectral density for ζ = 0.1, 0.3 and 1.0, but with C adjusted to give a constant groupiness factor of GF = 0.95. From these it may be concluded that smooth SIWEH spectral densities can be used to generate acceptable SIWEH functions.

Evidently much work needs to be done to explore prototype wave data in terms of these grouping characterizations.

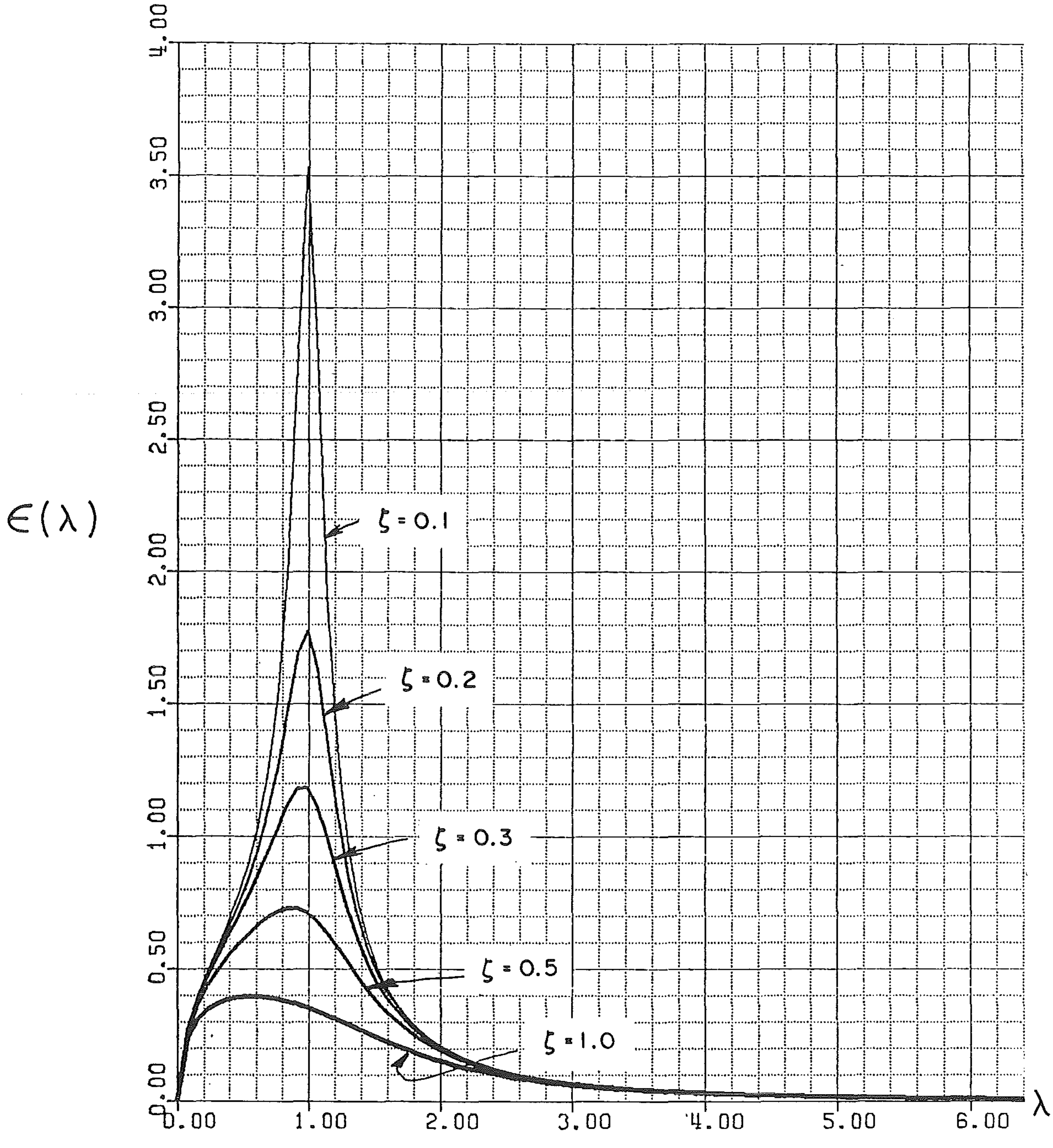


FIG. 11
A MODEL FOR A
SIWEH SPECTRAL DENSITY

$$\epsilon(\lambda) = \frac{1}{\sqrt{(1-\lambda^2)^2 + 4\zeta^2 \cdot \lambda^2}} \cdot \frac{\lambda}{\sqrt{1+\lambda^2}}$$

9.0 THE ISOLATION OF WAVE GROUPS

At this time of writing, the requirement to isolate wave groups within a wave record is only of academic interest. However, it may be that further analysis of isolated wave groups in prototype records will shed some new light on the properties of waves (Ref. 11).

The SIWEH lends itself extremely well to the operation of wave group isolation. This is accomplished in the following manner:

- Given the SIWEH E(t), as defined by equations 3.5 to 3.8, compute the mean energy, $\bar{E}$ as defined by equation 4.5.
- Use the mean energy, $\bar{E}$ as a threshold and determine the intersections between the SIWEH and $\bar{E}$.
- Calculate the slope of the SIWEH at the points of intersection and project the slope on both sides of the group towards the zero-energy line. These intersections with the zero-energy line define the extent of the group.
- Create a switching function which is 1 within the defined limits of all groups and zero everywhere else.
- Multiply the original water surface displacement $\eta(t)$ with the switching function to get the isolated groups. Fig. 13 illustrates the operation graphically.

