that could and would be mistaken for a flow of particles from point to point in space-time. Of the 136 components we easily pick out one which must correspond to some kind of stress in the x direction since it is covariant with $dx\,dx'$, and therefore with dx^2, in our four-dimensional world. This is evidently the spurious stress that has to be removed from E_1t_1; it has made it appear that particles have been rushing to and fro changing places, whereas all that has been interchanged is their labels. By the same argument three other spurious stresses are removed; and we reach equation (10) without any ambiguity, except that the two i's in the equation are not proved to be the same root of -1.

On the Theory of the Solar Diurnal Variation of the Earth's Magnetism.

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Introduction.

1. The most likely explanation of the solar diurnal magnetic variation (conveniently denoted by S) has for more than a generation seemed to be that afforded by the "dynamo" theory, originally proposed by Balfour Stewart and further developed by Schuster and the writer.* It attributes S to overhead electric currents, induced by convective motion of the air across the earth's magnetic field. The form and intensity of the current-system can be inferred from the spherical harmonic analysis of S; J. Bartels,† on the basis of the writer's analysis, has drawn the current-diagram reproduced in the figure (p. 383). It will be seen that the current-system consists of four circuits, two north and two south of the equator, each flowing round a centre or focus. At the equinoxes, to which the figure refers, the system is approximately symmetrical with respect to the equator. The principal pair of circuits are situated in the sunlit hemisphere, and each circuit of this pair carried 62,000 amperes at sunspot minimum; their foci lie approximately on the 11 hour meridian. The other pair of circuits, each carrying 32,000 amperes, are less intense but more

* For references *cf.* 'Phil. Trans.,' A, vol. 218, p. 1 (1919).

† J. Bartels, "Handbuch exp. Physik," vol. 25 (1928).

extensive, covering about 15 hours in longitude. The equatorial current-intensity, per centimetre measured along the meridian, has an eastward maximum of $2 \, . \, 10^{-5}$ e.m.u. at about 11h, and a westward maximum of 10^{-5} e.m.u. at about 17h.

2. The dynamo theory can give a good account of the general form of this current-system, but meets serious difficulties when it attempts to explain the phase and intensity of the currents. This is because the convective motion which induces the currents, and the total conductivity of the layer in which they flow, are unknown. But if it is assumed that the convective motion is the same as at ground level, it appears that the day current-foci should be on the meridian 13h or 14h, distinctly later than the observed position. Further, the total electric conductivity (or $\int \sigma \, dh$, the integral of the specific conductivity σ throughout the thickness of the layer) must be large, of the order 10^{-5} e.m.u.

3. Until recently no independent check on this estimate of $\int \sigma \, dh$ was available, but radio data now give information concerning the electron-density at great heights, and confirm the existence of a highly conducting layer. But it is still doubtful whether the total conductivity of the layer is so great as 10^{-5}; P. O. Pedersen, in his recent valuable and comprehensive discussion of radio propagation,* concludes (*loc. cit.*, p. 149) that it is only $5 \, . \, 10^{-7}$, or about 20 times too small.

4. The free charges in the upper air seem likely to consist mainly of electrons and singly ionised nitrogen or oxygen molecules. It is convenient to treat the latter as of equal mass, corresponding to a molecular weight of about 30. Symbols for quantities relating to electrons, positive or negative ions, or ions without distribution of sign, will be distinguished by the suffixes e, $+$, $-$, i. Let m be the mass of a free charge, n the number per cubic centimetre, ν the number of collisions which it undergoes per second, e its charge in electromagnetic units. Then we take

$$m_e = 9 \cdot 0 \, . \, 10^{-28}, \quad m_i = 5 \cdot 0 \, . \, 10^{-23}, \quad e = \pm 1 \cdot 59 \, . \, 10^{-20}.$$

The specific conductivity σ of the gas is due to contributions σ_e or σ_i of each individual electron or ion, *i.e.*,

$$(1) \qquad \sigma = n_e \sigma_e + n_i \sigma_i.$$

* P. O. Pedersen, "The Propagation of Radio Waves," Copenhagen, 1927.

In the absence of a magnetic field, approximately

(2) $$\sigma_e = e^2/m_e\nu_e, \quad \sigma_i = e^2/m_i\nu_i,$$

so that the rarer the gas, and the less frequent the collisions, the greater are σ_e and σ_i.

5. Pedersen made an important contribution to the theory of the conductivity by recognising that in a magnetic field the conductivity is different in different directions, its value for directions along the field being the same as if the field were absent, but its value (conveniently denoted by $\sigma_\perp$) for directions transverse to the field being reduced in the ratio

(3) $$K = \nu^2/(\nu^2 + \omega^2),$$

where

(4) $$\omega = eH/m,$$

ω being the angular velocity with which the charge spirals freely round the lines of magnetic force. Since ν and ω are different for ions and electrons, the corresponding ratios $\sigma_\perp/\sigma$ or K are likewise different; we have

(5) $$\sigma_\perp = n_e\sigma_{e\perp} + n_i\sigma_{i\perp} = n_eK_e\sigma_e + n_iK_i\sigma_i = \frac{n_e e^2}{m_e}\frac{\nu_e}{\nu_e^2 + \omega_e^2} + \frac{n_i e^2}{m_i}\frac{\nu_i}{\nu_i^2 + \omega_i^2}$$

so that as ν tends to zero $\sigma_\perp$ tends also to zero* instead of to infinity like σ.

