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ON THE THERMODYNAMICS OF ANGULAR PROPULSION

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ABSTRACT

In a recently published article by Polihronov and Straatman (Phys. Rev. Letters 109, 054504, 2012), the thermodynamics of angular propulsion was presented and a theoretical model of the energy transfer was proposed. This article will show that the theoretical model leads to the most basic element of a radial inflow device. It is shown that Euler's work equation reduces to the same theoretical result for this case. The system is then studied as a self-governed device moving in a medium posing external resistance. It is observed that the output power from the device exhibits a peak at a certain characteristic value of its peripheral velocity. In the presence of resistance or loading, the system has motion, characterized by the requirement of pre-rotation exhibiting maximum power output and a terminal state. The points of equilibrium and operational thresholds of the motion are discussed, accompanied by a theoretical model of the presented dynamical system. The presented angular propulsion theory is then utilized to provide better understanding of phenomena taking place in vortex tubes.

INTRODUCTION

In a recently published article [1], it was shown from first-principles thermodynamics that angular propulsion is a phenomenon able to drive fluid vortex rotation. An understanding of the physics of this phenomenon was achieved through simplification of the vortex flow to a discrete, uniformly rotating, radial adiabatic conduit, as shown in Fig 1. Radial temperature separation between the center and periphery of the vortex is due to radial flow and was derived to be [1]:

$$\Delta T = \frac{c^2}{c_p} \quad . \qquad (1)$$

This is the temperature separation observed in the stationary reference frame; it is a result of deceleration and decompression of the fluid mass, which is forced to move from the vortex periphery towards the vortex center by a radial pressure difference, which operates against an induced centrifugal gravitational potential. Cooling at the vortex center is only possible for compressible fluids, since they have the innate capability to store internal energy that can be released as thrust or propulsion to the rotating system. Deceleration or loss of kinetic energy alone is not sufficient to explain the magnitude of energy transfer that occurs in the rotation of the above system. Both the kinetic energy of the gas and the work stored as internal energy are invested into the rotation as propulsion. This is the physical phenomenon of angular propulsion through which the confined vortex operates, and the associated theory [1] completely reconciles the energy transfer and temperature separation that occurs.

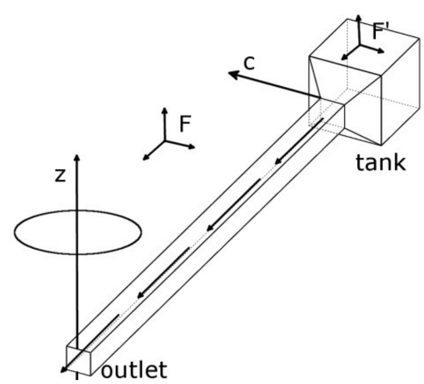

Figure 1 Schematic of an angular propulsion engine

The goal of this article is to revisit the propulsion physics of the flow system of Fig. 1 and to show that this is the simplest system exhibiting the fundamental laws which govern rotational cooling and its corresponding propulsion in radial inflow turbines. Another goal is to examine the behaviour of the flow system as an angular propulsion engine, exhibiting operational thresholds, peak output power and terminal state when it moves in a resistive medium. The points of equilibrium and output power characteristics are discussed, accompanied by a theoretical model of the presented dynamical system. Finally, the concept of rotational cooling is revisited with respect to its application in the Ranque-Hilsch vortex tube.

NOMENCLATURE

T	[K]	Total temperature of the gas
c	[m/s]	Peripheral tangential velocity
c_p	[J/kg.K]	Isobaric heat capacity of the gas
ΔT	[K]	Temperature separation
ΔP	[Pa]	Pressure difference
P_T	[W]	Thrust power output
P_R	[W]	Power, needed to overcome resistance to rotation
c_S	[-]	Unstable equilibrium
c_T	[-]	Terminal state
F, F'	[-]	Stationary/moving frame of reference
$\dot{m}$	[kg/s]	Mass flux
ω	[rad/s]	Angular velocity
r	[m]	Radial position
v	[m/s]	Tangential velocity
E_T	[J]	Propulsion energy
h	[J]	Total enthalpy

