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On the use of Youla–Kucera parametrisation in adaptive active noise and vibration control – a review — Source link

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Abstract
Youla-Kucera parametrization plays a very important role in adaptive active vibration control and adaptive active noise control. This concerns both vibration and noise attenuation by feedback as well as by feedforward compensation when a measurement of an image of the disturbance (noise or vibration) is available. The paper will review the basic algorithms and various extensions trying to emphasize the advantages of using Youla-Kucera parametrization. Specific aspects related to the use of this approach in adaptive active vibration and noise control will be mentioned. A brief review of applications and experimental testing will be provided.

KEYWORDS
Adaptive control, Feedback control, Feedforward control, Youla-Kucera parametrization, Active nois control, Active vibration control

Abbreviations:
ANC - Active noise control system
AVC - Active vibration control system
FIRYK - Youla-Kucera parameterized IIR adaptive feedforward compensator using a FIR Youla-Kucera filter
IIR - IIR adaptive feedforward compensator
IIRYK - Youla-Kucera parameterized IIR adaptive feedforward compensator using an IIR Youla-Kucera filter
IMP - Internal model principle
PAA - Parameter adaptation algorithm
QFIR - Youla-Kucera FIR filter
QIIR - Youla-Kucera IIR filter
SPR - Strictly positive real (transfer function)
YK - Youla-Kucera

1. Introduction

Active vibration control (AVC) and active noise control (ANC) are two very dynamic fields driven on one hand by environmental issues and on the other hand by technological issues.
From the point of view of automatic control, AVC and ANC raise the same type of problems even if the implementation aspects are very different and the dynamic mod-

The paper is dedicated to Professor A.L. Fradkov for his contributions to the field of adaptive control with the occasion of his $\boldsymbol{70^{th}}$ birthday.

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els of the systems involved do not have the same complexity.
One of the major control problems encountered both in ANC and AVC is the attenuation of disturbances (noise or vibration) using a secondary source of noise or vibration (the actuator in control terms). The term "active " is associated to this type of structures. The objective is to drastically reduce the level of residual noise or vibration. In practice one encounters two situations:

- No information available in real time about the disturbance
- An image of the disturbance is available

For the first situation a feedback solution has to be considered. For the second situation a feedforward compensation scheme can be considered, eventually combined with a feedback control.
The problem to be solved is a regulation problem, the objective being to minimize the residual noise (in ANC) or the residual acceleration (or force) in AVC. The regulated variable (i.e. the output of the system as well as the performance variable) is the residual noise or acceleration or force measurement. The control signal to be computed is the input to the hardware of the compensatory path (called "secondary path"). Fig. 1 gives the structure of the feedback control. The disturbance propagates through the so called primary path and compensation is done through the secondary path.
Fig.2 gives a view of a feedforward compensator (a feedback control can be added).

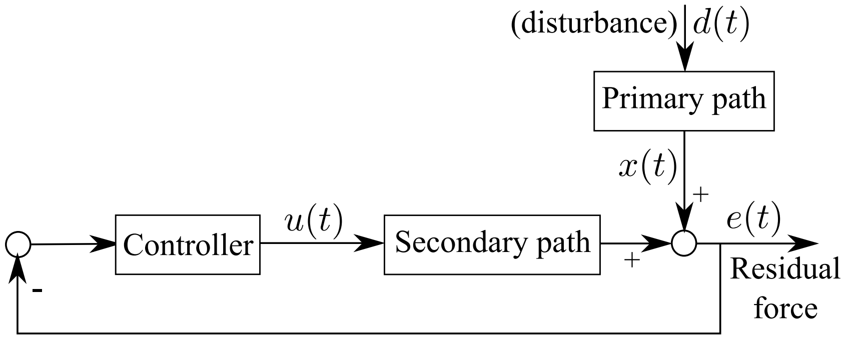

Figure 1. Active compensation by feedback.

The signal $d(t)$ is an image of the disturbance (called also "source"). Its measured value denoted $y(t)$ will be processed by the feedforward compensator in order to minimize the residual noise or acceleration. Unfortunately in most of the systems there is an unwanted positive coupling from the control signal $u(t)$ to the measured signal $y(t)$, since the effect of the compensator propagates toward the residual noise or acceleration measurement but also toward the measurement of the image of the disturbance (only in the absence of the compensator $y(t) = d(t)$). See I. Landau, Airimitoaie, Castellanos-Silva, and Constantinescu (2016) and Zeng and de Callafon (2006) for ilustrative examples. This block characterizing the internal positive feedback is denoted by M. Since this feedback effect is positive it raises of course stability problems. Therefore the design of the feedforward compensator involves two objectives:

- minimization of the residual noise or acceleration
- simultaneous stabilization of the internal positive feedback loop formed by the feedforward compensator and the positive coupling block.

