

Research Article

Open Access

L.E. Sjöberg*

On the topographic effects by Stokes' formula

Abstract: Traditional gravimetric geoid determination relies on Stokes' formula with removal and restoration of the topographic effects. It is shown that this solution is in error of the order of the quasigeoid-to-geoid difference, which is mainly due to incomplete downward continuation (dwc) of gravity from the Earth's surface to the geoid. A slightly improved estimator, based on the surface Bouguer gravity anomaly, is also biased due to the imperfect harmonic dwc the Bouguer anomaly. Only the third estimator, which uses the (harmonic) surface no-topography gravity anomaly, is consistent with the boundary condition and Stokes' formula, providing a theoretically correct geoid height. The difference between the Bouguer and no-topography gravity anomalies (on the geoid or in space) is the "secondary indirect topographic effect", which is a necessary correction in removing all topographic signals.

Keywords: Bouguer gravity anomaly; geoid; no-topography gravity anomaly; secondary indirect topographic effect; topographic correction

DOI 10.2478/jogs-2014-0014

Received June 4, 2014; accepted October 1, 2014

1 Introduction

Stokes' formula (Stokes 1849) is fundamental for gravimetric geoid determination. As it requires no masses outside the sphere of computation, traditionally (Heiskanen and Moritz 1967, Ch. 3) the topographic signal on gravity is removed, or reduced by a compensation mass below or by a density layer at sea-level (direct topographic effect; DITE on gravity). Another topographic correction is the free-air correction, which provides a downward continuation (dwc) of gravity from the Earth's surface to sea-level. By subtracting normal gravity at the reference ellipsoid a Bouguer type of gravity anomaly is obtained. Adding the so-called "secondary indirect topographic effect" (SITE), Stokes' operator yields the regularized geoid or co-geoid,

and finally, after adding also the primary indirect topographic effect (PITE) on the geoid, for restoration of the topographic signal, the geoid height follows.

More or less the same topographic corrections are applied in one way or another in the modern Remove-Compute-Restore (RCR) techniques for geoid determination (e.g., Forsberg 2001; Sideris 1994; Ellmann and Vanicek 2007, Tziavos and Sideris 2013; Sanso and Sideris 2013). Some methods start from the classical gravity anomaly on the geoid (as above), while others start from M.S. Molodensky's (Molodensky et al. 1962) surface gravity anomaly. In the Least Squares Modification of Stokes' formula with Additive Corrections (LSMSA; Sjöberg 2003a, 2003b), the surface anomaly is used, and all topographic corrections (the DITE, SITE, PITE as well as the dwc) of the surface gravity anomaly are added as total effects to the modified Stokes' formula.

Below we will discuss and compare some of the above principles for topographic corrections in geoid determination. To make the presentation more transparent, the methods for applying Stokes' formula are given without corrections for atmospheric masses and ellipsoidal effects. Also, for simplicity, if not explicitly mentioned, the topographic mass reductions are considered only for the case with removal of topography (Bouguer case) without mass compensation. The theoretical discussion is followed by a comparison using a simple Earth model.

2 The traditional solution to Stokes' formula

2.1 Basic formulas

In classical physical geodesy (Heiskanen and Moritz 1967, Ch. 3; Sanso and Sideris 2013, Ch. 8) the reduction for the topographic signal in the gravity anomaly warrants that the (effect of the) forbidden external topographic masses has been removed (or restored below sea-level), and the gravity anomaly, located at sea-level, needs an additional small correction, the SITE (denoted dg_I), to be suitable for Stokes' integration on the co-geoid. Finally, by adding the PITE (denoted dN_I^T) the geoid height is obtained. The

