192

On the Theory of Backscattering in Single-Mode Optical Fibers

ARTHUR H. HARTOG AND MARTIN P. GOLD

Abstract—A new theory of backscattering in single-mode fibers is presented. It allows backscatter waveforms to be predicted for fibers of any refractive-index profile or scattering-loss distribution. The results agree with experimental data and provide confirmation of an earlier, more restricted theory.

I. Introduction

The backscattering method [1]-[3] has been used increasingly in recent years to characterize loss and imperfections in single-mode fibers. The technique involves the launching of a pulse of light into the fiber and examining the temporal behavior of the return signal. The latter consists of energy which, having been scattered from the guided wave, is recaptured by the fiber in the reverse direction.

A theoretical analysis of backscattering in single-mode fibers appeared in 1980 [4] and was subsequently verified experimentally [5]. A series of experimental results has been published covering such aspects as long-range fault location [6]-[12], determination of structural parameter variations [13], measurements of splicing loss [7], [8], and the examination of polarization effects [14]-[16].

The backscatter factor, i.e., the ratio of the backscattered power to the energy launched into the fiber, is important in long-range fault location since it determines the magnitude of the signal and hence the range which the apparatus can cover. Moreover, for more quantitative measurements, such as the evaluation of splicing losses, or the determination of variations of fiber parameters along the length, it is essential to have an accurate model of the dependence of the backscatter signal on the parameters of the fiber. Without such a model, valid and accurate interpretations of the results cannot be made since one is usually interested in quite subtle changes in the received power.

It is, therefore, of some concern that doubts have been expressed [6], [12] as to the validity of the presently available wave-optics theory, due to Brinkmeyer [4]. Unfortunately, the only alternatives to the latter are either approximate [11] or based on geometric optics [17], an approach clearly inappropriate to the analysis of single-mode fibers. The resulting impression is one of a confusion which requires clarification.

Manuscript received June 6, 1983, revised October 26, 1983. This work was supported in part by the UK Science and Engineering Research Council and the Pirelli General Cable Company.

The authors are with the Department of Electronics, The University Southampton, Southampton, England S09 5NH.

In contrast, the theory of backscattering in multimode fibers is well established [18]-[19].

The purpose of the present contribution is, therefore, to resolve the disagreement in the literature. To this end, we present a new theory of backscattering in single-mode fibers which is derived using a different and more general approach from those proposed previously. The result is a simple expression for the backscatter factor which may be used in circularly symmetric fibers having arbitrary refractive-index profiles and an arbitrary distribution of scattering loss. Comparisons are made with previously published results. It is found that if the near-field distribution is approximated by a Gaussian function, our results agree exactly with those of Brinkmeyer [4].

II. Time-Dependence of the Backscatter Signal

For the purpose of the present argument, we assume that the pulse launched into the fiber is a Dirac function of energy $E(0)$ and of vanishingly narrow width. The effect of a finite pulsewidth W is to limit the distance resolution of the measurement to $\delta x \simeq v_g W/2$ where v_g is the group velocity in the fiber; for a constant input energy, varying the pulsewidth will not alter the backscatter signal level.

The forward pulse energy is attenuated at a rate $\alpha (np/m)$ and its dependence on position z is thus

$$E(z) = E(0) \exp(-\alpha z). \tag{1}$$

The energy scattered while the pulse travels a distance element dz situated at z is

$$dE_s(z, z + dz) = E(z)\, \alpha_s(z)\, dz \tag{2}$$

where α_s is the Rayleigh scattering-loss coefficient.

We define the capture fraction $B(z)$ as the proportion of the total energy scattered at z which is recaptured by the fiber in the return direction. The energy $dE_{BS}(z, z + dz)$ returning from the length element dz to the launching end is, therefore,

$$dE_{BS}(z, z + dz) = E(0)\, \alpha_s(z) B(z) \exp(-2\alpha z)\, dz. \tag{3}$$

This energy arrives spread over a time interval given by

$$dt = \frac{2}{v_g} dz \tag{4}$$

which is simply the time taken by the impulse to travel the distance dz in both directions. In the case of a Dirac pulse (which we have assumed), no energy will arrive from any other part of the fiber during this time interval. The power received at the

0733-8724/84/0400-0076$01.00 © 1984 IEEE

launch end of the fiber at time $t = 2z/v_g$ is, therefore,

$$P_{BS}(t) = \frac{dE_{BS}}{dt} = \frac{v_g}{2} E(0)\, \alpha_s(z) B(z) \exp(-2\alpha z). \tag{5}$$

A similar expression has been put forward recently [12] in which the factor of $\frac{1}{2}$ is omitted. Note that the result given above, in (5), and the derivation are virtually the same as for multimode fibers [18].

