# ON THE TEMPERATURE OF INTERSTELLAR GRAINS 

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## Summary

The temperatures of various grain models have been computed using the interstellar radiation field in the solar neighbourhood as given by Zimmermann and the best available optical data for graphite and ice. A graphite core of radius $0.05 \mu$ surrounded by an ice mantle of outer radius $0.15 \mu$ (a grain model which fits the extinction law) acquires a temperature of $\sim 20^{\circ} \mathrm{K}$. The presence of weakly bound impurity atoms in the crystal lattice can enhance the infrared absorption efficiency at $\lambda \approx 0 \cdot 1 \mathrm{~cm}$, and lower the temperatures of all grain models below $10^{\circ} \mathrm{K}$. Low grain temperatures favour molecule formation and mantle growth at the grain surfaces.
I. Introduction. The temperature of interstellar grains plays an important role in several astrophysical processes. It enters in the determination of conditions for molecule formation, the background radiation from grains, as well as in discussions concerning the magnetic alignment of grains. Reliable estimates of grain temperature are available only for a limited number of special cases considered by Greenberg (1), Stein (2) and Stecher \& Williams (3). In the present paper we shall determine the temperatures of graphite grains, graphite core_ice mantle grains and dirty ice grains of various sizes, using the best available refractive index data. We also include the effects of weakly bound impurity atoms, which, as it turns out, will drastically alter previous estimates of temperature.
2. Equation of energy balance. The method used to compute temperature is essentially that described by van de Hulst (4) (see also Wickramasinghe (5)). It is assumed that the grain temperature adjusts itself so that the energy absorbed from the interstellar radiation field balances the infrared re-radiation. The condition of balance for a grain of radius $a$ is:

$$
\begin{equation*}
\int_{0}^{\infty} Q_{\mathrm{abs}}(a, \lambda) F_{\lambda} d \lambda=\int_{0}^{\infty} Q_{\mathrm{abs}}(a, \lambda) B\left(\lambda, T_{g}\right) d \lambda \tag{I}
\end{equation*}
$$

where $Q_{\mathrm{abs}}(a, \lambda) \pi a^{2}$ is the absorption cross-section at wavelength $\lambda, F_{\lambda}$ is the flux of the interstellar radiation field, $T_{g}$ is the grain temperature and $B\left(\lambda, T_{g}\right)$ is the Planck function. This equation could be solved numerically for $T_{g}$ provided all the other quantities are specified. The method of solution is to compute the integral on the right hand side of equation (1) for a closely spaced grid of $T_{g}$ values and choose the $T_{g}$ value which gives a sufficiently near equality with the left hand side of the equation. In our calculations we shall adopt for $F_{\lambda}$ the interstellar radiation field in the solar vicinity estimated by Zimmermann (6). We shall discuss the effect of departures from this field in the concluding section. The $Q_{\text {abs }}$ occuring on the left-hand side of equation (1) is computed from the Mie

[^0]theory (or Güttler theory) for a given grain model, and that occuring on the right hand side is derived from the infrared absorption properties of the bulk material.
3. The temperature of pure grains. In the long wavelength limit $(2 \pi a / \lambda \ll 1)$ the absorption efficiency for a spherical particle of refractive index $m=n-i k$, radius $a$ is:
\[

$$
\begin{align*}
Q_{\mathrm{abs}} & =\frac{8 \pi a}{\lambda} \operatorname{Im} \frac{\mathrm{I}-m^{2}}{2+m^{2}}  \tag{2}\\
& =\frac{a}{\lambda} \frac{48 \pi n k}{\left(n^{2}-k^{2}+2\right)+4 n^{2} k^{2}}
\end{align*}
$$
\]

For pure graphite the values of $n$ and $k$ are taken from the data of Taft $\&$ Philipp (7), (8). For dirty ice grains the infrared $n$ and $k$ values used are those of Kislovskii (9); and in the visible spectral region we have used $m=n(\lambda)-0.05 i$ with $n(\lambda)$ taken from tables for pure ice.