This result is of importance to the theory of S because the currents illustrated in the figure flow transversely to the magnetic meridians over a considerable fraction of their path, and even when the flow is in the magnetic meridian planes it is partly transverse to the magnetic field, since the current-flow is nearly horizontal, while except in low latitudes the magnetic force has a considerable vertical component. Hence the total resistance round any current-line is determined mainly by $\sigma_\perp$, especially when $\sigma_\perp/\sigma$ is small. The conducting layer is therefore effectively limited by the value of $\sigma_\perp$, and this places a hitherto unsuspected limit on the upward extension of the layer: it will appear later that the layer must probably lie approximately between 100 and 170 km. height above the ground (§ 9).

6. Regarding the earth's magnetic field as that of a sphere uniformly magnetised along the axis of rotation,

(6) $$\omega = eH/m = (eH_0/m)(1 + 3\cos^2\theta)^{\frac{1}{2}},$$

* Ross Gunn, 'Phys. Rev.,' vol. 32, p. 133 (1928), has also remarked the same result, apparently without knowledge of Pedersen's work.

where H_0 denotes the equatorial magnetic intensity, and θ denotes the north-polar-distance in angle. Approximately

(7) $$H_0 = 0 \cdot 3 \text{ gauss.}$$

Inserting the values of e, H_0 and m,

(8) $$\omega_e = 5 \cdot 3 \, . \, 10^6 \, (1 + 3 \cos^2 \theta)^{\frac{1}{2}}, \quad \omega_i = 95 \, (1 + 3 \cos^2 \theta)^{\frac{1}{2}}.$$

7. The value of ν varies with the height h in a way which depends on the distribution of absolute temperature T and on the composition of the air; these are not definitely known at great heights. But it is probably not far from the truth to take T = 220° in the stratosphere up to 60 km., and 300° above 80 km., with a uniform transition between these heights; and, further, to assume that convection keeps the composition uniform up to 110 km.; and thereafter ceases. In this case the air will consist almost wholly of N_2 and O_2 up to 200 km., since even if helium be present at 110 km. in the proportion observed near the ground, it will only amount to 2 per cent. at 200 km.; the proportion of oxygen will be 21 per cent. at 110 km., 13 per cent. at 150 km. and 6 per cent. at 200 km., nitrogen being the only other important constituent.

The electrons and ions will be assumed to have speeds v corresponding to thermal equilibrium*; hence above 80 km., where T = 300°,

(9) $$v_e = 1 \cdot 1 \, . \, 10^7 \text{ cm./sec.}, \quad v_i = 4 \cdot 7 \, . \, 10^4 \text{ cm./sec.}$$

The mean free path l will be supposed related to the pressure p (in dynes/cm.^{2}) by the equations

(10) $$l_e = 43/p, \quad l_i = 6/p.$$

The above assumptions concerning the atmosphere enable p to be calculated at any height, and hence, using (9), (10) and

(11) $$\nu = v/l,$$

the values of ν given in Table I have been determined; the table also gives values of K_e and K_i, and of $1 - K_e$, $1 - K_i$, calculated from the values of ν and of ω (*cf.* 8), for the equator and the polar regions.

The above values of ν, and hence of K, differ from those given by Pedersen, because he assumed that T = 220° at all heights in the stratosphere, and that mixing by convection ceases at 12 km. The first assumption is contrary to three independent lines of evidence, which support a higher temperature

* Townsend and Tizard, 'Roy. Soc. Proc.,' A, vol. 88, p. 347 (1913).

Table I.

Height.	60.	70.	80.	90.	100.	110.	120.	130.	140.	150.
ν_e	$3\cdot3 . 10^7$	10^7	$3\cdot3 . 10^6$	$1\cdot1 . 10^6$	$3\cdot5 . 10^5$	$1\cdot2 . 10^5$	$3\cdot8 . 10^4$	$1\cdot3 . 10^4$	$4\cdot4 . 10^3$	$1\cdot5 . 10^3$
ν_i	10^6	$3 . 10^5$	10^5	$3\cdot5 . 10^4$	$1\cdot1 . 10^4$	$3\cdot6 . 10^3$	$1\cdot2 . 10^3$	$4 . 10^2$	$1\cdot3 . 10^2$	$4\cdot7 . 10$
(K_e) eq.	0·975	0·74	0·28	$4 . 10^{-2}$	$4 . 10^{-3}$	$5 . 10^{-4}$	$5 . 10^{-5}$	$6 . 10^{-6}$	$7 . 10^{-7}$	$8 . 10^{-8}$
$1 - (K_e)$ eq.	0·025	0·26	0·72	0·96	1·00	1·00	1·00	1·00	1·00	1·00
(K_e) pole	0·907	0·41	0·08	$1\cdot1 . 10^{-2}$	$1\cdot1 . 10^{-3}$	$1\cdot3 . 10^{-4}$	$1\cdot3 . 10^{-5}$	$1\cdot5 . 10^{-6}$	$1\cdot7 . 10^{-7}$	$2 . 10^{-8}$
$1 - (K_e)$ pole	0·093	0·59	0·92	0·99	1·00	1·00	1·00	1·00	1·00	1·00
(K_i) eq.	1·00	1·00	1·00	1·00	1·00	1·00	0·99	0·94	0·65	0·20
$1 - (K_i)$ eq.	10^{-8}	10^{-7}	10^{-6}	$8 . 10^{-6}$	$8 . 10^{-5}$	$8 . 10^{-4}$	$6 . 10^{-3}$	$6 . 10^{-2}$	0·35	0·80
(K_i) pole	1·00	1·00	1·00	1·00	1·00	1·00	0·98	0·81	0·32	0·06
$1 - (K_i)$ pole	$4 . 10^{-8}$	$4 . 10^{-7}$	$4 . 10^{-6}$	$3 . 10^{-5}$	$3 . 10^{-4}$	$3 . 10^{-3}$	$2\cdot5 . 10^{-2}$	0·19	0·68	0·94