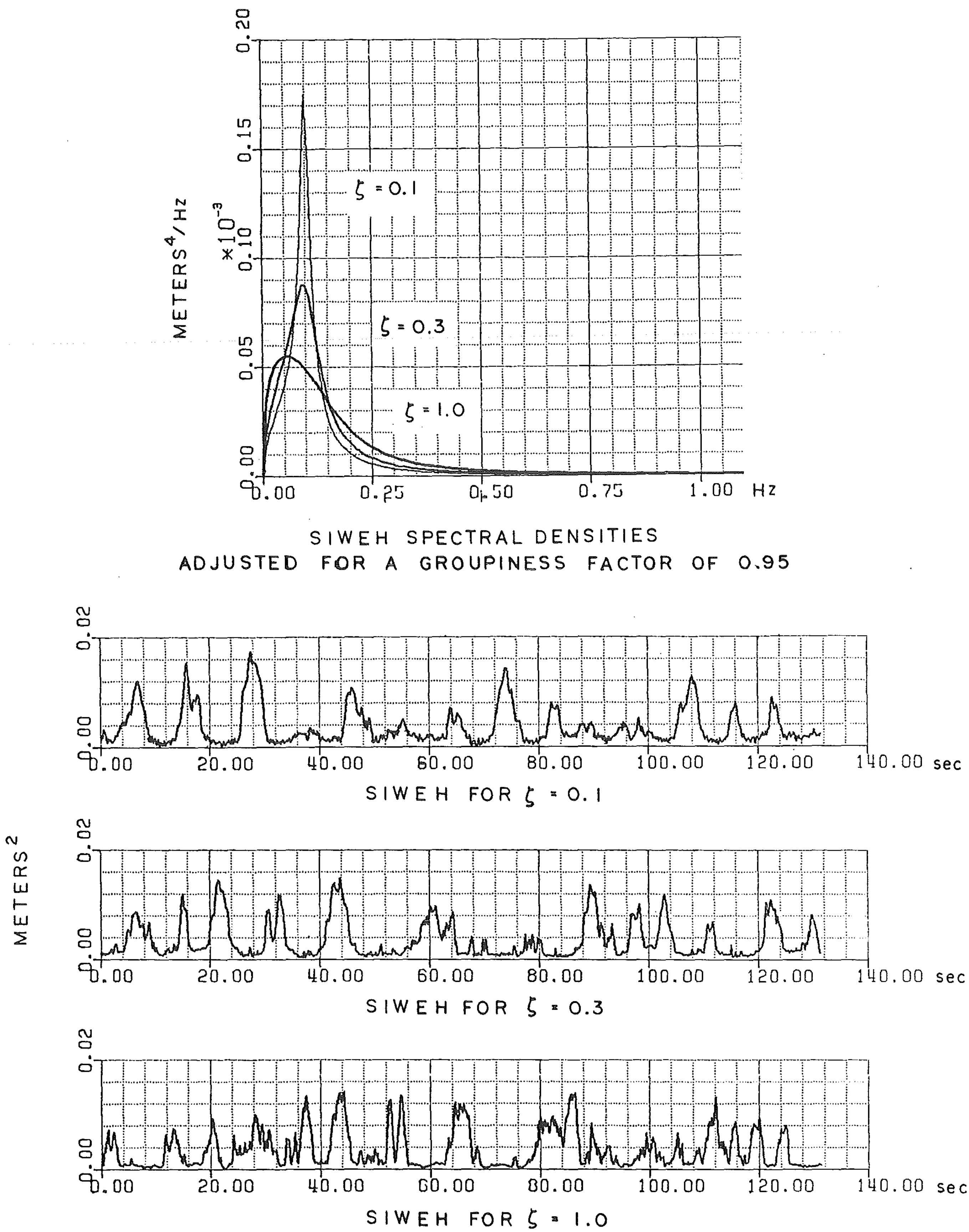


FIG. 12

SIWEHs OBTAINED FROM THEORETICAL SPECTRAL DENSITIES

10.0 ON THE MEASUREMENT OF ESSENTIAL PARAMETERS FOR SIMULATED SEA STATE

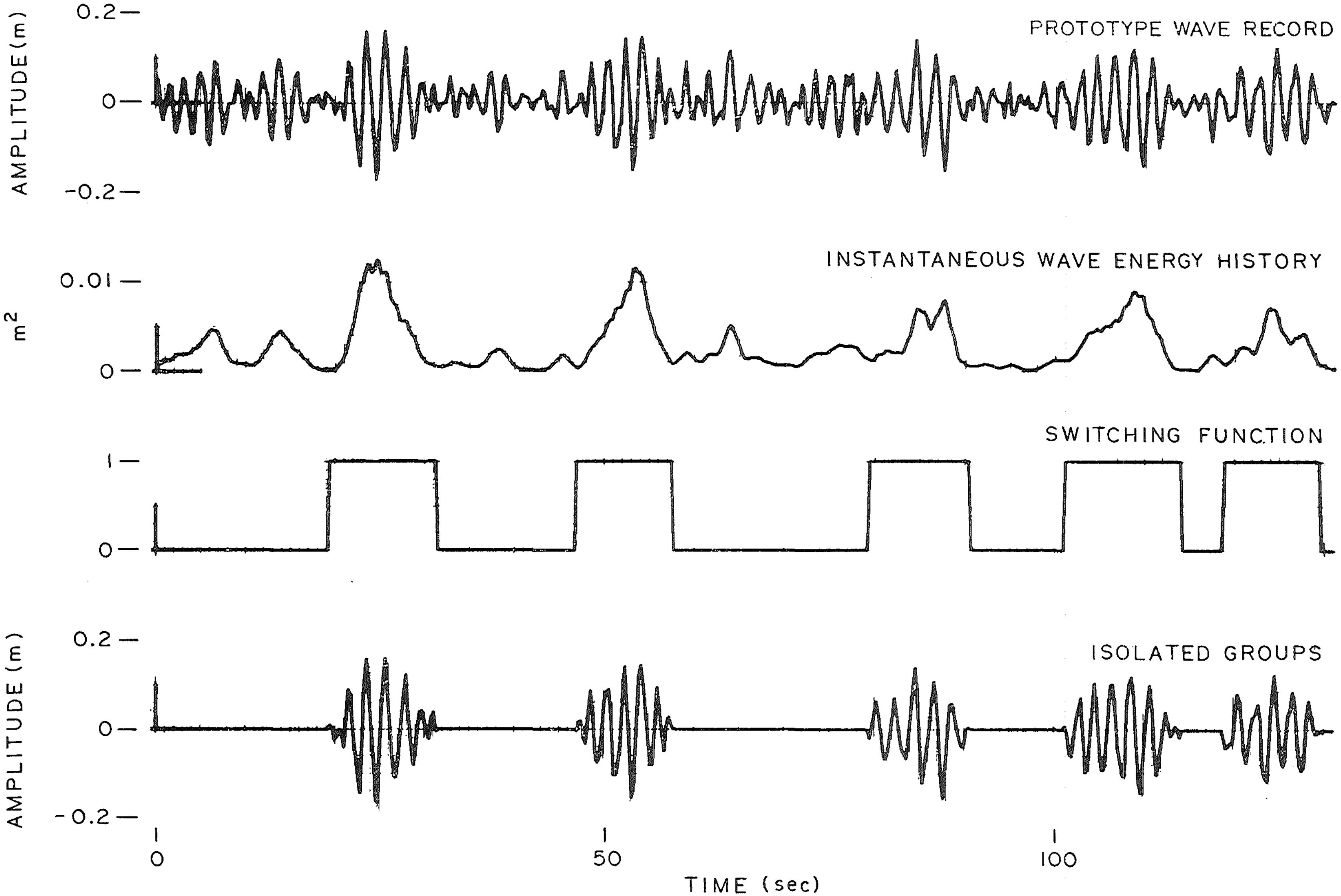
Having established that the amplitude spectrum of simulated 'random' sea states is not always a sufficient description of the test conditions, it remains to be decided what sea state parameters should be measured in order to assure that tests may be duplicated by other laboratories under similar conditions.

It will require a great deal of time-consuming and expensive research before it is established without doubt which sea state parameters are relevant for which tests. However, in the absence of this information it seems advisable to measure and compute as many parameters as seems possible and practical at this