The temperature of pure graphite and ice grains as a function of radius is plotted in the solid curves of Figs I and 2. The temperature of a graphite core of radius $0.05 \mu$ covered with an ice mantle of variable radius is plotted as the solid curve of Fig. 3. For 'pure' grains of all types with sizes that fit the extinction curve $T_{g}$ is in the range $20-45^{\circ} \mathrm{K}$.
4. The temperature of grains with weakly bound impurities. It is well known that a black sphere placed in the interstellar radiation field will take up a temperature $\approx 4{ }^{\circ} \mathrm{K}$. The much higher temperature emerging from the previous calculations is because $Q_{\text {abs }}$ given by equation (2) becomes very small at wavelengths $\lambda \approx 1 \mathrm{~mm}$.


Fig. I. The temperature of a graphite grain as a function of radius. Solid curve represents pure graphite grains. The dotted curves are for graphite with weakly bound impurity atoms giving rise to resonances at $\lambda_{0} \approx 0 \cdot 1$ and 0.2 cm .


Fig. 2. The temperature of a dirty ice grain as a function of radius. Solid curve is the case without weakly bound impurity atoms. The dotted curves are for dirty ice grains with weakly bound impurity atoms giving rise to resonances at $\lambda_{0} \approx 0.1$ and 0.2 cm .

Hoyle \& Wickramasinghe (10) have argued that grains may incorporate weakly bound impurity atoms such as He which give rise to broad absorption bands at $\sim$ I mm. The absorption arises due to bodily oscillations of the impurity atom in the electric field of the crystal and the typical $f$ value is of order unity. The effect of such oscillators would be to enhance the $Q_{\mathrm{abs}}$ value within the absorption band.

The absorption cross-section per atom near the centre $\nu_{0}$ of an atomic line is given by

$$
\begin{equation*}
\sigma_{\nu 0} \cong \frac{4}{\gamma} \frac{\pi e^{2}}{m c} f \tag{3}
\end{equation*}
$$

where $\gamma$ is the line width in frequency units and $f$ is the oscillator strength. For the usual case where the line is due to an electronic transition $m$ is the mass of an electron. In our case, however, the effective quantum oscillator is the entire atom so that $m$ must be set equal to the atomic mass. With $\nu_{0} \cong 10 \mathrm{~cm}^{-1}, \gamma / \mathrm{e} \nu_{0} \cong 0 \cdot 1$, $f \cong \mathrm{I}, m \cong 4 m_{\mathrm{H}}$ (appropriate for He ) we have $\sigma \cong 10^{-15} \mathrm{~cm}^{2}$. Thus $\sim 10^{5}$ impurity


Fig. 3. The temperature of a graphite core of radius $0.05 \mu$ surrounded by a dirty ice mantle of variable radius. Solid curve is for the case without weakly bound impurity atoms. Dotted curves are for grains with weakly bound impurity atoms giving rise to resonances at $\lambda_{0} \approx 0 \cdot 1$ and 0.2 cm .
atoms would give a total cross-section $\sim 10^{-10} \mathrm{~cm}^{2}$ at $\lambda \approx 0.1 \mathrm{~cm}$-comparable with the geometrical cross-section for a grain of radius $\sim 10^{-5} \mathrm{~cm}$. We may therefore assume that $Q_{\mathrm{abs}} \cong$ I within the absorption band.

The equation of energy balance (i) must now be modified to take into account the contribution from an absorption band. For an absorption band in the wavelength range $\lambda_{0}+\delta / 2 \geq \lambda \geq \lambda_{0}-\delta / 2$ we have

$$
\begin{align*}
\int_{0}^{\infty} Q_{\mathrm{abs}}(a, \lambda) F_{\lambda} d \lambda= & \int_{0}^{\infty} Q_{\mathrm{abs}}(a, \lambda) B\left(\lambda, T_{g}\right) d \lambda \\
& +\int_{\lambda_{0}-\delta / 2}^{\lambda_{0}+\delta / 2} B\left(\lambda, T_{g}\right) d \lambda \tag{4}
\end{align*}
$$

We set $\delta / \lambda_{0} \cong 0 \cdot 1$ and consider two cases $\lambda_{0}=0.1 \mathrm{~cm}, 0.2 \mathrm{~cm}$ for the central wavelength of the absorption band. The resulting grain temperatures for each of the models considered before are plotted in the dashed curves of Figs I-3.