above about 60 km., as here supposed; they are (i) the observations of meteor trails as interpreted by the theory of Lindemann and Dobson,* (ii) the observations of abnormal sound-propagation to great distances,† (iii) the theoretical calculation of upper air temperatures, taking into account the known layer of ozone in the stratosphere.‡ The second assumption is contrary to the evidence of mixing afforded by the contortions usually observed in long-enduring meteor-trails, and notably in the one, lasting for nearly 2 hours, observed by Mr. J. E. Clarke§ and others, on February 22, 1909. This trail, 150 miles long, sloping slightly downwards from a height of 90 km. above a point 45 miles 26° W. of S. from Beachy Head to a height of 77 km. above a point, 87 miles 22° E. of S. from Start Point, became distorted in such a way as to indicate high level winds showing the following remarkable differences of intensity and direction at heights of 50 to 55 miles (80 to 90 km.).

Approximate height.		Speed. miles/hour.	Mean direction.
Miles.	Km.		
55	88·5	111	NE by E
54		135	NE by N
53		135	NW
52		175	WNW
51·5		187	W by N
50	80·5	30	S
50		170	E

Such long-enduring trails are rare, but less exceptional instances are available which give similar evidence of intense non-uniform winds, of the order of 100 miles per hour, at heights extending up to 130 km. These winds are probably due to large temperature variations set up in the atmosphere, not only as between day and night, but also from day to day and from place to place. Large temperature variations will arise from variations (such as are actually observed) in the amount of ozone, which has so great an influence on the temperature of the upper stratosphere; and also from the considerable variations, up to 100 per cent., in the sun's ultra-violet radiation, as observed by Pettit.||

* 'Roy. Soc. Proc.,' A, vol. 102, p. 411 (1922).

† F. J. W. Whipple, 'Nature,' vol. 111, p. 187 (1923).

‡ Gowan, 'Roy. Soc. Proc.,' A, vol. 120, p. 655 (1928).

§ 'Q. J. R. Met. Soc.,' vol. 39, p. 115 (1913).

|| 'Proc. Nat. Acad. Sci.,' vol. 130, p. 380 (1927).

Such strong and non-uniform winds must promote mixing of the air up to heights exceeding 100 km., and it seems likely that they are capable of effectively counteracting the tendency towards diffusive separation of the atmospheric constituents according to their molecular weights ; this process is extremely slow, at least up to 100 km. It is here assumed that the mixing ceases at 110 km. Pedersen's assumption that it ceases at 12 km. results in a rapid increase in the proportion of helium above 90 km., until at 130 km. it is 90 per cent. of the whole air. This is rendered improbable by the fact that no spectral lines that can be attributed to helium are observed in auroræ, although the excitation would seem sufficient to produce such lines were helium present.

The low temperature assumed by Pedersen renders the upward decrease of pressure too rapid until heights are reached where helium is an important constituent, and thereafter the decrease of pressure is too slow. Hence his values of ν at 100 km. are only one-third those given in Table I, while at 150 km. they are 10 times as great. The effect of these differences on the values of K is easily inferred.

8. The number of electrons and ions at great heights is uncertain. Pedersen gives the following values (*loc. cit.*, Appendix, p. 10, fig. IX, 6), based partly on radio measurements and partly on a theory of ionisation and recombination ; they refer to noon in summer, at some latitude apparently not stated, which will here be assumed to be that of the equator.

Table II.

Height.	60.	70.	80.	90.	100.	110.	120.	130.
n_e	<1	<1	<1	1	$3 . 10^3$	$1{\cdot}6 . 10^4$	$5 . 10^5$	$7 . 10^5$
n_i	$7 . 10^3$	$6 . 10^3$	$3 . 10^3$	$5 . 10^4$	$2 . 10^6$	10^7	$8 . 10^6$	$4 . 10^6$
Height.	**140.**	**150.**	**160.**	**170.**	**180.**	**190.**	**200.**	
n_e	$5 . 10^5$	$4 . 10^5$	$2 . 10^5$	$1{\cdot}3 . 10^5$	$5 . 10^4$	$3 . 10^4$	$1{\cdot}2 . 10^4$	
n_i	$2 . 10^6$	$7 . 10^5$	$3 . 10^5$	$1{\cdot}3 . 10^5$	$5 . 10^4$	$3 . 10^4$	$1{\cdot}2 . 10^4$	

According to this table the electron density first becomes important at about 100 km. ; below this the electrons are supposed to attach themselves to oxygen or water molecules as soon as they are formed. Above 170 km. the amount of such recombination is supposed negligible, owing to the low density, and $n_e = n_i = n_+$, $n_- = 0$. At all heights the air is practically neutral, *i.e.*,

(12) $$n_+ = n_- + n_e, \quad n_i = n_+ + n_- = 2n_+ - n_e.$$

If each of the above values of n_e and n_i is taken to apply throughout a 10 km. layer (10^6 cm.) the total numbers (N_e, N_i) of each, per square centimetre column, are approximately

(13) $$N_e = 2{\cdot}6 \,.\, 10^{12} \quad , \quad N_i = 2{\cdot}8 \,.\, 10^{13}.$$

9. On substituting the above values of n_e, n_i, K_e, K_i into (2)–(4), we obtain the following values of $n_e\sigma_e$, $n_i\sigma_i$ and σ, both along and transverse to the magnetic field at the equator. The contributions of the layers below 100 km. are negligible in comparison with those of the higher strata, and are therefore not indicated.