*Corresponding Author: L.E. Sjöberg: Royal Institute of Technology (KTH) Stockholm, Sweden, E-mail: lsjo@kth.se

whole computational procedure can be expressed as

$$N_1 = \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) \Delta g_{red} d\sigma + dN_1^T, \quad (1)$$

where

$$dN_1^T = \frac{V_g^T}{\gamma_0} \quad (2)$$

is the PITE. Here R is the radius of the sphere of computation (which approximates sea level), σ is the unit sphere, $S(\psi)$ is Stokes' function with argument ψ being the geocentric angle between integration and computation points, γ_0 is normal gravity at the reference ellipsoid, V_g^T is the topographic potential on the geoid (denoted by subscript g), and the gravity anomaly suitable for Stokes' integration becomes:

$$\Delta g_{red} = \Delta \tilde{g}^B + dg_I. \quad (3)$$

The SITE is explicitly determined from

$$dg_I = 2 \frac{\gamma_0}{R} dN_1^T, \quad (4)$$

and $\Delta \tilde{g}^B$ is the estimated simple Bouguer gravity anomaly at the geoid level, which can be expressed as

$$\Delta \tilde{g}^B = g_P - B + F - \gamma_0. \quad (5)$$

Here g_P is the observed gravity at the Earth's surface point P , $B = 2\pi G\rho H$ is the Bouguer plate attraction (with G and ρ being the gravitational constant and density of topography, respectively) and $F = -H(\partial\gamma/\partial h) \approx 2\gamma_0 H/R$ is the free-air correction. Here h and H are the geodetic and orthometric heights.

[In the classical definition Δg_{red} is an estimator for the Bouguer gravity anomaly on the co-geoid. In a modern interpretation with the refined Bouguer correction applied, it is an estimator for the so-called no-topography gravity anomaly on the geoid (see below).]

The simple Bouguer gravity anomaly is often improved by replacing it by the refined Bouguer anomaly, which means (at least theoretically) that the attraction of all the topographic mass has been removed. Nevertheless, the main limitation with this approach for geoid determination is caused by the free-air correction F , which is supposed to represent the dwc of the surface Bouguer gravity to the geoid. This could only be correct if the topographic attraction were the only cause of the gravity disturbance.

According to Heiskanen and Moritz (1967, p. 142), Hoffmann-Wellenhof and Moritz (2005, p. 150) and Sanso and Sideris (2013, pp. 358 and 364) the correction dg_I is needed to correct the Bouguer gravity anomaly from the geoid to the co-geoid prior to Stokes' integration. We will revise this definition of the SITE in the concluding remarks (Sect. 7).

2.2 On the error of the traditional solution

The refined Bouguer attraction equals the total topographic attraction A_P^T , i.e.

$$A_P^T = - \left(\frac{\partial V^T}{\partial r} \right)_P, \quad (6)$$

and the refined Bouguer gravity anomaly, corresponding to the simple Bouguer anomaly of Eq. (5), can be written:

$$\Delta \tilde{g}_g^B = g_P - A_P^T - \gamma_Q + d\gamma = \Delta g_P^B + d\gamma, \quad (7a)$$

where Δg_P^B is the surface Bouguer gravity anomaly at surface point P , γ_Q is normal gravity at normal height (i.e. at the telluroid), and

$$d\gamma = \gamma_Q - \frac{\partial\gamma}{\partial h} H_P - \gamma_0 \approx 0. \quad (7b)$$

Hence, the traditional Bouguer anomaly of Eq. (7a) is practically equal to the surface Bouguer gravity anomaly.

Introducing the *no-topography gravity anomaly* (implying that all topographic signal has been removed; see Sect. 4) on the geoid:

$$\Delta g_g^{NT} = g_g - \gamma_0 - A_g^T + dg_I = \Delta g_g^B + dg_I, \quad (8a)$$

where

$$\Delta g_g^B = g_g - \gamma_0 - A_g^T, \quad (8b)$$

we see that the traditional Bouguer gravity anomaly of Eq. (7a) is related to the no-topography anomaly by the relation

$$\Delta g_{red} = \Delta \tilde{g}_g^B + dg_I = \Delta g_g^{NT} + Dg, \quad (9a)$$

or approximately

$$\Delta g_{red} \approx \Delta g_P^B + dg_I, \quad (9b)$$

where

$$Dg = \Delta g_P^B - \Delta g_g^B. \quad (9c)$$

Here Δg_P^B and Δg_g^B are the rigorous Bouguer gravity anomalies at the Earth's surface and geoid, respectively. Hence, Eq. (1) can be expressed

$$\begin{aligned} N_1 &\approx \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) \left(\Delta g_g^{NT} + Dg \right) d\sigma + dN_1^T \\ &= \frac{T_g^{NT}}{\gamma_0} + dN_1 + \frac{V_g^T}{\gamma_0} = N + dN_1, \end{aligned} \quad (10a)$$

where the error dN_1 follows from

$$\begin{aligned} dN_1 &= \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) Dg d\sigma \\ &= \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) \Delta g_P^B d\sigma - \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) \Delta g_g^B d\sigma \\ &\approx \frac{T_P^{NT} - T_g^{NT}}{\gamma_0} \approx \zeta - N + \frac{V_g^T - V_P^T}{\gamma_0} \approx -\frac{\Delta g^B}{\gamma_0} H. \end{aligned} \quad (10b)$$