ELEMENTAL RADIAL PASSAGE

In a classroom course of engineering physics [2] a radial turbine passage is generally analyzed through the application of energy conservation. The work extracted from the flow in the passage is then given by the turbine equation of Euler

$$h_2 - h_1 = c_p (T_2 - T_1) = \omega (r_2 v_2 - r_1 v_1) \quad , \qquad (2)$$

where $r_1 < r_2$. The delivered power is

$$P_T = \omega \dot{m} (r_2 v_2 - r_1 v_1) \quad . \qquad (3)$$

If we define

$$c \equiv \omega r_2 = v_2$$

and set r_1=0, then (2) is reduced to the simple expression (1), while the formula for the delivered power simplifies to

$$P_T = \dot{m} c^2 \quad . \qquad (4)$$

Eqs. (4) and (1) thus arise from Euler's turbine equation (2) and present a case with maximum output of propulsion power and its corresponding temperature drop, respectively. For a fixed value of c, they decrease if the exhaust radial position r_1 is non-zero, since this leads to the subtraction of a nonzero term in the right-hand side of (2) and (3).

The expressions (1) – (4) are thus fundamental relations, describing the physics of radial flow in a rotating adiabatic conduit where all losses have been neglected. Equation (1) is a limiting case of Euler's work equation (2) showing the upper limit of rotational cooling, while Eq. (4) is the upper limit of the thrust power given in Eq. (3).

Once the work equation of Euler has been derived, the analysis of the flow passage of a radial inflow turbine in an engineering textbook is typically devoted to the description of the loss processes rather than the basic underlying physics of angular propulsion [3]. Such engineering analysis comprises theories and estimates of stator passage losses; friction losses in a blade passage as well as nozzle, windage and hub blockage losses. It may contain sections on vaneless space and vane solidity with the purpose of describing the circumferential flow around the rotor periphery; or include discussions about minimum number of radial passages, blade spacing and clearance gap between rotor and shroud as well as materials analysis needed for the high-temperature applications used for power generation in turbine cycles. Since non-zero incidence angles provide optimum flow conditions at the rotor inlet of real systems, the zero-incidence angle is deemed an oversimplification for the purposes of achieving maximum efficiency [3].

While the above engineering considerations are justified in their attempt to describe the optimization of real systems, the physical phenomena taking place in the passage of a radial inflow turbine are best described by the flow system of Fig. 1 and Eqs. (1) – (4) to which we can add an expression for the maximum thrust energy

$$E_T = c_p m \Delta T = mc^2 \quad , \qquad (5)$$

where the rotational cooling ΔT is given by Eq. (1). This is the energy that a gas parcel with mass m delivers as angular propulsion to the radial conduit when the parcel transitions from a state of rest in reference frame F' to a state of rest in reference frame F, where F' is in uniform motion with velocity c with respect to the stationary frame F. In this simple setup where a rotating tank has been added to the inlet of an adiabatic duct, incidence and exit losses have been neglected such that the number of physical phenomena that take place is minimized. This then gives us the ability to follow the energy initially invested into the mass m by compression and its acceleration to velocity c all the way to the energy extraction through the deceleration of the gas parcel mass m to zero velocity and its simultaneous decompression. The simple expressions for the maximum thrust power (Eq. (4)) and energy (Eq. (5)) exhibited by the idealized radial inflow device of Fig. 1, are the upper limit of the power and energy output from any real radial inflow turbine passage.

It is thus seen that the flow system of Fig. 1 and its analysis affords clarity of the physical process and allows smooth logical transition from the idealized state of lossless radial flow to the state of realistic geometries which involve flow resistance and losses. The device of Fig. 1 is therefore the simplest flow system exhibiting the fundamental laws of thrust and rotational cooling observed in radial inflow turbines.