Table III.

Height.	100.	110.	120.	130.	140.	150.
$n_e\sigma_e$	$2{\cdot}4 \,.\, 10^{-15}$	$3{\cdot}7 \,.\, 10^{-14}$	$3{\cdot}7 \,.\, 10^{-12}$	$1{\cdot}5 \,.\, 10^{-11}$	$3{\cdot}2 \,.\, 10^{-11}$	$8 \,.\, 10^{-11}$
$n_i\sigma_i$	10^{-15}	$1{\cdot}6 \,.\, 10^{-14}$	$3{\cdot}8 \,.\, 10^{-14}$	$5{\cdot}6 \,.\, 10^{-14}$	$9 \,.\, 10^{-14}$	$8 \,.\, 10^{-14}$
σ	$3 \,.\, 10^{-15}$	$5 \,.\, 10^{-14}$	$3{\cdot}7 \,.\, 10^{-12}$	$1{\cdot}5 \,.\, 10^{-11}$	$3{\cdot}2 \,.\, 10^{-11}$	$8 \,.\, 10^{-11}$
$n_e\sigma_{e\perp}$	10^{-17}	$2 \,.\, 10^{-17}$	$2 \,.\, 10^{-16}$	$9 \,.\, 10^{-17}$	$2 \,.\, 10^{-17}$	$6 \,.\, 10^{-18}$
$n_i\sigma_{i\perp}$	10^{-15}	$1{\cdot}6 \,.\, 10^{-14}$	$3{\cdot}8 \,.\, 10^{-14}$	$5{\cdot}3 \,.\, 10^{-14}$	$5{\cdot}9 \,.\, 10^{-14}$	$1{\cdot}6 \,.\, 10^{-14}$
$\sigma_\perp$	10^{-15}	$1{\cdot}6 \,.\, 10^{-14}$	$3{\cdot}8 \,.\, 10^{-14}$	$5{\cdot}3 \,.\, 10^{-14}$	$5{\cdot}9 \,.\, 10^{-14}$	$1{\cdot}6 \,.\, 10^{-14}$

According to this table the conductivity along the magnetic field is almost wholly due to the electrons, and increases upwards to at least 150 km. The transverse conductivity, on the other hand, is almost wholly due to the ions, and attains its maximum value at about 135 km., and does not extend much above 150 km.

The estimate of $\int \sigma_\perp \, dh$ afforded by the table is found to be $2 \,.\, 10^{-6}$ e.m.u. Pedersen's estimate is $0{\cdot}5 \,.\, 10^{-6}$ e.m.u. or one-quarter the value here found, the difference resulting from his different assumptions concerning the temperature and composition of the stratosphere.

The value of $\int \sigma_\perp \, dh$ here found is of the same order as that (10^{-5}) required by the dynamo theory, though rather smaller, *i.e.*, in the ratio one-fifth. The difference is probably within the present limits of error of the data for n_e and n_i; more accurate knowledge later may either narrow or widen the gap between the two estimates of the conductivity. The estimated conductivity required by the dynamo theory is capable of modification by supposing that the convective motion in the conducting layer is different from that observed near the ground.

9. R. Gunn* has recently pointed out that in the layers where ν/ω is small, that is, where a free charge can execute many revolutions in its spiral paths between successive collisions, the spiral motion makes the charge equivalent to a small magnet opposed to the field, so that the ionised medium is diamagnetic. The contribution of each free charge to the intensity of magnetisation of the medium is the same for electrons and ions when ν/ω is very small, being $k\mathrm{T}/\mathrm{H}$, where k is Boltzmann's constant $(1\cdot37 . 10^{-16})$; when ν/ω is not small, it is easily shown that the contribution is reduced in the ratio

$$\omega^2/(\nu^2 + \omega^2) = 1 - \mathrm{K};$$

thus the fractional contributions of a free charge to the transverse conductivity $\sigma_\perp$ and to the diamagnetisation are complementary. Table I shows that the electrons contribute almost their full quota to the diamagnetism from 90 km. upwards, so that the conducting layer is rendered diamagnetic by the electrons; the ions only begin to contribute fully to the diamagnetism at about 150 km., near where the conducting layer ceases (according to the values of n_e and n_i in Table II).

Gunn has suggested that this diamagnetic layer of the upper atmosphere, of intensity

$$(14) \qquad (k\mathrm{T}/\mathrm{H}) \int \{(1 - \mathrm{K}_e)\, n_e + (1 - \mathrm{K}_i)\, n_i\}\, dh = (k\mathrm{T}/\mathrm{H})\mathrm{N}_d,$$

is responsible for the diurnal magnetic variations, owing to the daily variation of the magnetic field of the layer, as n_e and n_i vary. He has partly worked out the resulting magnetic variations, assuming a probable variation of N_d with the sun's zenith distance, and has shown that they are in fairly good qualitative agreement with the observed S. In another paper I have developed the theory further, by means of spherical harmonic analysis, and have confirmed his general conclusion. In order to bring the theory into approximate agreement with observation, only one unknown magnitude has to be assumed, namely, the number N_d defined by the above equation; N_d may be called the number of diamagnetically effective free charges per square centimetre column. According to Gunn, its maximum value, occurring at the equator at noon, must be $1\cdot5 . 10^{17}$; my own estimate, which seems likely to be more accurate, is $5 . 10^{16}$.

