On the Use of Linear Programming for Dynamic Subchannel and Bit Allocation in Multiuser OFDM

Inhyoung Kim, Hae Leem Lee, Beomsup Kim, and Yong H. Lee *

Department of Electrical Engineering and Computer Science
Korea Advanced Institute of Science and Technology
373-1, Kusong-dong, Yusong-gu, Taejon, 305-701, KOREA
Phone:+82-42-869-3437
Fax: +82-42-869-4030

E-mail: yohlee@ee.kaist.ac.kr

Abstract—**Adaptive subcarrier allocation and adaptive modulation for multiuser orthogonal frequency division multiplexing (OFDM) is considered. The optimal subcarrier and bit allocation problems, that have been formulated in [1] and [2] as nonlinear optimizations, are converted into linear ones and solved by integer programming (IP). A suboptimal approach that separately performs subcarrier allocation and bit loading is proposed. It is shown that subcarrier allocation in this approach can be optimized by the linear programming (LP) relaxation of the IP. Comparison through computer simulation indicates that performance of the suboptimal approach can be close to that of the optimal.**

Index Terms—**Subcarrier and Bit Allocation, Multiuser OFDM, Integer Programming, Linear Programming.**

I. Introduction

It has been suggested that multiuser orthogonal frequency division multiplexing (OFDM) systems employ adaptive subcarrier allocation as well as adaptive bit loading. By adaptively assigning subcarriers depending on channel characteristics, multiuser OFDM can take advantage of channel diversity among users in different locations. This approach allows efficient use of all the subcarriers.

Optimal bit loading and subcarrier allocation problems for multiuser OFDM have been formulated in [1], [2]: specifically, minimization of the overall transmit power under the data rate constraint [1] and maximization of the data rate under the power constraint [2] have been considered. These are nonlinear optimization problems with integer variables, which are referred to as the *margin adaptive* (MA, the former) and *rate adaptive* (RA, the latter) optimizations [3]. Solving these problems turned out to be extremely difficult; they were solved after relaxing the requirement regarding integer variables to allow real numbers. As a consequence, this approach cannot yield an optimal solution, yet it requires intensive computation due to the nature of nonlinear optimization.

In this paper, we shall show that the nonlinear optimization problems can be converted into linear optimization with integer variables, and that optimal subcarrier and bit allocation is achieved by *integer programming* (IP). In addition, to reduce the computational load, we develop a suboptimal approach that separately performs channel allocation and bit loading. This approach assigns subcarriers to each user under the assumption of constant bit loading to all subcarriers. It will be shown that such subcarrier allocation can be optimized through *linear programming* (LP) *relaxation*[1] of the IP. Due to this fact, the proposed suboptimal approach needs much less computation than the original IP. Application of the subcarrier and bit allocation algorithms to multiuser OFDM indicates that performance of the suboptimal approach can be close to that of the optimal.

II. System Model and Problem Formulation

The structure of the adaptive multiuser OFDM system under consideration is shown in Fig. 1. The system has K users and N subchannels (subcarriers). The base station receives downlink channel information from all users, and using this information, it assigns a set of subcarriers to each user and determines the number of bits per OFDM symbol to be transmitted on each subcarrier. It is assumed that sharing a subcarrier by different users is not allowed. Depending on the number of bits assigned to subcarriers,

* Corresponding author

[1] The LP obtained by omitting all integer constraints on variables is called the LP relaxation of the IP [4].

0-7803-7206-9/01/$17.00 © 2001 IEEE

each user's data are distributed to the subcarriers allocated to the user, and adaptive modulation is performed at each subchannel. The subcarrier and bit allocation information is sent to the receivers via a separate control channel. At each receivers, subcarriers assigned to the user are selected and the signals associated with the subcarriers are demodulated.