THE ANGULAR PROPULSION ENGINE

The angular propulsion engine (hereinafter called the APE) presented in [1] represents a confined vortex and consists of a compressed air tank attached to the inlet of a straight adiabatic duct, which rotates about a central axis, as depicted in Fig 1. Compressed air expands adiabatically as it flows through the duct towards the rotation center, experiences a temperature drop and imparts angular propulsion, or rotary thrust, to the already rotating system. How is energy imparted to the rotating system? A rotating parcel of air — like any element of mass — being driven towards the rotation axis of a system attempts to increase its rotational speed to conserve angular momentum. Since in the confined APE the path of the air is constrained by the duct, the air is resisted to speeding up so it applies a torque to the system by pushing against the duct wall. This torque can either accelerate the system, or maintain the rotational speed against a resistance. It is important to emphasize that the confined APE is not a *heat* engine, since its operation does not depend on the temperature difference between two thermal reservoirs. Rather, it is an engine that extracts work from a compressed gas as it flows between two points with different potential energy due to their different pressures. The temperature difference that is established in the flow is a byproduct of the motion. Thus, the APE is a fluid system, which has the capability to deliver rotational work with efficiency not bounded by Carnot performance limits applicable to heat engines.

A key to the operation of the APE is that the system must be in motion for angular propulsion to take place; rotational motion is required first, and only then can the radially moving air mass release energy as rotary thrust. This requirement, known as pre-rotation, is crucial in the physics of the APE. In a radial-inflow turbine, such rotation is initiated by the impingement of gas on the blades. Once rotation has been initiated, energy is delivered continuously from the gas to the rotating system. In the absence of external resistance, the system accelerates to the point where the induced centrifugal gravitational potential balances the potential difference created by the pressures at the duct terminals and the mass flow through the duct stops. The duct will continue to rotate indefinitely at this constant angular velocity, since angular propulsion is no longer imparted to the system. To illustrate the power output and the terminal state of an APE, computational fluid dynamics (CFD) calculations were performed for an adiabatic rotating duct of length 0.5 [m], and cross-section 0.03 × 0.04 [m]. The simulations were conducted using the commercial software FLUENT [4] and the computational procedure used is described in detail in [1]. Fig. 2 shows the dependence of the power output

$$P_T = \dot{m} c_p \Delta T$$

which, by substitution of Eq. (1) yields Eq. (4) for P_T delivered as angular propulsion or rotary thrust, as a function of peripheral velocity c. Each power curve in Fig. 2 represents a locus of steady-states that an APE can operate in as it powers-up from a low rotational speed to its terminal state in the absence of resistance. The curves shown are for pressure differences of ΔP = 40 [kPa] and 60 [kPa]; the higher pressure potential has a higher output power and is able to achieve a higher terminal rotational speed, as might be expected. The power output curves are also noted to be highly non-linear due to the nature of the competing effects that produce rotary thrust. At low peripheral velocity, the radial mass flow is high due to the large imbalance between the pressure and centrifugal potentials, but because c is small, P_T is relatively low. As the system gains rotational speed, the mass flow rate is reduced, but P_T is high due to increasing c. Beyond the local maximum in rotary thrust, the APE continues to gain rotational speed, but the centrifugal gravitational potential approaches a balance with the pressure potential rendering the radial mass flow, and consequently the rotational thrust, to zero: the condition defined as the unresisted terminal state.

Now, let the APE rotate in a resistive medium or against an applied torque described by the arbitrary resistance curve, P_R, indicated in Fig. 2. In this case, after the phenomenon has been initiated, the APE delivers rotational energy, a portion of which is spent to overcome the external resistance and a new terminal state is reached. In this manner, the APE can be looked upon as a self-governed dynamical system, wherein the pressure potential and the induced centrifugal potential come to an appropriate imbalance which permits radial mass flow that delivers rotary thrust to maintain the rotation of the