* 'Phys. Rev.,' vol. 32, p. 133 (1928).

This value, which includes practically all the electrons and a fraction of the ions, namely, those above about 145 km., is far larger than that to be deduced from Pedersen's discussion, according to which N_e is about $2 \cdot 6 . 10^{12}$, and $(N_i)_d$, or $\int (1 - K_i)\, n_i\, dh$, is about 10^{12}, so that N_d is less than $4 . 10^{12}$. The discrepancy between the ionisation as estimated by Pedersen, and that required by the diamagnetic theory of S, is far more extreme than in the case of the dynamo theory, the ratio (required N/estimated N) being 10^4 instead of about 10.

10. Pedersen's estimates of n_e and n_i seem likely to be more accurate for heights up to about 150 km. (that is, within the conducting layer) than at greater heights. Since his discussion was written, E. V. Appleton has found that in the early morning during the three hours before sunrise, when the ionisation in the conducting layer has become much reduced by recombination, 400 metre waves that are usually reflected in this layer penetrate it and pass upwards till they meet another reflecting ionised layer at about 250 km. height. This suggests that n_e and n_i at heights above 150 km. may be much greater than Pedersen supposes, though the change in his figures would need to be revolutionary if S is to be attributed to the diamagnetism of the upper atmosphere. It will be shown, however, that the ions in the diamagnetic region can contribute to S far more effectively in another way, and that N_d must be of the order $2 . 10^{14}$ at most.

11. Consider the motion of a free charge in the presence of a uniform magnetic field of intensity H. Let the axis of z be taken along H. Suppose that a constant force F acts upon the charge, and take the x axis in the plane of F and H, and so that the x-component (X) of F is positive; let the third axis $0y$ be chosen so that the axes form a right-handed system. Then $F = (X, 0, Z)$.

The equations of motion of the particle are

(15) $$m\ddot{x} = X + eH\dot{y},$$

(16) $$m\ddot{y} = -eH\dot{x},$$

(17) $$m\ddot{z} = Z,$$

of which the integrals are

(18) $$\dot{x} = \dot{x}_0 \cos \omega t + \dot{y}_0 \sin \omega t - (X/eH) \sin \omega t,$$

(19) $$\dot{y} = -\dot{x}_0 \sin \omega t + \dot{y}_0 \cos \omega t - (X/eH)(1 - \cos \omega t),$$

(20) $$\dot{z} = \dot{z}_0 + (Z/m)t,$$

where the suffix 0 refers to the initial values.

The motion along the field is unaffected by H, but the mean value of $\dot{x}$ is zero, despite the force X; this is why $\sigma_{\perp}$ tends to zero with ν/ω. But the force X renders the mean transverse velocity $\dot{y}$, which would otherwise be zero, equal to

$$-(\mathrm{X}/e\mathrm{H}). \tag{21}$$

This mean transverse velocity will be called the drift-velocity V of the free charge.

12. If the free charge forms part of a gas, and makes ν collisions per second with other particles, after which its velocity takes a new direction at random, it is easy to show that the mean y-velocity is

$$(1-\mathrm{K})(\mathrm{X}/e\mathrm{H}), \tag{22}$$

where K is defined by (3). Thus the less the contribution (Kσ) made by the charge to $\sigma_{\perp}$, the greater the mean drift-velocity which it experiences, and *vice versa*.

13. The drift-motions of the charges give rise to a current of intensity i, where

$$i = (1/\mathrm{H})\{(1-\mathrm{K}_i)(n_+\mathrm{X}_+ + n_-\mathrm{X}_-) + (1-\mathrm{K}_e)\,n_e\mathrm{X}_e\}, \tag{23}$$

assuming that the positive and negative ions are of equal mass.

14. In the ionised atmosphere the light electrons tend to rise above the heavier ions, but this sets up an electrostatic field which prevents all but a very slight separation, so that the medium remains practically neutral, in accordance with (12). The field is smaller, the greater the proportion of negative ions to electrons, because the field due to the rise of the electrons tends to depress the negative ions, and therefore to produce a compensating opposing field.

When the conditions are steady, the equations of partial pressure for the three kinds of free charge are as follows,

$$\frac{\partial p_+}{\partial h} = -n_+(m_+g - \mathrm{E}e), \tag{24}$$

$$\frac{\partial p_-}{\partial h} = -n_-(m_-g + \mathrm{E}e), \tag{25}$$

$$\frac{\partial p_e}{\partial h} = -n_e(m_eg + \mathrm{E}e), \tag{26}$$

where $p = kn\mathrm{T}$, k being Boltzmann's constant $1{\cdot}37 \,.\, 10^{-16}$, and E denotes the electric field, reckoned positive when upwards. We may write $m_+ = m_- = m_i$, and ignore m_e, in (24) to (26). By (12)

(27) $$\frac{\partial p_+}{\partial h} = \frac{\partial(p_- + p_e)}{\partial h},$$

so that

(28) $$n_+ (m_i g - \mathrm{E}e) = n_- (m_i g + \mathrm{E}e) + n_e \mathrm{E}e = n_- m_i g + n_+ \mathrm{E}e,$$

or

(29) $$\mathrm{E}e = \tfrac{1}{2} (n_e/n_+) m_i g.$$

The vertically downward forces F on the free charges are therefore given by

(30) $$\mathrm{F}_e = \frac{n_e}{n_i + n_e} m_i g, \quad \mathrm{E}_+ = \frac{n_i}{n_i + n_e} m_i g, \quad \mathrm{F}_- = \frac{n_i + 2n_e}{n_i + n_e} m_i g.$$