time. However, one may perhaps use some intuition and speculate on these matters, and Table 1 is an attempt to relate various studies to what may be considered, essential measurement parameters.

The following comments must be added to clarify the monitoring requirements for the simulated sea states:

10.1 Comments Related to A of Table 1

The sea state parameters related to the water surface displacement spectra are commonly monitored and calculated by most laboratories at this time. Since the sea state is



(SCALES IN MODEL UNITS)

FIG. 13

A PROTOTYPE WAVE RECORD (STATION 90, 17-2-009) AND ITS DECOMPOSITION INTO WAVE GROUPS frequently synthesized from a smooth amplitude spectrum, it is not generally required to use variance spectral density analysis. However, for the sake of standardization the single sided variance spectral density function $S_\eta(f)$, computed with a resolution of approximately 0.05 Hz, may perhaps suffice. For test of shorter repetition periods it may not be advisable to utilize the Welch method (Ref. 12) which subdivides the record into many shorter subrecords. Instead the square of the periodogram of the entire record should be averaged in the frequency domain.

The peak frequency is easily obtained by peak detection and the RMS-value may be reliably obtained from

$$RMS = \sqrt{m_0} = \sqrt{\int_0^\infty S_\eta(f) \cdot df}$$

The broadness factor, on the other hand is difficult to obtain since the presently used methods employ the higher moments of $S_\eta(f)$. For this reason the authors have experimented with other techniques of defining broadness (e.g. the 90% power bandwidth) and it is hoped that a suitable broadness factor may be evolved.