III. Evaluation of the Backscatter Capture Fraction

Rayleigh scattering is described by classical theory in terms of electric dipoles driven by an electromagnetic wave traveling through the material [20]. In a homogeneous medium, interference between the radiation patterns of the dipoles results in cancellation of the secondary waves in all but the forward direction. A localized inhomogeneity of the refractive index, however, results in a dipole moment whose radiation is not canceled by adjacent dipoles. A portion of the incident wave is then radiated in all directions and power is lost to Rayleigh scattering. The scattering process may, therefore, be represented by a large number of dipoles oscillating with a fixed phase relative to the incident wave, but whose amplitude is proportional to the random local deviation $\Delta\chi$ of the electric susceptibility from its mean value $\bar{\chi}$. With coherent illumination, the phase relationship between the light scattered by separate dipoles is fixed. The scattered light, therefore, suffers interference, a phenomenon akin to laser speckle. Some aspects of these coherence effects have been discussed in the context of frequency-domain reflectometry by Eickhoff and Ulrich [21]. They can also be observed in time-domain reflectometry [13] with pulsed lasers of sufficiently narrow spectral width.

In the present analysis, we assume that the source used is sufficiently incoherent that such interference effects are eliminated [13]. The contribution of all dipoles to the scattered light can, therefore, be added in intensity. The calculation of the backscatter capture fraction B is then reduced to the evaluation of the power coupling between the electric dipoles and the fundamental mode of the fiber.

The power coupled into the HE_{11} mode is calculated with the aid of an overlap integral in the far field following an approach used by Marcuse and Marcatili [22] for slab waveguides and, more recently, by Wagner and Tomlinson [23] to study components for single-mode fibers.

For simplicity, we assume linearly polarized mode fields. For weakly guiding fibers there is no loss of generality since an arbitrary state of polarization may be described as a combination of linearly polarized waves. The orientation of the electric-field vector of the backscattered wave at the point of capture is that of the dipole moment. For isotropic materials this orientation coincides with that of the incident electric field, and the state of polarization is thus preserved by the scattering process. Practical fiber materials are locally anisotropic which results in a small degree (~5 percent) of depolarization of the scattered light. It is interesting to note that this depolarized component of the scattered light recaptured in the forward direction imposes a fundamental limitation on the ability of high-birefringence fibers to maintain a linear state of polarization.

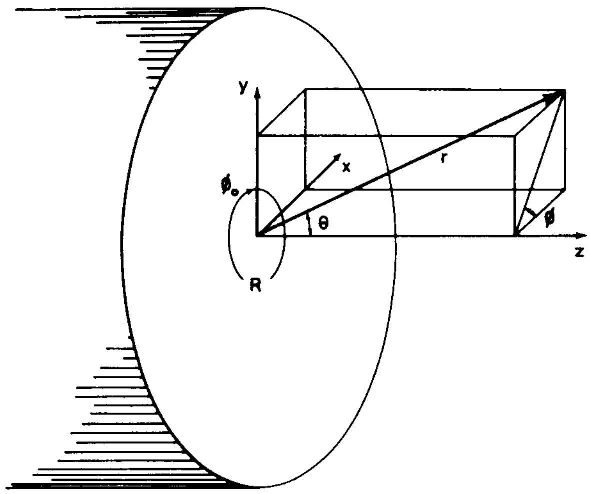

Fig. 1. Coordinates used in the near and far field.

In order to calculate the power coupled to the fiber mode, we consider one direction of propagation only and perform the coupling overlap integral over a hemisphere centered on the fiber axis. The coupling efficiency b, of a scatter point situated at a distance R_s from the fiber axis is

$$b(R_s) = \frac{1}{2} \frac{\left| \iint_{2\pi} \psi_F\, \psi_S\, d\Omega \right|^2}{\iint_{2\pi} |\psi_F|^2\, d\Omega \iint_{2\pi} |\psi_S|^2\, d\Omega} \tag{6}$$

where ψ_F and ψ_S are the far-field distributions of the HE_{11} mode and of the dipole, respectively, and $d\Omega$ is a solid angle element. Distance R_s has been normalized to the core radius a. The dipole, in fact, radiates the same amount of power into the complementary hemisphere, which couples to the HE_{11} mode in the forward direction. The factor of $\frac{1}{2}$ appearing in (6) is, therefore, required to give the overall power coupling efficiency between the dipole and the backscattered guided wave.