Also, ζ is the height anomaly and T^{NT} is the no-topography disturbing potential (see, e.g., Vanicek et al. 2004 or Sjöberg 2010) defined by

$$\zeta = T_P/\gamma_Q \text{ and } T^{NT} = T - V^T, \text{ respectively,} \quad (11)$$

and T is the disturbing potential. Eq. (10b) means that the error of the traditional geoid solution, Eq. (1), is of the order of the difference between the height anomaly and the geoid height, which is negligible for low elevation topography, but may range to several decimetres in high mountains. One may argue that this error should decrease when applying topographic compensation for the topographic reduction. Theoretically, the compensation has no effect on a correct application of Stokes' formula, but it will affect the bias obtained by Eq. (10b) such that

$$dN_1^c \approx \zeta - N + \frac{dV_g^c - dV_P^c}{\gamma_0}, \quad (12)$$

where dV^c is the difference between the topographic and compensation potentials. If one uses isostatic or Helmert compensation, one can expect that the last term of Eq. (12) is negligible in most cases. Hence, the bias remains on the same order when using a strategy with topographic compensation.

3 A minor refinement of the traditional technique

We now start from the surface gravity anomaly defined as

$$\Delta g_P = g_P - \gamma_Q, \quad (13)$$

where P is a point on the Earth's surface and Q is the corresponding point at normal height (i.e. on the telluroid) along the ellipsoidal normal through P . The Bouguer correction is again assumed as the rigorous topographic correction on gravity:

$$(dg_{dir})_P = -A_P^T = \left(\frac{\partial V^T}{\partial r} \right)_P \quad (14)$$

so that the surface Bouguer anomaly becomes

$$\Delta g_P^B = \Delta g_P + (dg_{dir})_P. \quad (15)$$

The dvc of the Bouguer gravity anomaly to the geoid is performed by a harmonic reduction, denoted $()^*$, e.g. by solving Poisson's integral equation (Heiskanen and Moritz 1967, p. 35; Hofmann-Wellenhof and Moritz 2005, p. 28-29). [One example where one starts from the surface Bouguer anomaly and downward continue it in free-air is given

in Sanso and Sideris (2013, Sect. 10.2).] Hence, by this technique one obtains Stokes' integral solution with topographic corrections as

$$N_2 = \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) \left[(\Delta g^B)^* + dg_I \right] d\sigma + dN_I^T. \quad (16)$$

Already at this stage we may point out that this solution is biased, as $r\Delta g^B$ is not harmonic below the topographic surface, as a topographic signal still remains (see also the next section). In Sect. 5 we estimate the bias of N_2 .

4 A modern approach to Stokes' formula

We now start from the decomposition of the disturbing potential into the NT and topographic potentials (Vanicek et al. 2004; Vajda et al. 2007):

$$T = T^{NT} + V^T. \quad (17)$$

Considering also the boundary condition of physical geodesy (Heiskanen and Moritz 1967, Sect. 2-14):

$$\Delta g = -\frac{\partial T}{\partial r} + \frac{\partial \gamma}{\partial h} \frac{T}{\gamma} \approx -\frac{\partial T}{\partial r} - 2\frac{T}{r}, \quad (18)$$

where the approximation is the result of the spherical approximation of the derivative of γ w.r.t. the geodetic height (h), it follows that the surface gravity anomaly can be decomposed into a NT gravity anomaly (Δg^{NT}) and a topographic gravity anomaly (Δg^T):

$$\Delta g = \Delta g^{NT} + \Delta g^T, \quad (19a)$$

where

$$\Delta g^{NT} = -\frac{\partial T^{NT}}{\partial r} + \frac{\partial \gamma}{\partial h} \frac{T^{NT}}{\gamma} \approx -\frac{\partial T^{NT}}{\partial r} - 2\frac{T^{NT}}{r} \quad (19b)$$

and

$$\Delta g^T = -\frac{\partial V^T}{\partial r} + \frac{\partial \gamma}{\partial h} \frac{V^T}{\gamma} \approx -\frac{\partial V^T}{\partial r} - 2\frac{V^T}{r}. \quad (19c)$$