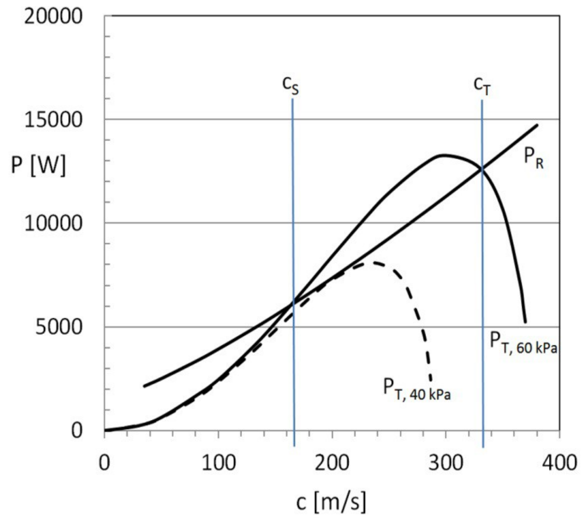

Figure 2 Operation plot for the angular propulsion engine

vortex. In Fig. 2, the intersection points between the upper power curve and the resistance curve define thresholds, or equilibria, where the energy released by the radial airflow as rotary thrust, P_T, balances the power required to overcome the resistance to rotation, P_R. In the context of a dynamical system, the first intersection defines an unstable equilibrium, c_S, while the second intersection defines the stable equilibrium or terminal state, c_T. Below c_S, $P_T < P_R$ and the APE would power-down if the external *startup* torque on the APE were removed. Between c_S and c_T (the operation region), $P_T > P_R$ and the APE will power-up under its own influence until it reaches c_T. Beyond c_T, $P_T < P_R$, thereby not allowing further acceleration. As such, for a given pressure difference, once the APE is pre-rotated past its unstable or *startup* threshold, c_S, it will approach its terminal state as P_T approaches its balance with P_R. While the system can theoretically operate at either threshold, systems in nature generally do not operate at their unstable equilibria since any slight perturbation in either direction will cause the system to either power-down or accelerate. The terminal state c_T is a stable equilibrium since perturbations in either direction are corrected back to c_T. The lower power curve in Fig. 2 characterizes an APE that could not operate against the resistance P_R since there is no point where $P_T < P_R$. Another important concept to be taken from Fig. 2 is that of *loading*. The APE (in its characterization as a radial inflow turbine) can only release energy against an applied load; i.e., in the absence of resistance, the system accelerates to a terminal state where no rotary thrust is delivered, whereas under load, the system achieves a state where there is a constant transfer of energy as rotary thrust from the induced radial inflow. Thus, regardless of whether the desired output of such a system is rotational work or cold air, the APE must be loaded.

Finally, it is seen that the total power delivered in the process is $\dot{m}c^2$, which is independent of system size, but is dependent only on the mass flow and the peripheral vortex velocity c. Therefore, a confined APE is able to accelerate its rotational motion and sustain itself irrespective of the presence or absence of external resistance provided that the potential pressure difference between center and periphery is sufficient to overcome the resistance. The confined APE is able to achieve a terminal state irrespective of its radial size.

THE RANQUE-HILSCH VORTEX TUBE

Though already alluded to in [1], it remains relevant to discuss the cooling achieved in a Ranque-Hilsch vortex tube (RHVT) in light of our refined description of the APE. In essence, we describe the RHVT as a radial-inflow fluid machine that has all of the elements of the APE system described above. The discovery of the so-called vortex-tube effect is attributed to Ranque [5] and Hilsch [6] and devices that utilize this effect are often referred to as Ranque-Hilsch vortex tubes (RHVTs). While many variations of the RHVT have emerged since its introduction some 80 years ago, the typical configuration consists of a narrow tube of circular cross-section that is 5-50 diameters in length from inlet to hot outlet. Air, or another compressible gas, is injected tangentially at one end of the tube producing a rotating flow inside. An outlet located on the central axis at the inlet end of the tube sees a fraction of the inlet gas leave at a temperature lower than the inlet temperature, while the remainder of the gas exits at the periphery of the opposite end of the tube at a temperature that is higher than the inlet temperature. The temperature difference or *separation* and the mass flow split between the hot and cold exit streams are affected by their exit pressures relative to the inlet pressure, by the length to diameter aspect ratio and by the inlet swirl velocities, among other factors (see, for example, [7]).