The far field $\psi_F(r, \theta, \phi)$ of the HE_{11} mode may be obtained from the near-field distribution $\psi_N(R, \phi_0)$ using the Fraunhofer diffraction formula. The spherical polar coordinates system used is represented schematically in Fig. 1. ψ_F is given by [24], [25]

$$\psi_F(r, \theta, \phi) = j\, \frac{Cna^2}{\lambda r}\, e^{-jknr} \int_0^{\infty} \int_0^{2\pi} \psi_N(R, \phi_0) \cdot \exp\left(jkanR \sin\theta \cos(\phi - \phi_0)\right) d\phi_0 R\, dR \tag{7}$$

where C is a normalization constant and R is the radial coordinate normalized to the core radius a. λ is the wavelength of the incident light measured in free space and $k = 2\pi/\lambda$. On the assumption of weakly guiding fibers, we make the approximation: $n_1 \simeq n_2 = n$.

Assuming circular symmetry and using the well-known integral representation of J_0 [26], the Bessel function of the first kind, (7) may be rewritten in the form

$$\psi_F(r,\theta) = j\,\frac{Ck_1a}{r}\,e^{-jknr}\int_0^\infty \psi_N(R)J_0(k_1R\sin\theta)R\,dR \tag{8}$$

where $k_1 = kan$.

Similarly, the far field of the dipole may be expressed as [20]

$$\psi_S(r,\theta,\phi) = \psi_{s0}e^{-j[knr+\delta(R_s)]}(1-\sin^2\theta\cos^2\phi)^{1/2} \tag{9}$$

where ψ_{s0} is a normalization constant. $\delta(R_s)$ is a phase shift resulting from the displacement R_s of the scatter point from the fiber axis. It carries the only information which remains in the far field as to the value of R_s. Using a simple geometric argument, δ may be shown to be

$$\delta(R_s) = k_1R_s\sin\theta\sin\phi. \tag{10}$$

The numerator $|I_1|^2$ of (6) then becomes

$$|I_1|^2 = \left| j\,\frac{Ck_1a}{r}\,\psi_{s0}\,e^{-2jknr} \cdot\int_0^{\pi/2}\int_0^\infty \psi_N(R)J_0(k_1R\sin\theta)R\,dR \cdot\int_0^{2\pi}(1-\sin^2\theta\cos^2\phi)^{1/2}e^{-jk_1R_s\cdot\sin\theta\sin\phi} \cdot d\phi\sin\theta\,d\theta\right|^2. \tag{11}$$

For the values of refractive-index difference normally used in single-mode fibers, the far-field distribution only has significant intensity for small values of θ. We can, therefore, make the approximation

$$1-\sin^2\theta\cos^2\phi \simeq 1.$$

For the same reason, the upper limit of the integral over θ in (11) may be taken to infinity and $\sin\theta$ replaced by θ. The accuracy of these approximations has been checked numerically and found to be better than 0.5 percent for numerical apertures of 0.2 or less and for V-values of 1 or more. Using these approximations and the integral representation of J_0, (11) may be rewritten as

$$|I_1|^2 = \left| j\,\frac{2\pi Ca}{k_1r}\,\psi_{s0}e^{-2jknr}\int_0^\infty xJ_0(xR_s) \cdot\int_0^\infty \psi_N(R)J_0(xR)R\,dR\,dx\right|^2 \tag{12}$$

where $x = k_1\theta$.

From the definition of the Hankel transform [27], $|I_1|^2$ simplifies to

$$|I_1|^2 = \frac{4\pi^2C^2a^2}{k_1^2r^2}\,\psi_{s0}^2\psi_N^2(R_s). \tag{13}$$

Similarly, the first term in the denominator of (6) is given by

$$\iint_{2\pi}|\psi_F|^2\,d\Omega = \frac{2\pi C^2k_1^2a^2}{r^2}\int_0^{\pi/2} \cdot\sin\theta\left[\int_0^\infty \psi_N(R)J_0(k_1R\sin\theta)R\,dR\right]^2 d\theta \tag{14}$$

and, making the same approximation as previously, namely $\sin\theta \simeq \theta$ and $\pi/2 \to \infty$, (14) becomes

$$\iint_{2\pi}|\psi_F|^2\,d\Omega = \frac{2\pi C^2a^2}{r^2} \cdot\int_0^\infty x\left[\int_0^\infty \psi_N(R)J_0(xR)R\,dR\right]^2 dx. \tag{15}$$

From Parseval's equation, as applied to Hankel transform pairs [27], it follows that the integration over x can be performed in the near field, (15) is the equivalent to

$$\iint_{2\pi}|\psi_F|^2\,d\Omega = \frac{2\pi C^2a^2}{r^2}\int_0^\infty R\,\psi_N^2(R)\,dR. \tag{16}$$

Finally, it is found that

$$\iint_{2\pi}|\psi_S|^2\,d\Omega = \frac{4\pi}{3}\,\psi_{s0}^2. \tag{17}$$

Substituting (13), (16) and (17) into (6), the following expression for the capture fraction at R_s (the local capture fraction) is obtained

$$b(R_s) = \frac{3}{4k^2a^2n^2}\,\frac{\psi_N^2(R_s)}{\int_0^\infty R\,\psi_N^2(R)\,dR}. \tag{18}$$