10.2 Comments Related to B of Table 1

Since most structures under test affect the sea state in front of the structure, it is usually difficult to know precisely what the test conditions are. The experimenter usually has two choices:

SEA STATE PARAMETERS IN COASTAL AND MARINE ENGINEERING STUDIES

TYPES OF STUDIES	SEA STATE PARAMETERS				
	A	B	C	D	E
1. STABILITY OF RUBBLE MOUND BREAKWATERS	X	X	X	X	X
2. FORCES AND PRESSURES ON OFFSHORE STRUCTURES	X	X	X	X	X
3. HARBOUR RESPONSE	X	X		X	
4. STABILITY OF FLOATING STRUCTURES	X		X	X	X
5. RESPONSE OF FLOATING STRUCTURES	X	X		X	X
6. STRESSES IN SHIP HULLS	X	X	X	X	X
7. MOORING FORCES FOR MOORED VESSELS	X	X	X	X	X
8. BEACH PROCESSES	X	X		X	
9. WAVE ENERGY EXTRACTION	X	X	X	X	X

A. TOTAL SPECTRUM, $S(f)$; PEAK FREQUENCY, f_p;
PEAK PERIOD, $T_p = 1/f_p$; RMS = $\sqrt{m_o}$; BROADNESS FACTOR

B. INCIDENT AND REFLECTED SPECTRA, $S_I(f)$ AND $S_R(f)$;
COEFFICIENT OF REFLECTION $C_R(f)$

C. WAVE HEIGHT DISTRIBUTION PARAMETERS, H_s, H_{MAX};
PERIODS, T_{H_s}, $T_{H_{MAX}}$

D. SIWEH, $E(t)$; AVERAGE WAVE ENERGY, $\bar{E}$;
SIWEH SPECTRAL DENSITY, $\varepsilon(f)$; PEAK FREQUENCY OF ε, $f_{p\varepsilon}$;
GROUPINESS FACTOR, GF

E. WAVE SLOPE DISTRIBUTION, $D(S)$;
MAXIMUM WAVE SLOPE, S_{MAX}

- either he measures the sea state before the structure is inserted into the test flume and then assumes that the same conditions prevail after the structure is placed, or
- he uses the required measurement and analysis technology that allows him to separate the incident and the reflected spectra.

It is this latter option which is proposed as the better alternative. Ref. 13 describes a technique which performs this task with excellent results. The reflection coefficient is also a useful by-product.

10.3 Comments Related to C of Table 1

Time domain analysis may still have a place in modern sea state analysis. Wave height analysis in terms of zero-upcrossing or -downcrossing techniques are also easily implemented by computer and, as a result, the significant and the maximum wave height can readily be obtained. It is likely that particularly the maximum wave height can have a significant impact on the outcome of the experiment. Evidently such observations must be made in reasonable proximity to the structure under test because there is evidence that the maximum observed wave height depends on the spatial point of observation in the flume.

10.4 Comments Related to D of Table 1

In order to get information on groupiness, the SIWEH should be obtained as well as all its related functions. The mean value of the SIWEH should, of course, be equal to the m_0 as obtained from the integration of $S_\eta(f)$. However, its separate computation from the SIWEH serves as a check.

10.5 Comments Related to E of Table 1

A parameter which is not being measured now is the wave slope. It is extremely costly, if not impossible, to measure the wave slope at sea, and, therefore it is usually derived by calculation. In the laboratory there are no longer serious obstacles to wave slope measurement particularily since the same wave probe array which is being used for reflection analysis may double up for the slope measurement. A probability distribution analysis may, at this time best describe the characteristics of this parameter. Above all, the maximum wave slope should be given.

11.0 CONCLUSIONS

A novel technique for the synthesis of realistic sea states is described. With this technique, it is now possible to create data strings representing waves which have well defined amplitude spectra and wave group characteristics.