While numerous studies have been conducted over the past 80 years, the physical phenomenon responsible for energy separation in the RHVT remains an open question [8]. To this end, operation of the RHVT is often attributed to the work of Maxwell's demon, an entity that functions at the molecular level selectively separating hot and cold molecules. Recent reviews by, for example [8-9] provide summaries of existing studies on the vortex tube effect; in the interest of brevity, these reviews are not reproduced here, except to state that prevailing theories cite sudden expansion as the main mechanism for cooling. We propose herein that energy transfer in the RHVT is via angular propulsion, which is released from the working cold stream to power the vortex, and moved to the hot exit stream by a collective resistance to rotation. While the theories of Ranque [5], Hilsch [6] and Deissler & Perlmutter [10] all contribute to the explanation, no single theory has been able to reconcile the magnitude of the energy separation that is observed to occur. Angular propulsion is an important pathway for energy transfer in the RHVT and, combined with other mechanisms, serves to fully reconcile the observed energy separation between the cold and hot gas streams.

To describe the operation of the vortex tube, CFD was used to simulate the airflow in the generic RHVT system shown in Fig. 3. The system consists of a circular tube of diameter D = 11.4 mm and aspect ratio, L/D = 9.3 [11]. While this represents a relatively short vortex tube, significant temperature separation occurs, and it is a system for which previous computational and experimental results exist. The tube inlet and the cold outlet are located at the leftmost end of the tube, while the hot exit is located at the periphery of the rightmost face. Air is assumed to enter uniformly around the periphery at the inlet end at a total temperature of 300 [K], and the inlet flow angle relative to the local tangent is uniform and fixed. The cold outlet is specified as an opening centered on the tube axis with area 30.25 mm^2 and pressure P = 100 [kPa]. The hot exit is specified as an annular opening at the periphery of the rightmost face of the domain with an area 35.81 mm^2, and pressure P = 100 [kPa].

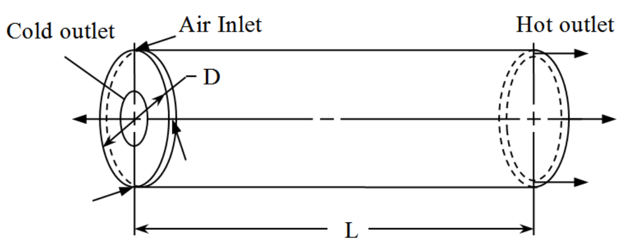

Figure 3: Computational domain for RHVT simulations.

The three-dimensional domain was discretized using 1,060,500 hexahedral volumes, with fine resolution at the tube walls gradually increasing towards the tube axis. The flow was solved using CFX-ANSYS, wherein the compressible flow formulation, combined with the ideal gas law and the k-ω turbulence model with automatic wall treatment. A grid-independence study confirmed resolution to within 2% based on temperature separation. Computations were conducted for inlet mass flows of 1.0 and 7.0 g/s, which resulted in pressure differences (between the inlet and the outlets) of ΔP_1 = 12.6 and ΔP_2 = 276.5 kPa, respectively. The computed results for the highest pressure difference replicate the experimental results of Skye et al. [11] to within 5% in terms of both mass split and temperature separation, thereby illustrating the accuracy of the present solution.

Figure 4 shows streamlines and static temperature contours on the center plane of the tube under the two conditions considered. The plot at the top of Fig. 4 shows the wall shear stress along the tube axis and is, perhaps, the most compelling evidence of the existence of a driven vortex in the RHVT.