It is evident that impurity induced absorption bands have the effect of drastically lowering the grain temperatures for all the models considered. This feature is not strongly dependent on the precise value of $\lambda_{0}$. The requirement is that $\lambda_{0} \sim 0 \cdot 1 \mathrm{~cm}$, which may be expected to hold for He atoms at lattice sites (10). The low temperature of $\lesssim 10^{\circ} \mathrm{K}$ obtained for the grains has several important consequences. For graphite grains it would permit both molecule formation at the grain surfaces as well as growth of ice mantles (Knaap et al. (11), Williams (12)). Low grain temperatures would also explain the observed excitation of the rotational levels of the CN molecule on the basis of a purely galactic process (13).
5. Effect of varying the interstellar radiation field. Throughout the preceding calculations we have adopted the interstellar radiation field given by Zimmermann for the solar neighbourhood. This radiation field is essentially derived by summing the contributions of stars of various spectral types, with a cut-off in intensity at the Lyman limit. Although there are some uncertainties arising due to corrections for the interstellar extinction and also to the uncertain stellar flux in the far ultraviolet, it is probably the best available determination of the mean interstellar radiation field. The energy density of this field over the spectral range $1000-5000 \AA$ is $\sim 4 \cdot 10^{-13} \mathrm{erg} \mathrm{cm}^{-3}$. Any other radiation field with a comparable energy density over the same spectral region would be expected to produce grain temperatures similar to those computed for the Zimmermann field.

Estimates of grain temperature are available for graphite and dirty ice grains placed in a black-body radiation field of temperature $T=10^{4} \mathrm{~K}$ diluted by factors $10^{-13}, 10^{-14}$ and $10^{-15}(\mathbf{r}),(3)$. In Table I we set out these temperatures along with the values derived for the Zimmermann field. The case where the dilution factor is $10^{-14}$ corresponds to an energy density of visible and ultraviolet radiation closely similar to that in Zimmermann's field; the grain temperatures in these two cases are also seen to be similar.

For grains with weakly bound impurity atoms, the grain temperature is hardly affected by slight changes in the interstellar radiation field. Such grains would tend to take up the effective black body temperature of interstellar space $\sim W^{1 / 4} T$, which ranges from $\sim 2-6^{\circ} \mathrm{K}$ over the range of dilution factors $W$ considered in Table I.

Table I
Temperatures of interstellar grains without weakly bound impurities in various radiation fields

|  | Radius | Black-body field with $T=10^{4} \mathrm{~K}$ |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: |
| dilution, $W$ |  |  |  |  |$\quad$ Zimmermann

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## References

(1) Greenberg, J. M., 1963. Ann. Rev. Astr. Astrophys., 1, 267.
(2) Stein, W., 1966. Astrophys. F., 144, 318.
(3) Stecher, T. P. \& Williams, D. A., 1966. Astrophys. F., 146, 88.
(4) van de Hulst, H. C., 1949. Rech. Astron. Obs. Utrecht, 11, Part 2.
(5) Wickramasinghe, N. C., 1967. Interstellar Grains, p. 91 Chapman and Hall, London.
(6) Zimmermann, H., 1964. Astr. Nachr. 288, 95 and 99.
(7) Taft, E. A. \& Phillipp, H. R., r965. Phys. Rev., r38A, 197.
(8) Taft, E. A. \& Phillipp, H. R., 1966. Private communication of computer print outs.
(9) Kislovskii, L. D., 1959. Opt. Spectrosc., 7, 201.
(10) Hoyle, F. \& Wickramasinghe, N. C., 1967. Nature, Lond., 214, 969.
(II) Knaap, H. F. P., van den Meijdenberg, C. J. N., Beenakker, J. J. M. \& van de Hulst, H. C., 1966. Bull. astr. Insts Neth., 18, 256.
(12) Williams, D. A., 1967. Astrophys. F., in press.
(13) Narlikar, J. V. \& Wickramasinghe, N. C., 1967. Nature, Lond., 216, 43.


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