15. Regarding the earth's magnetic field as that of a sphere uniformly magnetised along the axis of rotation, the downward vertical and northward horizontal components of H at north-polar-distance θ are

(31) $$2\mathrm{H}_0 \cos\theta, \quad \mathrm{H}_0 \sin\theta,$$

so that

(32) $$\mathrm{H}^2 = \mathrm{H}_0^2 (1 + 3\cos^2\theta).$$

The z axis, as defined in § 11, is therefore northward and downward, while the x axis is southward and downward, and the y axis is horizontal and westward. The x components of the forces F on the free charges are equal to

(33) $$\mathrm{F} \sin\theta/(1 + 3\cos^2\theta)^{\frac{1}{2}}.$$

Hence, by (23, 30, 33), the drift-current intensity is given by

(34) $$i = -\frac{m_i g \sin\theta}{\mathrm{H}_0 (1 + 3\cos^2\theta)} \left\{ (1 - \mathrm{K}_i) \frac{n_i^2 + n_i n_e - n_e^2}{n_i + n_e} + (1 - \mathrm{K}_e) \frac{n_e^2}{n_i + n_e} \right\}$$
$$\equiv -\frac{m_i g \sin\theta}{\mathrm{H}_0 (1 + 3\cos^2\theta)} n'.$$

Since i is essentially negative, the drift-current is everywhere eastward.

16. The drift-velocity V of a free charge is greatest at the equator, where $\mathrm{X} = \mathrm{F}$ and H has its minimum value. At heights where $\mathrm{K} = 1$, and $n_e = n_i$, $\mathrm{V}_e = \mathrm{V}_i = \frac{1}{2} m_i g/e\mathrm{H}$, which at the equator is 5·1 cm./sec. If $\mathrm{K} = 1$ but $n_e < n_i$, V_e lies between 0 and 5·1, while V_+ lies between 5·1 and 10·2, and V_- lies between 10·2 and 15·3 cm./sec. For given values of K, n_e, n_i, these velocities decrease from the equator to the pole in proportion to the factor $\sin\theta/(1 + 3\cos^2\theta)$, the value of which is given for various latitudes in the following table.

Table IV.

Latitude	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
θ	90°	80°	70°	60°	50°	40°	30°	20°	10°	0°
$\sin\theta/(1+3\cos^2\theta)$	1	0·904	0·696	0·495	0·342	0·233	0·153	0·094	0·044	0

17. The equatorial values of $(1 - K_e)$ and $(1 - K_i)$ are practically unity from 160 km. upwards, so that from Table I, II the following table giving the value of n', corresponding to Pedersen's data for n_e and n_i, can be constructed.

Table V.

Height (km.)	100	110	120	130	140	150
$(1-K_e)\frac{n_e^2}{n_i+n_e}$	4	26	$3 \cdot 10^4$	10^5	10^5	$1{\cdot}5 \cdot 10^5$
$(1-K_i)\frac{n_i^2+n_in_e-n_e^2}{n_i+n_e}$	160	$8 \cdot 10^3$	$5 \cdot 10^4$	$2 \cdot 10^5$	$7 \cdot 10^5$	$4 \cdot 10^5$
n'	164	$8 \cdot 10^3$	$8 \cdot 10^4$	$3 \cdot 10^5$	$8 \cdot 10^5$	$5{\cdot}5 \cdot 10^5$

Height (km.)	160	170	180	190	200	
$(1-K_e)\frac{n_e^2}{n_i+n_e}$	$8 \cdot 10^4$	$6 \cdot 10^4$	$2{\cdot}5 \cdot 10^4$	$1{\cdot}5 \cdot 10^4$	$6 \cdot 10^3$	
$(1-K_i)\frac{n_i^2+n_in_e-n_e^2}{n_i+n_e}$	$2 \cdot 10^5$	$6 \cdot 10^4$	$2{\cdot}5 \cdot 10^4$	$1{\cdot}5 \cdot 10^4$	$6 \cdot 10^3$	
n'	$3 \cdot 10^5$	$1{\cdot}2 \cdot 10^5$	$5 \cdot 10^4$	$3 \cdot 10^4$	$1{\cdot}2 \cdot 10^4$	

The resulting value of $N'\left(\equiv \int n'\,dh\right)$ is $2{\cdot}3 \cdot 10^{12}$, of which one-quarter is due to the electrons; it is slightly less than the estimate of N_d $(4 \cdot 10^{12}$—§ 9) the effective number of diamagnetic free charges.

This value of N′, by (34), shows that at the equator at noon, if Pedersen's values of n_e and n_i are correct, $I\left(\equiv -\int i\,dh\right) = 4 \cdot 10^{-7}$ e.m.u. This is about 1/50 of the equatorial current-intensity associated with S, so that the free charges in and above the conducting layer affect the earth's field far more by their drift-motion than by their diamagnetism. It will be shown that if $N' = 2 \cdot 10^{14}$, or about 100 times the number according to Pedersen, the drift-currents are capable of producing a diurnal magnetic variation very similar to S.