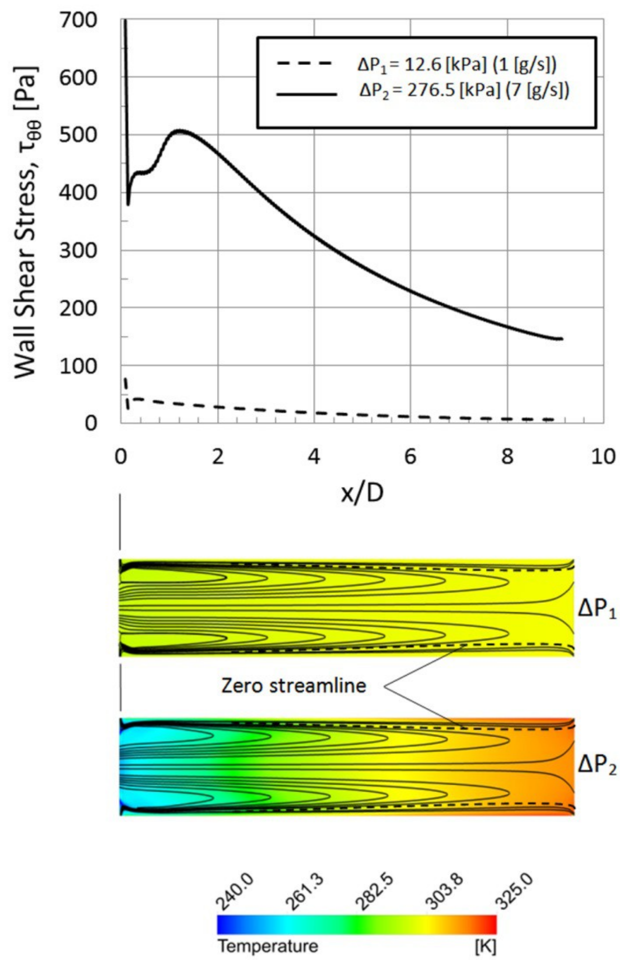

Figure 4: Plot showing the tangential component of the wall shear stress as a function of axial position, and streamline and static temperature contour plots for the RHVT considered.

In the low flow case, the swirling flow enters the tube (at the left), but simply weakens along the axis, as evidenced by the wall shear stress, producing a negligible cooling effect. The mass flow ratio is $\dot{m}_c / \dot{m}_{in}$ = 0.504 and the temperature separation is small (ΔT = 4.75 K). For the high pressure case, which corresponds to near-sonic inlet flow, the swirling inlet flow is more energetic and the wall shear stress plot indicates that strong rotation exists through a significant portion of the tube. In fact, the wall shear stress is shown to increase following a drop just past the tube inlet. This is evidence that the vortex is being powered from the inside through this region of the tube, since no other process can explain the observed increase and persistence of wall shear stress along the tube axis. In this case, the cold fraction has risen to $\dot{m}_c / \dot{m}_{in}$ = 0.554 and the total temperatures of the cold and hot streams are T_c = 277.4 K and T_h = 329.3 K (ΔT = 51.9 K).

So how is thrust imparted to the vortex in the RHVT? While in the confined APE, the radial flow exerts a force on the duct wall, in the RHVT, the resistance to the air parcel speeding up as it flows inward is shear. The importance of viscous friction and turbulent shear work has been alluded to in previous work by Hilsch [6] and Deissler and Perlmutter [10], and the concept of inner friction and turbulence in driving the vortex is discussed in Xue *et al.* [8] with many references to other studies. Herein, we propose that while it is inner friction or turbulent shear work that facilitates energy transfer to the vortex, it is both kinetic and internal energy from the radial flow stream that is transferred, and that the system can be envisioned in a manner similar to a radial-inflow turbine.

The flow structure depicted in Fig. 4 is typical of a short RHVT with no axial stagnation point, and shows a series of nested, dilated stream surfaces that characterize the axial motion of fluid particles as they progress along high-speed helical paths from the inlet to the cold central outlet or the hot annular outlet; the paths outside the dashed region are those that progress directly to the hot outlet. None of these paths intersect, and communication between them is via shear, which facilitates the outward transfer of energy via work. This structure evolves naturally in the RHVT to maximize the area over which shear work can occur and leads to the operation of this device as a type of radial inflow turbine removing energy from the inlet stream to drive a vortex that operates against a resistance, which in this case is the hot outflow stream. Energy transferred radially to the annular (hot) outlet stream as shear work in converted to internal energy as the waste stream progresses axially towards the exit, since the work is not being removed in any other manner.