In order to derive the overall capture fraction B it is now only necessary to average $b(R_s)$ over the entire near field, weighted by the intensity distribution of the scattered light. For a uniform distribution of the scattering loss, B is, therefore,

$$B = \frac{\int_0^\infty R\,b(R)\,\psi_N^2(R)\,dR}{\int_0^\infty R\,\psi_N^2(R)\,dR} \tag{19}$$

i.e.,

$$B = \frac{3}{4k^2a^2n^2} \frac{\int_0^\infty R\,\psi_N^4(R)\,dR}{\left[\int_0^\infty R\,\psi_N^2(R)\,dR\right]^2} \tag{20}$$

or, in terms of the normalized frequency V

$$B = \frac{3}{4V^2} \frac{(\mathrm{NA})^2}{n^2} \frac{\int_0^\infty R\,\psi_N^4(R)\,dR}{\left[\int_0^\infty R\,\psi_N^2(R)\,dR\right]^2}. \tag{21}$$

In the case of nonuniform scattering-loss distributions, B is given by

$$B = \frac{3}{4V^2} \frac{(\mathrm{NA})^2}{n^2} \frac{\int_0^\infty R\,\alpha_s(R)\,\psi_N^4(R)\,dR}{\int_0^\infty R\,\alpha_s(R)\,\psi_N^2(R)\,dR \int_0^\infty R\,\psi_N^2(R)\,dR} \tag{22}$$

It is then necessary to replace, in (5), α_s with the intensity-weighted mean value $\overline{\alpha_s}$ defined by

$$\overline{\alpha_s} = \frac{\int_0^\infty R\,\alpha_s(R)\,\psi_N^2(R)\,dR}{\int_0^\infty R\,\psi_N^2(R)\,dR}. \tag{23}$$

Equation (22) is a general expression for the capture fraction for arbitrary refractive-index profiles and scattering-loss distributions. Hence the backscatter factor may be evaluated from the near field of the HE_{11} mode and the radial scattering-loss distribution without resorting to the equivalent-step approach.

IV. Results

A. Gaussian Approximation

Brinkmeyer's theory of backscattering in single-mode fibers [4] is based on the Gaussian approximation, i.e., it is assumed that the near-field distribution is of the form

$$\psi_G(R) = \psi_0 \exp\left(-\frac{R^2a^2}{\omega_0^2}\right) \tag{24}$$

where ω_0 is the spot size. Substituting ψ_G into (21) leads to (for a uniform distribution of the scattering loss)

$$B = \frac{3}{2} \frac{1}{V^2\left(\frac{\omega_0^2}{a^2}\right)} \frac{(\mathrm{NA})^2}{n^2}. \tag{25}$$

Equation (25) is precisely that which is derived by Brinkmeyer and thus confirms, within the limitations of the Gaussian approximation, the validity of [4]. However, in those cases where the detailed behavior of the function $B(V)$ is of interest, (21) and (22) are more useful since the accuracy of the Gaussian approximation is itself sensitive to the V-value.

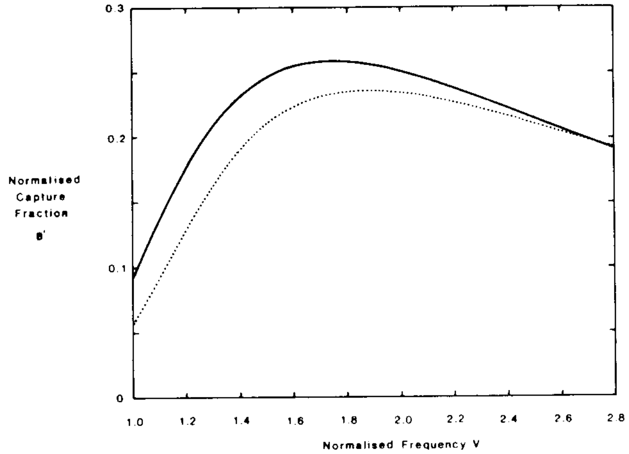

Fig. 2. Calculated values of the normalized capture fraction $B' = B \times (\mathrm{NA}/n)^{-2}$ for a step-index fiber having a uniform scattering-loss distribution. Solid line: Present theory (27). Dotted line: Gaussian approximation (25).