To describe the optimization problems, we introduce some notations[2]. Denote the data rate of the kth user by R_k and the number of bits of the kth user that are assigned to the nth subcarrier by $c_{k,n}$. It is assumed that $c_{k,n} \in \mathbf{D}$ where $\mathbf{D}$ is a set of nonnegative integers which are less than or equal to M and M is the maximum number of bits/symbol that can be transmitted by each subcarrier. The data rate R_k can be expressed as $R_k = \sum_{n=1}^{N} c_{k,n}$, or equivalently

$$R_k = \sum_{n=1}^{N} c_{k,n}\rho_{k,n} \tag{1}$$

where $\rho_{k,n}$ is an indicator variable defined as

$$\rho_{k,n} = \begin{cases} 0, & \text{if } c_{k,n} = 0 \\ 1, & \text{otherwise.} \end{cases} \tag{2}$$

As a subcarrier can be occupied by at most one user,

$$\sum_{k=1}^{K} \rho_{k,n} = 1. \tag{3}$$

The transmission power allocated to user k's subcarrier n is expressed as

$$P_{k,n} = \frac{f_k(c_{k,n})}{\alpha_{k,n}^2} \tag{4}$$

where $f_k(c_{k,n})$ is the required received power in the n-th subcarrier for reliable reception of $c_{k,n}$ bits/symbol when the channel gain is equal to unity, and $\alpha_{k,n}$ is the channel gain of user k's subchannel n. Using these notations, the MA and RA optimizations are stated as follows.

Margin Adaptive (MA) Optimization. Suppose that the user data rate $\{R_1, ..., R_K\}$ are fixed and given. The MA procedure minimizes the total transmission power required for transmitting the data with rate $\{R_1, ..., R_K\}$:

$$\min_{c_{k,n},\rho_{k,n}} P_T = \min_{c_{k,n},\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}} \cdot \rho_{k,n} \tag{5}$$

$$\text{subject to} \quad R_k = \sum_{n=1}^{N} c_{k,n}\rho_{k,n}, \quad \text{for all } k \tag{6}$$

where P_T denotes the total transmission power.

Rate Adaptive (RA) Optimization. Suppose that available total transmission power is limited. The RA procedure maximizes the minimum of user's throughput subject to the power constraint[3]:

$$\max_{c_{k,n},\rho_{k,n}} \min_k R_k = \max_{c_{k,n},\rho_{k,n}} \min_k \sum_{n=1}^{N} c_{k,n}\rho_{k,n} \tag{7}$$

$$\text{subject to} \quad \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2}\rho_{k,n} \le P_T. \tag{8}$$

Due to the "max·min" criterion in (7), the RA procedure tends to assign more transmission power to users with poor channel responses; as a result, the throughputs of the users allocated by this procedure usually become close to each other.

The MA and RA optimizations are nonlinear because $f_k(c)$ in (4) is nonlinear. For example, in the case of M-ary quadrature amplitude modulation (M-QAM), $f_k(c)$ can be represented as

$$f_k(c) = \frac{N_o}{3}[Q^{-1}(\frac{p_e}{4})]^2(2^c - 1) \tag{9}$$

where p_e denotes the required bit error rate (BER), $N_o/2$ denotes the variance of the additive white Gaussian noise (AWGN), and

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt \tag{10}$$

[1]. In the following section, the nonlinear optimization problems are converted into linear ones by utilizing the fact that $c_{k,n}$ takes only integer values.

III. Integer Programming Formulation

Suppose that $c_{k,n} \in \{0, 1, ..., M\}(= \mathbf{D})$. Then

$$f_k(c_{k,n}) \in \{0, f_k(1), ..., f_k(M)\} \tag{11}$$

where $f_k(0) = 0$ and $\{f_k(c)\}$ are constants that can be precalculated: for example, (9) can be used for M-QAM. We define a new indicator variable $\gamma_{k,n,c}$ as follows:

$$\gamma_{k,n,c} = \begin{cases} 1, & \text{if } c_{k,n} = c \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

[2] The notations in [1] will be adopted throughout this paper

[3] In the single user case, the maximization of the total data rate has been considered [5]. This criterion will not be adopted in the multiuser case, because it tends to exclude some users with poor channel responses.