We notice that the topographic correction $-\Delta g^T$ (yielding the NT anomaly) differs from the gravity correction dg_{dir} (yielding the Bouguer gravity anomaly). This is because the NT anomaly applies an additional correction ("the SITE at point level") for the change of the normal height due to the removal of the topographic signal. See also Heiskanen and Moritz (1967, Sect. 8-11) and Hofmann-Wellenhof and Moritz (2005, Sect. 8-9). The NT anomaly, but not the Bouguer anomaly, can be correctly downward

continued from the surface to the geoid as a harmonic function (when pre-multiplied by r). In this case the solution to Stokes' integral becomes:

$$N_3 = \frac{R}{4\pi\gamma} \iint_{\sigma} S(\psi) (\Delta g^{NT})^* d\sigma + dN_I^T, \quad (20)$$

The first term on the right side of Eq. (20) is the NT geoid height:

$$\frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) (\Delta g^{NT})^* d\sigma = \frac{T_g^{NT}}{\gamma_0} = N^{NT}, \quad (21)$$

so that Eq. (20) becomes

$$N_3 = N^{NT} + dN_I^T = \frac{T_g^{NT} + V_g^T}{\gamma_0} = \frac{T_g}{\gamma_0} = N. \quad (22)$$

This shows that the solution N_3 yields the correct geoid height. Hence, the NT anomaly is a rigorous gravity anomaly that suits Stokes' formula in contrast to the Bouguer gravity anomaly.

This method (with topographic compensation by Helmert condensation) is used in the UNB geoid model (Ellmann and Vanicek 2007; Vanicek et al. 2013). Eq. (20) is also the basis in the LSMSA geoid modelling technique (Sjöberg 2003a and 2003b), which has been applied by several authors (e.g., Kiamehr 2006, Ulotu 2009, Ågren et al. 2009 and Abbak et al. 2012).

5 The bias in the refined traditional solution

The solution N_2 of Eq. (16) can be expressed as

$$N_2 = N + \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) \frac{2}{R} \left[(V^T)^* - V_g^T \right] d\sigma \quad (23)$$

where the last term is a bias in this solution. As the topographic potential is not harmonic inside the topographic masses, it follows that $(V^T)^* \neq V_g^T$ and the bias follows. As shown by Sjöberg (2007)

$$(V^T)^* - V_g^T = 2\pi G\rho \left(H^2 - \frac{2H^3}{3R} \right), \quad (24)$$

so that the bias of N_2 can be written

$$dN_2 = \frac{G\rho}{2\gamma_0} \iint_{\sigma} S(\psi) \left(H^2 - \frac{2H^3}{3R} \right) d\sigma, \quad (25)$$

which can also be expressed in the spectral form as

$$dN_2 = \frac{G\rho}{2\gamma_0} \sum_{n=0}^{\infty} \frac{1}{n-1} \left[(H^2)_n + \frac{2}{3R} (H^3)_n \right], \quad (26)$$

where $(H^k)_n$ are Laplace harmonics of H^k , $k=2,3$. This error can reach several decimetres in the highest mountains, and it does not change with topographic compensation.

6 An example for a simple Earth Model

Let us assume that the Earth is a homogeneous sphere of radius R and mass M and a topography with potential V^T . In addition there is a mass homogeneity within the sphere that generates a space variable potential dV and gravity v . Then the geopotential can be written

$$W = U + dV + V^T, \quad (27)$$

where

$$U = GM/r \quad (28)$$

is the chosen normal potential (equal to the potential of the homogeneous sphere). The disturbing potential becomes

$$T = W - U = dV + V^T, \quad (29)$$

and the PITE and geoid height can be expressed:

$$dN_I^T = \frac{V_g^t}{\gamma_0} \quad \text{and} \quad N = \frac{dV_g + V_g^T}{\gamma_0}. \quad (30)$$