With the availability of this tool, more attention should now be directed to the following topics:

- the development of a suitable wave group model in terms of the SIWEH spectral density or the SIWEH auto-correlation function
- a description of the prototype wave in terms of this new model
- a clear definition of sea state parameters to be monitored in the laboratory, and
- the establishment of relevance factors between the various sea state parameters and the hydrodynamic or marine dynamic phenomenon under investigation.

It is hoped that the material presented here will invite critical comments from the engineering community.

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APPENDIX A1

Derivation of an Expression for the SIWEH Spectral Density

The SIWEH may be given by the 1st and 4th term in equation (9) of Ref. 4 as

$$E(t) = \frac{1}{2} \sum_{i=1}^{N} C_i^{\,2} + \sum_{i,j} C_i \cdot C_j \cdot \cos\left[(\omega_j - \omega_i)t + (\phi_j - \phi_i)\right]$$

$$\text{for } 1 \leq i < j \leq N$$

where C_i are the Fourier coefficients of the wave train. The expansion may be re-ordered so that all terms of common frequency are grouped together. Therefore let

$$j = i + k$$

for $1 \leq k \leq N-i$

and $1 \leq i \leq N-1$.

Also define:

$$\omega_{i+k} - \omega_i = k \cdot \Delta\omega$$

and $\phi_{i+k} - \phi_i = \theta_{i,k}$

and $C_i \cdot C_{i+k} = \Pi_{i,k}$

Therefore the SIWEH becomes:

$$E(t) = \frac{1}{2} \sum_{i=1}^{N} C_i^{\,2} + \sum_{k=1}^{N-1} \sum_{i=1}^{N-k} \Pi_{i,k} \cdot \cos[k \cdot \Delta\omega \cdot t + \theta_{i,k}]$$

In order to find the contribution made by each frequency $k \cdot \Delta\omega$ one must sum over i, i.e.

$$A_k \cdot \cos(k \cdot \Delta\omega \cdot t + \gamma_k) = \sum_{i=1}^{N-k} \Pi_{i,k} \cdot \cos[k \cdot \Delta\omega \cdot t + \theta_{i,k}]$$

$$= \sum_{i=1}^{N-k} \Pi_{i,k} [\cos(k \cdot \Delta\omega \cdot t) \cdot \cos \theta_{i,k} - \sin(k \cdot \Delta\omega \cdot t) \cdot \sin \theta_{i,k}]$$

$$= \cos(k \cdot \Delta\omega \cdot t) \cdot \sum_{i=1}^{N-k} \Pi_{i,k} \cdot \cos \theta_{i,k} - \sin(k \cdot \Delta\omega \cdot t) \cdot \sum_{i=1}^{N-k} \Pi_{i,k} \cdot \sin \theta_{i,k}]$$

$$= \sqrt{\left(\sum_{i=1}^{N-k} \Pi_{i,k} \cdot \cos\, \theta_{i,k}\right)^2 + \left(\sum_{i=1}^{N-k} \Pi_{i,k} \cdot \sin\, \theta_{i,k}\right)^2} \cdot \cos(k \cdot \Delta\omega \cdot t + \gamma_k)$$

where:

$$\gamma_k = \text{atan}\left(\frac{\sum_{i=1}^{N-k} \Pi_{i,k} \cdot \sin\, \theta_{i,k}}{\sum_{i=1}^{N-k} \Pi_{i,k} \cdot \cos\, \theta_{i,k}}\right)$$

Therefore the SIWEH spectral density may be given as:

$$\varepsilon(k \cdot \Delta f) = \frac{1}{2 \cdot \Delta f}\left(\left(\sum_{i=1}^{N-k} \Pi_{i,k} \cdot \cos\, \theta_{i,k}\right)^2 \quad \left(\sum_{i=1}^{N-k} \Pi_{i,k}\ \sin\, \theta_{i,k}\right)^2\right)$$

where $\Delta f = \Delta\omega / 2\pi$.