In a large number of applications, the dynamic characteristics of the secondary

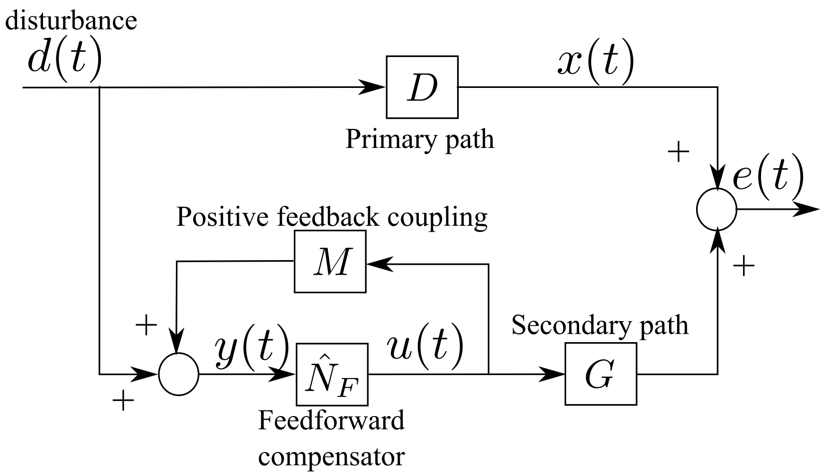

Figure 2. Active compensation by feedforward.

path (the "plant" in control terms) are constant or subject to small variations. In addition using system identification methodology an accurate model of the secondary path (as well as of the reverse path in feedforward control) can be obtained from on site experimental data (see I. Landau et al. (2016), Melendez, Landau, Dugard, and Buche (2017)). Therefore one can assume that the plant model is constant and known and the eventually small variations of the plant model are quantifiable and bounded and therefore, their effect can be handled by the control design.
The challenging problem for control design comes from the fact that the characteristics of the disturbances (defined in the frequency domain) are in general unknown and time-varying. However some information upon the range of variations and the structure of the disturbance are in general available. It is convenient to classify the type of disturbances as:

- single or multiple narrow-band disturbances
- broad (finite)-band disturbances[2]

One has to solve an adaptive regulation problem (see I. D. Landau, Alma, Constantinescu, Martinez, and Noë (2011)for details on adaptive regulation) characterized by the presence of unknown and time varying disturbances while the plant dynamics is constant and known [3]. It turns out that in this context, the Youla-Kucera parametrization plays a fundamental role for implementing efficient adaptive control algorithms.
The objective of the paper is to present the fundamental issues in using Youla-Kucera parametrization in Adaptive ANC and AVC and to illustrate its advantages. Some open problems will also be mentioned. Before going to technical details let us point out some of the pertinent advantages of using Youla-Kucera parametrization:

- Use of Youla-Kucera parametrization allows to derive direct adaptive control schemes for adaptive feedback regulation
- As a consequence a substantial reduction of the computer load is obtained
- Even when indirect adaptive regulation schemes are used, a significant reduction

[2]Of course, combination of the two types of disturbances is possible
[3]The basic adaptive control paradigm deals with unknown and time-varying plants while making assumptions on the disturbances and reference

of the computer load related to the solution of a Bezout equation is obtained

- In adaptive feedforward compensation it allows to decouple the problem of stabilizing the internal positive loop from the problem of minimizing the residual noise or acceleration
- When using YK parametrization in the context of adaptive feedforward compensation, the design of filters operating on the regressor vector allowing to satisfy a strictly postive real condition (SPR) for stability is much easier

The use of YK parametrization in the context of adaptive ANC and AVC raised also a number of interesting design problems (the design of the central controller, the size of the parametrization, use of FIR or IIR structures for the YK filter) which have not been considered in the traditional control literature. These issues, even if they may be considered as minor from a control theoretical point of view, have a tremendous impact on the final performance of the ANC or AVC systems. A basic reference for the Youla-Kucera parametrization is the survey paper Anderson (1998) published in 1998. However the specific problems encountered in ANC and AVC are not covered (these results have been published basically after 1998).
The paper consider a discrete-time representation of the AVC and ANC systems since most of the applications have been done using a discrete-time formulation. Of course a continuous time representation for the Youla Kucera parametrization can be used. An example of continuous time approach is Wang, Aranovski, and Bobtsov (2017).
The paper is organized as follows: Section 2 reviews the basic system's equations. Section 3 is dedicated to the use of YK parametrization in adaptive feedback regulation. Section 4 illustrates the use of YK parametrization for adaptive feedforward compensation. Section 5 provides a brief summary of experiments and applications of YK parametrization in adaptive active noise and vibration control.