B. Step-Index Fibers Having Uniform Distributions of Scattering Loss

In the case of the step-index profile, the field distribution is the well-known Bessel function expression

$$\psi_N(R) = \begin{cases} J_0(UR)/J_0(U), & R \leq 1 \\ K_0(WR)/K_0(W), & R \geq 1 \end{cases} \tag{26}$$

where U is the eigenvalue and $W^2 = V^2 - U^2$. B is then given by

$$B = 3\frac{W^4}{V^6} \frac{J_0^4(U)}{J_1^4(U)} \frac{(\mathrm{NA})^2}{n^2} \cdot \left[\int_0^1 R\frac{J_0^4(UR)}{J_0^4(U)}\,dR + \int_1^\infty R\frac{K_0^4(WR)}{K_0^4(W)}\,dR\right]. \tag{27}$$

Hence, as is the case for the Gaussian approximation, it is possible to separate the dependence of B on V-value and on numerical aperture. A general curve of $B' = B(\mathrm{NA}/n)^{-2}$ which depends only on V may, therefore, be produced.

In Fig. 2, the values of the function $B'(V)$ have been compared for the two above results. The solid curve is obtained from (27); it has a peak of 0.258 at $V = 1.75$ and decreases gradually for larger V-values. At low V-values, the fall of B' is more dramatic and reflects the spreading of the power into the cladding as the frequency decreases. The dotted line is calculated with the aid of the Gaussian approximation, (25), and using the expression for ω_0/a due to Marcuse [28]

$$\omega_0/a = 0.65 + 1.619V^{-1.5} + 2.879V^{-6}. \tag{28}$$

The peak value of B' in this case is some 10 percent below that obtained from the rigorous expression (solid line). Moreover, the maximum backscatter factor occurs at a larger V-value in

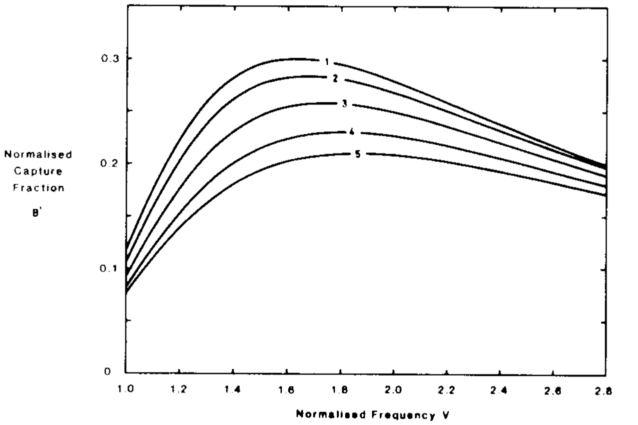

Fig. 3. Calculated values of the normalized capture fraction B' for various distributions of scattering loss in a step-index fiber. Labeling parameter corresponds to the fiber numbers of Table I.

TABLE I

Fibre No.	Scattering loss (dB $km^{-1} \mu m^4$) core	cladding	$\Delta = \frac{n_1 - n_2}{n_1}$
1	1.4	0.7	1%
2	1.05	0.7	0.5%
3	1.05	1.05	
4	0.7	1.05	0.5
5	0.7	1.4	1

Note: Fibers 1 and 5 have unusually large index differences for single-mode fibers, fibers 2 and 4 have a more typical numerical aperture, and the scattering loss of fiber 3 is uniform for comparison.

the Gaussian approximation and the detailed shapes of the two curves are thus significantly different. The difference is scarcely important in the prediction of the range of backscatter apparatus; use of the Gaussian approximation could, however, lead to errors in the interpretation of features in backscatter waveforms.

C. Step-Index Fibers Having Nonuniform Distributions of Scattering Loss

In most fibers the scattering loss is a function of radial position. In particular, the admixture of dopants such as GeO_2 or B_2O_3 to silica in order to alter the refractive index is known to modify the scattering level. In general, such additives lead to increased scattering loss owing to small-scale fluctuations of the glass composition and hence of its refractive index. In order to evaluate the effect of nonuniform scattering loss on the backscatter factor, the function $B'(V)$ has been calculated from (22) for step-index fibers having the scattering losses given in Table I. The right-hand column in the table shows approximate corresponding values of refractive-index difference, obtained from [29] and [30], and assuming that either the core or the cladding is made from pure silica.

Fig. 3 shows the resulting $B'(V)$ curves. It is immediately clear from the figure that the capture fraction increases as the proportion of the scattering loss occurring in the core increases. We note also that the peak values of the capture fraction occur at different V-values. The curves come together at large V-values, as the power confinement improves and the effect of the cladding loss is less important.

V. Discussion

A. Interpretation of Backscatter Measurements

The interpretation of features in backscatter waveforms relies on the availability of a theory of backscattering applicable to any refractive-index profile or distribution of scattering loss. If the backscatter waveform of a fiber exhibits departures from the simple exponential decay, one or more of the structural parameters must be a function of position. It would be useful to determine which of the fiber properties are nonuniform in order to provide feedback to the manufacturing process.