Moreover, from Eqs. (13), (15) and (19a) the surface, Bouguer and NT gravity anomalies become:

$$\Delta g_P = \Delta v_P + \Delta g_P^T, \quad \Delta g_P^B = \Delta v_P - 2 \frac{V_P^T}{r_P} \quad \text{and} \quad \Delta g_P^{NT} = \Delta v_P. \quad (31)$$

where

$$\Delta v_P = v_P - 2 \frac{dV_P}{r_P} \quad (32)$$

is the gravity anomaly caused by the mass homogeneity in the sphere of radius R , which implies that

$$(dV)^* = dV_g \quad \text{and} \quad (\Delta v)^* = \Delta v_g. \quad (33)$$

Hence, by inserting Eq. (31) into Eqs. (16) and (20) one arrives at the biased and unbiased geoid solutions

$$\begin{aligned} N_2 &= \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) \left[\left(\Delta v - 2 \frac{V^T}{r} \right)^* + 2 \frac{V_g^T}{R} \right] d\sigma + \frac{V_g^T}{\gamma_0} \\ &= N + \frac{1}{2\pi\gamma_0} \iint_{\sigma} S(\psi) \left[V_g^T - (V^T)^* \right] d\sigma \end{aligned} \quad (34)$$

and

$$N_3 = \frac{dV_g + V_g^T}{\gamma_0} = N. \quad (35)$$

Finally, we also consider the solution N_1 given by Eqs. (1) - (5). In this case we have

$$\Delta \tilde{g}^B = \gamma_P + v_P + g_P^T - B + F - \gamma_0. \quad (36)$$

Moreover, if we assume that B is the refined Bouguer attraction as in Eq. (6), i.e. $B = g_P^T$, and $\gamma_P + F \approx \gamma_g$, it follows that

$$\begin{aligned} \Delta g_{red} &\approx v_P + \gamma_g - \gamma_0 + 2 \frac{V_g^T}{R} \approx v_P + \frac{\partial \gamma}{\partial r} N + 2 \frac{V_g^T}{R} \\ &= v_P - 2 \frac{dV_g}{R} = \Delta g_g^{NT} + v_P - v_g, \end{aligned} \quad (37)$$

so that Eq. (1) yields the biased solution

$$\begin{aligned} N_1 &= \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) \Delta g_{red} d\sigma + \frac{V_g^T}{\gamma_0} \\ &\approx N + \frac{R}{4\pi\gamma_0} \iint_{\sigma} S(\psi) (v_P - v_g) d\sigma. \end{aligned} \quad (38)$$

7 Concluding remarks

The traditional Bouguer gravity anomaly, Eq. (5), includes approximations that are too crude for today's accurate demands on geoid determination. The error of the resulting geoid solution, of the order of the quasigeoid-to-geoid difference, is significant in mountainous areas. The bias remains of the same order when adding a compensation for the topographic reduction. Also, the second estimator, based on a surface Bouguer gravity anomaly, yields a biased geoid estimate due to the erroneous harmonic downward continuation of a non-harmonic gravity anomaly. Only the third geoid estimator, based on the surface NT gravity anomaly, is unbiased.

The traditional assumption that the SITE is needed only for geoid determination as a change of normal gravity from the geoid to the co-geoid is too narrow minded. Instead, the SITE is always needed as a part of the topographic reduction (with or without topographic compensation) of the surface, as well as the classical gravity anomaly to a rigorous gravity anomaly, free from topographic signal. Only such a gravity anomaly is consistent with the boundary condition of physical geodesy and Stokes' formula. It is also a requirement for analytical continuation of the anomaly.

Another view of the SITE is as follows (Vajda et al. 2006). The gravity disturbance is physically an attraction. Accordingly, it is completely reduced for the topographic signal by the DITE ($-A^T$), which is the removal of the topographic attraction. In contrast, the gravity anomaly

is not an attraction, implying that it needs both the DITE and SITE to fully remove the influence of the topographic masses. In other words, the direct topographic effect on the gravity anomaly is the sum of the traditional DITE and SITE.