18. In both the dynamo and diamagnetic theories of S, it is supposed that

n_e and n_i, or the quantities σ, N_d which depend on them, are greatest at noon at the equator (considering the equinoctial season, for simplicity) and decrease with increasing zenith distance of the sun. The same reasonable assumption applies to N′, on which the total drift-current intensity I depends ; this may be indicated by writing N′ as N′ (θ, φ), where φ denotes local time, or longitude measured eastward from the midnight meridian. Hence

$$I(\theta, \phi) = -\int i\,dh = \frac{m_i g}{H_0} N'(\theta, \phi) \frac{\sin\theta}{1 + 3\cos^2\theta}.$$

Let I (θ) denote the mean value of I (θ, φ) round a circle of colatitude θ, *i.e.*,

$$2\pi I(\theta) = \int I(\theta, \phi)\, d\phi.$$

Let

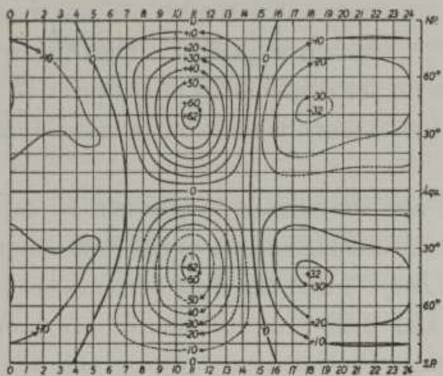
$$I(\theta, \phi) = I(\theta) + \Delta I(\theta, \phi).$$

The term I (θ) denotes the mean eastward total drift-current intensity round the circle of colatitude θ ; this part satisfies the equation of continuity, and affects the daily mean value of the north and vertical components of H, but does not contribute to S. The term ΔI (θ, φ) denotes the difference between the local and the mean current-intensity in this latitude ; this part will not satisfy the equation of continuity, but will tend to produce an accumulation of charge at the rate $-dI(\theta, \phi)/a \sin\theta d\phi$, where a denotes the earth's radius, or rather that of the ionised layer ; the rate of accumulation of charge is here reckoned per square centimetre area of a vertical column. Since I increases towards noon, and decreases after noon, positive charge tends to accumulate over the post meridiem hemisphere, and negative over the ante meridiem hemisphere.

Any accumulation of charge would set up potential differences opposing the accumulation, and tending to equalise the drift-current in each latitude. But owing to the long free paths of the charges in the drift-current layer, a potential gradient along the circles of latitude, that is, transverse to the magnetic meridians, will have little direct effect in opposing the accumulations ; instead of this it would tend to make both the positive and negative charges move upwards* or downwards normal to the magnetic field. But the medium is highly conducting along the lines of force, and electricity can flow freely in this direction to or from the conducting layer below ; the drift-current layer is, as it were, connected to the conducting layer by a sheaf of laterally insulated conductors lying along the lines of force. Hence no serious accumulation of

* Upwards over the sunlit hemisphere.

charge can be set up, because this would produce large potential gradients not only in the drift-current layer, but also in the conducting layer below. Thus the drift-currents would produce closed electrical circuits partly traversing the conducting layer; charge would accumulate until the potential gradient just enabled the current, the amount of which is determined by the value of N′ in the drift-current layer, to overcome the resistance of the remainder of the circuit. The resultant current-system would resemble that shown in the figure, except that the current foci would be on the noon meridian, or slightly later, where N′ has its maximum value.



Over the night hemisphere the drift-current $\Delta I(\theta, \phi)$ is westward, as in fig. 1, but the whole current $I(\theta, \phi)$ is everywhere eastward. Hence $I(\theta)$ must at least equal the maximum westward value of $\Delta I(\theta, \phi)$, so that the noon current $I(\theta) + \Delta I(\theta, \phi)$ must be not less than the numerical sum of the maximum eastward and westward current-intensities in the day and night hemispheres; this is approximately $3 \cdot 10^{-5}$ e.m.u. (§ 1). The value of N′ required to make $I_{max.} = 3 \cdot 10^{-5}$ is $2 \cdot 10^{14}$, or about 1/250 times the number required by the diamagnetic theory. It therefore seems clear that the diamagnetism can account for at most less than 1 per cent. of S.

19. The positive ions will move eastward and the negative ions and electrons will move westwards, with velocities lying between 5 and 15 cm./sec. It may be shown that these opposing drifts will scarcely affect the distribution of the various kinds of charge, when the processes of ionisation and recombination are taken into account. Consider, for example, the change of n_+; the equation is

$$\frac{\partial n_+}{\partial t} = \beta - \alpha_{+-} n_+ n_- - \alpha_{+e} n_+ n_e - \frac{\partial (n_+ V_+)}{a \sin \theta \, \partial \phi},$$

where β denotes the rate of formation of ions per cubic centimetre, and the α's are coefficients of recombination, while the last term represents the rate of accumulation owing to the drift of the ions; V_+ is independent of ϕ. We may replace $\partial/\partial t$ by $\partial/\partial\phi$, reckoning ϕ in time-units (seconds); if ϕ is thus reckoned also in the last term, where as written it is supposed reckoned in circular measure, we must multiply the last term by $86{,}400/\pi$. Hence the equation may be written

$$\left(1 + \frac{8\cdot 64\ .\ 10^4}{2\pi a \sin\theta} V_+\right)\frac{\partial n_+}{\partial\phi} = \beta - \alpha_{+-}n_+n_- - \alpha_{+e}n_+n_e.$$

Since V_+ is of order 10, while $a = 6\cdot 8\ .\ 10^8$, the first factor on the left differs from 1 by only $2\ .\ 10^{-4}$, which is negligible. Thus the drift has only a minute proportional influence on the distribution of the positive ions (and likewise for the negative ions and electrons), which will be most numerous on or near the noon meridian.

20. The type of S and its current-system can be explained either by the dynamo theory, or by the diamagnetic theory of Ross Gunn, or by the hypothesis (the "drift-current theory") that the drift-currents are the main cause.