Experimentally, the local backscatter factor [5] defined as

$$\eta(z) = \tfrac{1}{2} \alpha_s(z) B(z) v_g \tag{29}$$

is obtained by combining backscatter waveforms measured from each end of the fiber [13], [31]. In general, it is not possible to interpret this information unambiguously in terms of the fiber properties, since the backscatter factor depends on all of the major fiber parameters, even under the simplifying assumption of a Gaussian near-field distribution.

In the case of a step-index fiber, the parameters which affect the backscatter factor are the core radius, the index difference and the scattering loss in the core and cladding. The importance of the distribution of the scattering loss, as well as its mean value has been emphasized by the present theory. Since all of these parameters are potentially length-dependent, it is clearly not possible to distinguish their individual contribution without additional information. However, we propose a method whereby the origin of the observed nonuniformities may be determined under certain conditions, as follows.

In order to calculate the backscatter factor, it is necessary first to obtain the near-field distribution. This may either be measured directly on the fiber, or calculated from the refractive-index profile (by numerical solution of the wave equation). A third possibility is to adopt one of the equivalent step-index approaches. For the sake of clarity, the remainder of the present discussion is restricted to the case of a step-index fiber whose core scattering loss is allowed to be different from that of the cladding.

We assume that the nonuniformity of interest is a perturbation of one or more fiber parameters about their nominal values. The sensitivity of η to changes in fiber parameters has been calculated and the results are given in Figs. 4 and 5. Fig. 4 shows the relative sensitivity $(a/\eta \cdot d\eta/da)$ of η to core radius variations, for the same distributions of scattering loss as used in Fig. 3. For variations of the numerical aperture, the curves are the same except for a constant offset of 2 which results from the $(NA)^2$ factor in (27). The exact value depends to some extent on the distribution of scattering and varies substantially with V-value, even over the limited range where low microbending-loss single-mode operation is feasible.

From (29), the relative sensitivity of η to variations of the mean scattering loss is unity. This is not the case if α_s varies differently in the core or cladding regions as might be expected for defects in MCVD fibers. The relative sensitivity of η to core scattering variations is shown in Fig. 5. The effect of variations of α_s in the cladding is exactly complementary to the curves of Fig. 5. It is, therefore, found that, if V is greater than ~1.5, η is sensitive to variation of the scattering

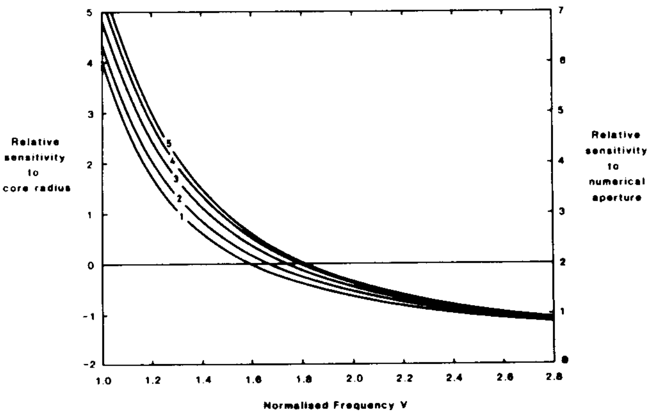

Fig. 4. Relative sensitivity of the backscatter factor η to variations of the core radius (left-hand axes) and of the numerical aperture (right-hand axes) for the distributions of α_s given in Table I.

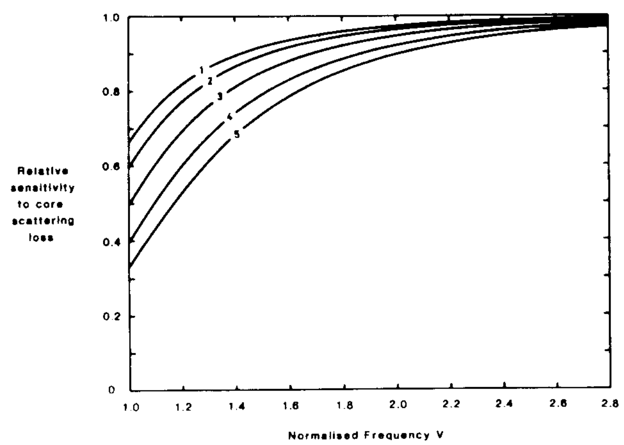

Fig. 5. Relative sensitivity of the backscatter factor to the scatter loss in the core, for the distributions of α_s given in Table I.

loss in the core but largely unaffected by changes of α_s occurring in the cladding region.

The interesting point to note from Figs. 4 and 5 is that the relative sensitivities of the backscatter factor to the different fiber parameters vary in markedly different ways as the V-value changes. This characteristic can be exploited by performing backscatter measurements at 2 or 3 well-spaced wavelengths. Comparison of the behavior of the measured features at the different V-values with the parameter sensitivities predicted by the theory for the nominal fiber characteristics will then reveal which parameters are changing at the observed backscatter features.