Consider first particles that progress to the cold outlet. Air parcels initially cool at the tube inlet due to the sudden decompression across the inlet nozzle, but then heat as they progress axially along the tube, as indicated in Fig. 4 by an increase in static temperature. No air leaves the tube directly from the inlet region, so sudden decompression cannot be the sole mechanism for cooling. While the particles spiral in the positive x direction along a given stream surface, they take on energy from their adjacent inner surface while delivering energy to their adjacent outer surface, both via shear work. The production of shear, which is necessary for outward work transfer is a result of the fluid particles being forced radially inward along their stream surface by the pressure gradient between the inlet and outlet. The particles attempt to speed up to conserve angular momentum, but are resisted by the adjacent, slower outer surface. In this manner, kinetic energy is invested to propel the vortex system via shear work. In addition, the pressure energy stored in the gas as internal energy that is released to drive the air across the induced centrifugal acceleration field is also invested as propulsion to the vortex system resulting in further cooling.

In terms of overall operation, it is the pressure difference that drives radial airflow across the induced centrifugal gravitational field releasing energy as rotary thrust to drive the vortex against the collective resistance of the device. Thus, increasing the pressure difference changes everything about the operation of the RHVT, making it extremely complicated to analyze. Here, we divide the flow field inside the tube into three regions based on their function in the overall system: the *pre-rotation region* near the inlet where the swirling flow enters, the *APE region* outlined by the zero streamline that separates the axial from the peripheral exit flow, and the *loading region* that defines the remainder of the flow. Each region plays an important role in producing the low temperature stream, which is the useful output of the RHVT. The function of the pre-rotation region is to set the flow into rotational motion, a requirement for initiation of an APE. The higher the pressure difference, the higher the (tangential) inlet velocity and rotational speed, and the higher the induced centrifugal gravitational field at the inlet. The APE region is where energy is released from the radial airstream as rotary thrust to power the rotating flow, as described above. The loading region is a large part of the RHVT and provides the resistance to absorb the energy released from the APE region.

The best way to understand the importance of the loading region is to consider what would happen if it were removed. In this case, we need to imagine having a frictionless RHVT with no hot exit. Then, for a given pressure difference, the vortex inside the tube would accelerate to the point where the induced centrifugal potential comes into balance with the pressure potential, eventually rendering a zero mass flow through the device, similar to the unresisted terminal state of the confined APE. In any real case, the resistance to rotation and the deceleration of the flow towards the hot exit provide the load for the APE to work against. When loaded, the APE slows down rendering an imbalance between the pressure and centrifugal potentials, which produces radial mass flow and releases rotary thrust which is then absorbed in the loading region as internal energy, thereby providing the pathway for energy transfer in the RHVT and cooling of the central outlet airstream. With this description, it is easy to see that the highest pressure case represents the case with the highest driving potential and the highest loading, thereby producing the highest energy transfer from the cold to the hot airstreams.

CONCLUSION

In conclusion, we have presented the theory of the angular propulsion engine (APE) by outlining its governing equations. Analysis of the APE demonstrates clarity of the physical process which allows realistic conduit geometries and loss processes to be introduced gradually and methodically. The APE is therefore the simplest flow system exhibiting the fundamental laws of thrust and rotational cooling observed in radial inflow turbines.

The function of the APE within or outside a resistive environment leads to its description as a dynamic system exhibiting operational thresholds and a working regime. Having thus described the APE, we discover its presence in a Ranque-Hilsch vortex tube (RHVT), operating as a result of an imposed radial pressure difference. Angular propulsion in the RHVT is manifested in an observed peak of the tangential shear stress found away from the inlet injection. The exhaust of the APE in the RHVT is responsible for its cold flow, while its output propulsion is converted to internal energy of the flow at the far end of the tube. The mechanism for radial transfer of momentum is described as one dependent on viscous and turbulent shear. Computational fluid dynamics simulations of RHVT flow are presented in support of the proposed models.

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