The diamagnetic and drift-current theories are more definite than the dynamo theory, because they depend only on the distribution of ions and electrons in the upper atmosphere, while the dynamo theory involves this and also the unknown convective motion of the upper air. The two former theories imply that the S-current foci should be on or slightly after the noon meridian, instead of 1 or $1\frac{1}{4}$ hour before, as the observations indicate. The dynamo theory is not committed to any definite prediction of the phase, so long as we remain ignorant of the convective motion in the upper atmosphere; but in so far as the attempt has been made, first by Schuster and then, rather differently, by the writer, to connect these motions with those observed in the lower atmosphere, the result has not been entirely favourable to the dynamo theory. Schuster considered both the diurnal and semidiurnal atmospheric motions as indicated by the barometer, and found that, taking self-induction into account, the predicted variations were from 2 to $2\frac{3}{4}$ hours later than the observed; the writer, considering only the fundamental semidiurnal oscillation, found a similar phase difference between the prediction and observations. The discrepancy is therefore about twice as great as on the drift-current hypothesis.

If, as I have recently suggested, the convective motion is an independent one associated with a large diurnal variation of temperature in the upper atmosphere, and if the corresponding pressure variation is not complicated by resonance, but has its maximum at the same time as the temperature, then the current foci for the circuit which is eastward at the equator should be on or after the 18h meridian. Thus the phase of S constitutes a real difficulty in all the three theories, and possibly most of all for the dynamo theory.

It may, however, be possible to explain the phase of S by the dynamo theory, if it can be shown that there is an inversion or large change of phase of the upper-atmospheric diurnal convection. Alternatively, S may be mainly due to drift-currents, but also partly to dynamo-action, the latter being called upon to account for the advance of the current-foci from noon to about 11h; this hypothesis likewise requires a complete or partial inversion of the high-level diurnal convection. The second of these theories may be called the "drift + dynamo" hypothesis. Because of the definite small discrepancy of phase, it does not seem possible to account for S by drift-currents alone.

21. The ultimate choice between the various proposed theories will depend largely on the determination of the distribution of ions and electrons in the upper atmosphere. The drift-current and diamagnetic theories are concerned mainly with the diurnal variation in this distribution; they would be unaffected by a layer in which the electron or ion content, however great, was constant throughout the day and night. The dynamo theory, on the other hand, would be affected by an addition of conductivity uniform throughout the 24 hours; however, the radio evidence suggests that in the layer, from 100 to about 170 km., where the main conductivity exists, the free charges are much reduced by the recombination during the night.

Since the number of free charges cannot be negative, the minimum values of N_d or N' at the equator at noon required to explain S by the drift-current or diamagnetic theories are equal to the difference between the midnight and noon values of N; as has been seen, they are approximately $5 . 10^{16}$ and $2 . 10^{14}$. Since N_d and N' must be nearly equal, by their definition, it is clear that if N_d were $5 . 10^{16}$, the drift-currents would be great enough to produce a diurnal magnetic variation 250 times as intense as is observed. This consideration is fatal to the diamagnetic theory, despite its attractive character from a qualitative standpoint. It seems possible to conclude that the diamagnetism of the outer atmosphere makes only an insignificant contribution to S.

Further, unless we are to admit the unlikely hypothesis that S is due to two or more independent causes producing effects which nearly balance one another, it seems possible to conclude that the difference between the noon maximum and night minimum of N' on the equator cannot exceed $2 . 10^{14}$.

The diamagnetic theory having been eliminated, the choice seems narrowed down to the dynamo theory, or the "drift + dynamo" theory. The latter can be excluded if it can be shown that in the actual atmosphere the difference between the noon maximum and night minimum of N′ on the equator is much smaller than $2 \cdot 10^{14}$ (*e.g.*, only one-tenth as great); if, however, this difference is shown to be about equal to $2 \cdot 10^{14}$, the drift theory will be substantially confirmed,* but the dynamo theory must probably still be called upon to account for the advance of phase. The dynamo theory will remain a possible explanation either of the major part of S, or of an appreciable fraction of S (in conjunction with the drift-currents as the major cause), unless both the conductivity of the layer, and the phase or intensity of the diurnal convection, can be proved inadequate.

22. Besides setting an upper limit to the diurnal variation of N′, the magnetic observations place a restriction upon its mean value, because the latter affects the part of the earth's mean magnetic field which is due to external causes. The various spherical harmonic analyses of the field have shown that the external portion is only a small fraction of the whole, too small, indeed, to be inferred at all accurately from the magnetic data. This implies that the earth's magnetic field above the ionised atmosphere is nearly the same as if the atmosphere were absent; in this respect the earth is very different from the sun, whose atmosphere almost completely imprisons the internal magnetic field, so that no lines of force extend beyond the drift current layer existing in and below the reversing layer.†

According to L. A. Bauer's analysis of the earth's field, for 1922, the part of the field of external origin is directed northwards and upwards (in the northern hemisphere), the horizontal component at the equator being about 450γ; this corresponds to a current-intensity I in the atmosphere of $5 \cdot 4 \cdot 10^{-4}$ e.m.u., or, if the currents are supposed to be "drift-currents," to a mean equatorial value $6 \cdot 6 \cdot 10^{15}$ for N′. This may be taken as an upper limit of the mean value of N′ round the equator.

* In this case the general discussion of the current-system given in § 18 must be supplemented by a mathematical investigation.

† Chapman, 'M.N.R.A.S.,' vol. 89, p. 57 (1928).