B. Comparison of the Present Theory with Experiment

Measurements of the backscatter factor in single-mode fibers have been published by the present authors [5]. The results are reproduced in Table II. The measurements were shown to agree with the predictions of the Gaussian-approximation theory of Brinkmeyer. Comparison of the experimental results with the present theory shows an agreement slightly worse than was found with the Gaussian-approximation theory but nevertheless within the accuracy to which the fiber parameters are known.

If, however, the factor of $\frac{1}{2}$ had been omitted from (5), as [12] suggests it ought, then there is no longer agreement between theory and experiment (see Table II). In [12], this lack

TABLE II

Fibre number	V-value at 1.06μm	Scattering loss* at 1.06μm	Backscatter factor			
			Experiment	Values predicted using:		
				Ref.4	Present work	Ref.12
		dB/km	W/J	W/J		
347	1.88	0.59	14.5(± 0.5)	15.1	17.2	30.2
348	1.99	0.62	19.6(± 0.6)	20.3	22.4	40.6

* obtained from published scattering loss data [29] on the basis of measured fibre parameters.

of agreement is put down to uncertainty in the estimates of the scattering loss in the fibers. We note, however, that in order for our previous experimental results to agree with the theory of [12], the scattering loss at $\lambda = 1.06$ μm in our fibers would have to be about 0.3 dB/km. Such a low scattering loss is, sadly, unrealistic in silica-based fibers at this wavelength. The present theory and, by extension, that of [4] are thus the only ones of those presented to date to agree accurately with the experimental evidence.

VI. Conclusions

A new approach to the theory of backscattering in single-mode fibers has been presented. A simple expression has been derived for the backscatter capture fraction which involves only the near-field distribution of the HE_{11} mode. This result enables the backscatter power to be calculated for a fiber of arbitrary refractive-index profile and scattering-loss distribution.

The predictions of the theory were found to agree with previously published experimental data and also to confirm the validity of an earlier, less general, result derived by a separate argument.

Finally, the theory presented in this paper allows the effect of fiber imperfections to be modeled so that the features observed in backscatter waveforms may be interpreted more accurately.

Acknowledgment

The authors would like to thank D. N. Payne and C. M. Ragdale for helpful discussions, and W. A. Gambling for his guidance.

References

[1] F. P. Kapron, R. D. Maurer, and M. P. Teter, "Theory of backscattering effects in waveguides," *Appl. Opt.*, vol. 11, no. 6, pp. 1352-1356, 1972.

[2] M. K. Barnoski and S. M. Jensen, "Fiber waveguides: A novel technique for investigating attenuation characteristics," *Appl. Opt.*, vol. 15, no. 9, pp. 2112-2115, 1976.

[3] S. D. Personick, "Photon probe–An optical-fiber time-domain reflectometer," *Bell Syst. Tech. J.*, vol. 56, no. 3, pp. 355-366, 1977.

[4] E. Brinkmeyer, "Analysis of the backscattering method for single-mode optical fibers," *J. Opt. Soc. Amer.*, vol. 70, no. 8, pp. 1010-1012, 1980.

[5] M. P. Gold and A. H. Hartog, "Measurement of backscatter factor in single-mode fibers," *Electron. Lett.*, vol. 17, no. 25/26, pp. 965-966, 1981.

[6] P. Healey, "OTDR in monomode fibers at 1.3 μm using a semiconductor laser," *Electron. Lett.*, vol. 17, no. 2, pp. 62-64, 1981.

[7] —, "Multichannel photon-counting backscatter measurements on monomode fiber," *Electron. Lett.*, vol. 17, no. 20, pp. 751-752, 1981.

[8] S. Heckmann, E. Brinkmeyer, and J. Streckert. "Long-range backscattering experiments in single-mode fibers," *Opt. Lett.*, vol. 6, no. 12, pp. 634–635, 1981.

[9] M. Nakazawa, M. Tokuda, K. Washio, and Y. Morishige, "Marked extension of diagnosis length in optical time domain reflectometry using 1.32 μm YAG laser," *Electron. Lett.*, vol. 17, no. 21, pp. 783–784, 1981.

[10] M. P. Gold and A. H. Hartog, "Ultra-long range OTDR in single-mode fibers at 1.3 μm," *Electron. Lett.*, vol. 19, no. 13, pp. 463–464, 1983.

[11] K. Aoyama, K. Nakagawa, and T. Itoh, "Optical time domain reflectometry in a single-mode fiber," *IEEE J. Quantum Electron.*, vol. QE-17, no. 6, pp. 862–868, 1981.