for all $c \in \{0, 1, ..., M\}$. Using $\gamma_{k,n,c}$, (11) is rewritten as

$$f_k(c_{k,n}) = \sum_{c=1}^{M} \gamma_{k,n,c} f_k(c). \tag{13}$$

Since $f_k(c)$ are constants, (13) indicates that $f_k(c_{k,n})$ is a linear combination of the indicator variables $\{\gamma_{k,n,c}\}$. Using (12) in (5), the MA cost function is rewritten as

$$P_T = \sum_{k=1}^{K} \sum_{n=1}^{N} \{\sum_{c=1}^{M} \gamma_{k,n,c} \frac{f_k(c)}{\alpha_{k,n}^2}\} \rho_{k,n}. \tag{14}$$

The indicators $\rho_{k,n}$ and $\gamma_{k,n,c}$ are related as follows:

$$\rho_{k,n} = \begin{cases} 0, & \text{if } \gamma_{k,n,c} = 0 \text{ for all } c \\ 1, & \text{otherwise.} \end{cases} \tag{15}$$

From (15), it can be seen that

$$\gamma_{k,n,c} \cdot \rho_{k,n} = \gamma_{k,n,c}. \tag{16}$$

Therefore, (14) is rewritten by

$$P_T = \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{c=1}^{M} \gamma_{k,n,c} \frac{f_k(c)}{\alpha_{k,n}^2}. \tag{17}$$

This is a linear cost function. In a similar manner, the constraint in (8) can be converted into a linear form. Summarizing these results, the MA and RA problems are redescribed as follows.

Margin Adaptive (MA) Optimization

$$\min_{\gamma_{k,n,c}} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{c=1}^{M} \frac{f_k(c)}{\alpha_{k,n}} \gamma_{k,n,c}, \quad \text{for } \gamma_{k,n,c} \in \{0,1\} \tag{18}$$

$$\text{subject to} \quad R_k = \sum_{n=1}^{N} \sum_{c=1}^{M} c \cdot \gamma_{k,n,c} \quad \text{for all } k, \tag{19}$$

$$\text{and} \quad 0 \le \sum_{k=1}^{K} \sum_{c=1}^{M} \gamma_{k,n,c} \le 1, \quad \text{for all } n. \tag{20}$$

Rate Adaptive (MA) Optimization

$$\max_{\gamma_{k,n,c}} \min_{k} \sum_{n=1}^{N} \sum_{c=1}^{M} c \cdot \gamma_{k,n,c} \quad \text{for } \gamma_{k,n,c} \in \{0,1\} \tag{21}$$

$$\text{subject to} \quad \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{c=1}^{M} \frac{f_k(c)}{\alpha_{k,n}} \gamma_{k,n,c} \le P_T \tag{22}$$

and the constraint in (20).

These optimization problems can be solved by IP having $\gamma_{k,n,c}$ as variables. It can be observed that the IP requires considerably less computation than does the nonlinear optimization algorithms in [1], [2]. However, in general, IP needs an exponential time algorithm whose complexity increases exponentially with the number of constraints and variables. In the following section, some suboptimal, polynomial time algorithms will be developed.

IV. LP-Based Suboptimal Algorithms

In an attempt to simplify the subcarrier and bit allocation problem, we consider the following two step approach: in the first step, subcarriers are assigned under the assumption that bits loaded to the subcarriers are constant; in the second, bits are distributed to the subcarriers which are assigned in the first step. By separately performing subcarrier allocation and bit loading, this approach yields a suboptimal algorithm which is considerably simpler to implement than the IP-based approach.

A. Subcarrier Allocation

Suppose, for the time being, that the data rate R_k is constant for all k and that modulation schemes for all subcarriers are identical and fixed. Let S_k be the set of indices of subcarriers assigned to user k: $\{S_k\}$ are disjoint and $\bigcup_{k=1}^{K} S_k \subset \{1, 2, ..., N\}$. Then, for all k and n,

$$c_{k,n} = \begin{cases} c, & \text{if } n \in S_k \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

and

$$\sum_{n=1}^{N} \rho_{k,n} = r \tag{24}$$

where r is an integer satisfying $K \cdot r \le N$ (r subchannels are assigned to each user). In addition, it is assumed that quality-of-service (QoS) requirements of the users are the same. This results $f_k(c) = f(c)$ for all k. Then from (5), the subcarrier allocation problem is described as follows.