Geoid determination is an example of gravity inversion, and for such operations the NT gravity anomaly is preferred to the Bouguer gravity anomaly. Accordingly, the isostatic gravity anomaly needs compensations not only for the DITE but also for the SITE. See Sjöberg (2013).

References

- Abbak R., Sjöberg L. E., Ellmann A., Ustun A., 2012, A precise gravimetric geoid model in a mountainous area with scarce gravity data: a case study in central Turkey, *Stud Geophys Geod* 56: 909-927
- Ågren J., Sjöberg L. E., Kiamehr R., 2009, The new gravimetric geoid model KTH08 over Sweden, *J Appl. Geod.* 3: 143-153
- Ellmann A., Vanicek P., 2007, UNB application of Stokes-Helmert's approach to geoid computation, *J Geodyn* 43: 200-2013
- Forsberg R. 2001, Developments of a Nordic cm-geoid with basics of geoid determination. In: Harson B G (ED.) Lecture notes for autumn school organized by the NKG, Fevik, Norway, 28 August-2 September, 2000, Statens Kartverk, Hoenefoss, Norway.
- Heiskanen A. W., Moritz H., 1967, *Physical Geodesy*, W H Freeman and Co., San Francisco and London
- Hofmann-Wellenhof B., Moritz H., 2005, *Physical Geodesy*. Springer, Wien and New York.
- Sanso F., Sideris M., 2013, *Geoid determination - Theory and Methods*, Lecture Notes in Earth System Sciences, Springer
- Kiamehr R., 2006, *Precise Gravimetric Geoid Model for Iran Based on GRACE and SRTM Data and the Least-Squares Modification of Stokes' Formula: with Some Geodynamic Interpretations*. Doctoral Thesis in Geodesy, Royal Institute of Technology, Stockholm, Sweden
- Molodensky M.S., Eremeev V.F., Yurkina M.I., 1962, *Methods for Study of the External Gravitational Field and Figure of the Earth*. Transl. from Russian (1960), Israel program for Scientific Translations, Jerusalem, Israel
- Sideris M., 1994, Regional geoid determination. In: Vanicek P, Christou N T (Eds.) *The geoid and its geophysical interpretation*. CRC Press, Boca Raton, FL, pp. 77-94
- Sjöberg L. E., 2003a, A computational scheme to model the geoid by the modified Stokes's formula without gravity reductions. *J Geod* 77: 423-432
- Sjöberg L. E., 2003b, A general model of modifying Stokes' formula and its least-squares solution, *J Geod* 77: 459-464
- Sjöberg L. E., 2007, The topographic bias by analytical continuation in physical geodesy. *J Geod* 81: 345-350
- Sjöberg L. E., 2010, A strict theory for geoid-to-quasigeoid separation, *J Geod* 84: 699-702
- Sjöberg L. E., 2013, On the isostatic gravity anomaly and disturbance and their applications to Vening Meinesz-Moritz inverse problem of isostasy, *Geophys J Int* 193: 1277-1282
- Stokes G. G., 1849, On the variation of gravity on the surface of the earth, *Trans Cambridge Phil Soc*, 8: 672-695

- Tziavos I. N., Sideris M., 2013, Topographic reductions in gravity and geoid modeling, In: Sanso and Sideris, M (Eds.) Geoid determination - Theory and Methods, Lecture Notes in Earth System Sciences, Springer, Ch. 8.
- Ulotu P.E., 2009, Geoid model of Tanzania from sparse and varying gravity data density by the KTH method. Doctoral Thesis in Geodesy, Royal Institute of Technology, Stockholm
- Vajda P., Vanicek P., Meurers B., 2006, A new physical foundation for anomalous gravity. *Stud Geophys et Geod* 50(2): 182-216
- Vajda P., Vanicek P., Novak P., Tenzer R., Ellmann A, 2007, Secondary indirect effects in gravity anomaly data inversion or interpretation. *JGR* 112, B06411: 1-11
- Vanicek P., Tenzer R., Sjöberg L. E., Martinec Z., Featherstone W. E., 2004, New views of the spherical Bouguer gravity anomaly, *Geophys J Int* 159: 460-472
- Vanicek P., et al., 2013, Testing Stokes-Helmert geoid model computation on a synthetic gravity field: experiences and shortcomings, *Stud Geophys Geod* 57: 369-400