[12] D. L. Philen, I. A. White, J. F. Kuhl, and S. C. Mettler, "Single-mode fiber OTDR: Experiment and theory," *IEEE J. Quantum Electron.*, vol. QE-18, no. 10, pp. 1499–1508, 1982.

[13] M. P. Gold and A. H. Hartog, "Determination of structural parameter variations in single-mode optical fibers by time-domain reflectometry," *Electron. Lett.*, vol. 18, no. 12, pp. 489–490, 1982.

[14] A. H. Hartog, D. N. Payne, and A. J. Conduit, "Polarization optical time-domain reflectometry: Experimental results and application to loss and birefringence measurements in single-mode optical fibers," in *Proc. 6th European Conf. Opt. Commun.* (York, England), 1980 (post-deadline paper).

[15] J. N. Ross, "Measurement of magnetic field by polarization optical time-domain reflectometry," *Electron. Lett.*, vol. 17, no. 17, pp. 596–597, 1981.

[16] M. Nakazawa, T. Horiguchi, M. Tokuda, and N. Uchida, "Polarization beat length measurement in a single-mode optical fiber by backward Rayleigh scattering," *Electron. Lett.*, vol. 17, no. 15, pp. 513–515, 1981.

[17] D. L. Philen, "Optical time domain reflectometry on single-mode fibers using a *Q*-switched Nd:YAG laser," in *Tech. Dig. Symp. Opt. Fiber Measurements* (Boulder, CO), 1980, pp. 97–100.

[18] E.-G. Neumann, "Analysis of the backscattering method for testing optical fiber cables," *Electron. Commun.* (AEU), vol. 34, no. 4, pp. 157–160, 1980.

[19] A. R. Mickelson and M. Eriksrud, "Theory of the backscattering process in multimode optical fibers," *Appl. Opt.*, vol. 21, no. 11, pp. 1898–1909, 1982.

[20] B. Chu, *Laser Light Scattering.* New York: Academic, 1974.

[21] W. Eickhoff and R. Ulrich, "Statistics of backscattering in single-mode fiber," in *Tech. Dig. 3rd Int. Conf. Integrated Opt. Opt. Fiber Commun.* (San Francisco, CA), 1981, pp. 76–78.

[22] D. Marcuse and E.A.J. Marcatili, "Excitation of waveguides for integrated optics with laser beams," *Bell Syst. Tech. J.*, vol. 50, no. 1, pp. 43–57, 1971.

[23] R. E. Wagner and W. J. Tomlinson, "Coupling efficiency of optics in single-mode fiber components," *Appl. Opt.*, vol. 21, no. 15, pp. 2671–2688, 1982.

[24] M. Born and E. Wolf, *Principles of Optics.* Oxford, England, Pergamon, 1975.

[25] W. A. Gambling, D. N. Payne, H. Matsumura, and R. B. Dyott, "Determination of core diameter and refractive-index difference of single-mode fibers by observation of the far-field pattern," *Microwaves, Opt. Acoustics*, vol. 1, no. 1, pp. 13–17, 1976.

[26] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions.* New York: Dover, 1972.

[27] V. A. Ditkin and A. P. Prudnikov, *Integral Transforms and Operational Calculus.* Oxford, England: Pergamon, 1965.

[28] D. Marcuse, "Loss analysis of single-mode fiber splices," *Bell Syst. Tech. J.*, vol. 56, no. 5, pp. 703–718, 1977.

[29] H. Murata, S. Inao, Y. Matsuda, and T. Kuroha, "Optimum design for optical fiber used in optical cable system," in *Proc. 4th European Conf. Opt. Commun.* (Genoa, Italy), 1978, pp. 242–248.

[30] S. E. Miller and A. G. Chynoweth, Eds., *Optical Fiber Telecommunications.* New York: Academic, 1979.

[31] P. DiVita and U. Rossi, "The backscattering technique: Its field of applicability in fiber diagnostics and attenuation measurements," *Opt. Quantum Electron.*, vol. 12, no. 1, pp. 17–22, 1980.

*

Arthur H. Hartog was born in Toronto, Canada in 1955. He received the B.Sc. degree and Ph.D. degrees from the University of Southampton, Southampton, England.

He has worked in the Optical Fiber Group at Southampton University since 1976, initially as a Research Student and presently as a Research Fellow. His research interests are in propagation studies, and measurement techniques in optical fibers and fiber sensors.

*

Martin P. Gold was born in London, England in 1959. He received the B.A. degree in physics from Oxford University, England in 1980.

In 1980 he joined the Optical Fiber Group in the Department of Electronics, University of Southampton, Southampton, England as a Research Student working for the Ph.D. degree on reflectometry techniques for optical fibers. In October 1983 he was appointed as a Research Fellow at the University of Southampton.