Subcarrier Allocation Under Constant Bit Loading

$$\min_{\rho_{k,n}} P_T/f(c) = \min_{\rho_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{\rho_{k,n}}{\alpha_{k,n}^2} \tag{25}$$

$$\text{subject to} \quad \sum_{n=1}^{N} \rho_{k,n} = r. \tag{26}$$

Obviously, this is an IP problem. However, it can be solved through LP relaxation of the IP (the proof is omitted due to page length limitation). Therefore, (25) can be optimized using an efficient polynomial time LP algorithm. In the proposed approach, the subcarrier allocation based in (25) is applied to both MA and RA problems.

B. Bit Loading

Now let us consider bit loading under the assumption that channel allocation is completed. In this case, bits can be allocated following the greedy approach for single user OFDM [1], [5]: a greedy algorithm assigns bits to subcarrier one bit at a time, and in each assignment the subcarrier that requires the least additional power is selected. Let $\Delta P_{k,n}(c)$ denote the additional power needed for transmitting one additional bit through the kth user's subcarrier n, when the number of bits loaded to the subcarrier is c. It is given by

$$\Delta P_{k,n}(c) = [f_k(c+1) - f_k(c)]/\alpha_{k,n}^2, \text{ for } n \in S_k. \quad (27)$$

A greedy algorithm for the MA problem can be stated as follows:

Bit Loading To Given Subcarriers (MA-Type)

Initialization

Let $c_{k,n} = 0$ for all k and n.

Evaluate $\Delta P_{k,n}(0)$ for each k and $n \in S_k$.

Bit Assignment Iteration

For each k, repeat the following R_k times:

$\hat{n} = \arg\min_{n \in S_k} \Delta P_{k,n}(c_{k,n})$

$c_{k,\hat{n}} = c_{k,\hat{n}} + 1$

evaluate $\Delta P_{k,\hat{n}}(c_{k,\hat{n}})$.

This algorithm provides the optimal bit loading policy for the MA problem. Developing an optimal greedy algorithm for the RA case appears to be difficult. The algorithm described below provides a suboptimal solution.

Bit Loading To Given Subcarriers (RA-Type)

Initialization

Let $c_{k,n} = 0$ for all j and n, and $\Delta P_{k,n}(0)$ is evaluated for all k and $n \in S_k$.

Denote the current transmission power by P_c and let $P_c = 0$.

Bit Assignment Iteration

Repeat the following unless $P_c > P_T$:

For $k = 1$ to K

$\hat{n} = \arg\min_{n \in S_k} \Delta P_{k,n}(c_{k,n})$

$P_c = P_c + \Delta P_{k,\hat{n}}(c_{k,\hat{n}})$

$c_{k,\hat{n}} = c_{k,\hat{n}} + 1$

evaluate $\Delta P_{k,\hat{n}}(c_{k,\hat{n}})$.

The throughputs of the users allocated by this algorithm are almost identical: their difference is at most one.

In summary, for both the MA and RA problems, subcarriers are allocated by solving (25). Then one of the greedy algorithms are used for bit loading.

V. Performance Comparison

The proposed algorithms were applied to an adaptive multiuser OFDM system with the following parameters: the number of subcarriers $N = 64$; the number of users, K, was in between 2 and 8; the maximum number of loaded bits $M = 12$; the data rate after appending the cyclic prefix at the transmitter was 20M sample/sec; the required BER $p_e = 10^{-4}$. For comparison, the methods in [1] and [2] were also considered. The channel was a frequency selective Rayleigh fading channel with an exponentially decaying delay profile. The root mean square delay spread was 90nsec and the span of channel taps was 8. During the simulation, 100 channels were independently generated; the results presented in this section are the average of 100 trials.

Fig. 2 shows the results for the MA case when $\sum_{k=1}^{K} R_k = 256$ and $R_1 = R_2 = ... = R_K$. Due to the effect of channel diversity among users in different locations, the total transmission power P_T decreased as the number of users increased. The results from the optimal algorithm and the proposed suboptimal algorithm are close each other, and the proposed techniques outperformed the method in [1].

Fig. 3 shows the minimum data rates for the RA case when the number of users $K = 4$. The results from the method in [2] were obtained under the assumption that the number of loaded bits was limited to an integer value. It is seen, as in the MA case, that the performance of the pro-

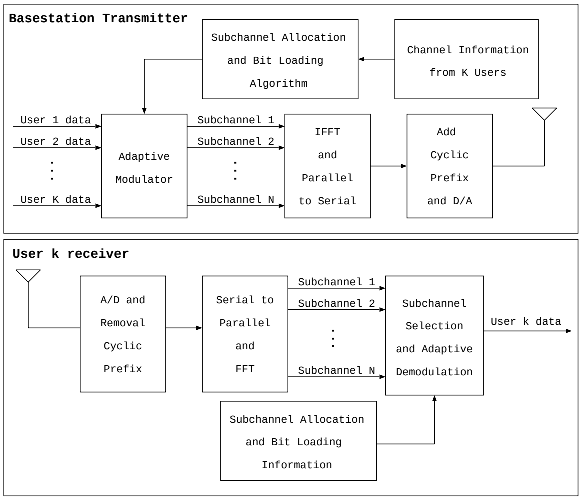

Fig. 1. A multiuser OFDM system.

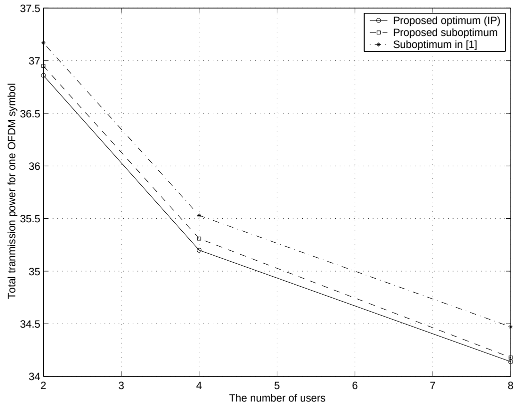

Fig. 2. Comparison of the total transmission power.

posed suboptimal algorithm is close to that of the optimal. The proposed method performed considerably better than the method in [2].

VI. Conclusion

It was shown that subcarrier allocation and bit loading for multiuser OFDM can be optimized by using IP, and that a useful suboptimal algorithm can be developed via the LP relaxation of the IP. Simulation results indicated that the proposed suboptimal method can perform like the optimal method, and outperform existing techniques.

References

[1] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1747–1758, Oct. 1999.

[2] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE Vehicular Technology Conf. (VTC'2000)*, 2000, pp. 1085–1089.

[3] J. M. Cioffi, *Lecture Notes for Advanced Digital Communication*, Stanford, Fall 1997.

[4] Laurence A. Wolsey, *Integer Programming*, John Wiley & Sons, Inc., 1998.

[5] Jorge Campello, *Discrete Bit Loading for Multicarrier Modulation Systems*, Ph.D dissertation, Stanford, 1999.

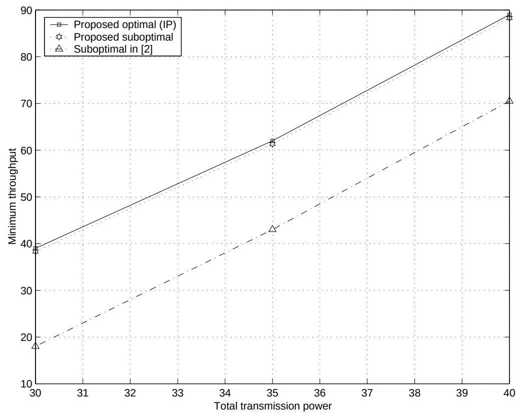

Fig. 3. Comparison of the minimum throughput when the number of users $K = 4$.