2. Plant representation and feedback controller structure

The structure of a linear time invariant discrete time model of the plant- the secondary path- used for controller design is:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^*(z^{-1})}{A(z^{-1})} \ , \tag{1}$$

with:

$$d = \text{the plant pure time delay in number of sampling periods}$$

$$\begin{aligned} A &= 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A} \ ; \\ B &= b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} = z^{-1} B^* \ ; \\ B^* &= b_1 + \cdots + b_{n_B} z^{-n_B+1} \ , \end{aligned}$$

where $A(z^{-1})$, $B(z^{-1})$, $B^*(z^{-1})$ are polynomials in the complex variable z^{-1} and n_A, n_B and $n_B - 1$ represent their orders[4]. The model of the plant may be obtained by

[4] The complex variable z^{-1} will be used for characterizing the system's behavior in the frequency domain and the delay operator q^{-1} will be used for describing the system's behavior in the time domain.

system identification. Details on system identification of the models considered in this paper can be found in Constantinescu (2001); Constantinescu and Landau (2003); Karimi (2002); I. Landau, Karimi, and Constantinescu (2001); I. Landau and Zito (2005); Melendez et al. (2017).

Since in this paper we are focused on regulation, the controller to be designed is a RS-type polynomial controller (I. Landau and Zito (2005); I. D. Landau, Lozano, M'Saad, and Karimi (2011)).

The output of the feedback system $e(t)$ and the plant input $u(t)$ when using a RS polynomial controller may be written as:

$$\begin{aligned} e(t) &= \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + x(t) \ ; & (2) \\ S(q^{-1}) \cdot u(t) &= -R(q^{-1}) \cdot e(t) \ , & (3) \end{aligned}$$

where q^{-1} is the delay (shift) operator ($x(t) = q^{-1}x(t+1)$) and $x(t)$ is the resulting additive disturbance on the output of the system. $R(z^{-1})$ and $S(z^{-1})$ are polynomials in z^{-1} having the orders n_R and n_S, respectively, with the following expressions:

$$\begin{aligned} R(z^{-1}) &= r_0 + r_1 z^{-1} + \ldots + r_{n_R} z^{-n_R} = R'(z^{-1}) \cdot H_R(z^{-1}) \ ; & (4) \\ S(z^{-1}) &= 1 + s_1 z^{-1} + \ldots + s_{n_S} z^{-n_S} = S'(z^{-1}) \cdot H_S(z^{-1}) \ , & (5) \end{aligned}$$

where H_R and H_S are pre-specified parts of the controller (used for example to incorporate the internal model of a known disturbance or to open the loop at certain frequencies).

We define the following sensitivity functions:

- Output sensitivity function (the transfer function between the disturbance $x(t)$ and the output of the system $y(t)$):

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{P(z^{-1})} \ ; \tag{6}$$

- Input sensitivity function (the transfer function between the disturbance $x(t)$ and the input of the system $u(t)$):

$$S_{up}(z^{-1}) = -\frac{A(z^{-1})R(z^{-1})}{P(z^{-1})} \ , \tag{7}$$

where

$$\begin{aligned} P(z^{-1}) &= A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \\ &= A(z^{-1})S'(z^{-1}) \cdot H_S(z^{-1}) + z^{-d}B(z^{-1})R'(z^{-1}) \cdot H_R(z^{-1}) & (8) \end{aligned}$$

defines the poles of the closed loop (roots of $P(z^{-1})$). In pole placement design, $P(z^{-1})$ is the polynomial specifying the desired closed loop poles and the controller polynomials $R(z^{-1})$ and $S(z^{-1})$ are minimal degree solutions of (8) where the degrees of P, R and S are given by: $n_P \leq n_A + n_B + d - 1$, $n_S = n_B + d - 1$ and $n_R = n_A - 1$. $P(z^{-1})$ has usually the following structure:

$$P(z^{-1}) = P_0(z^{-1})P_{aux}(z^{-1}) \tag{9}$$

where $P_0(z^{-1})$ defines the dominant poles and $P_{aux}(z^{-1})$ represents the auxiliary poles

added for robustenss reasons.
Using the equations (2) and (3), one can write the output of the system as:

$$e(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot x(t) = S_{yp}(q^{-1}) \cdot x(t) \ . \quad (10)$$

For more details on RS-type controllers and sensitivity functions see I. Landau and Zito (2005).

Suppose that $x(t)$ is a deterministic disturbance, so it can be written as

$$x(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \ , \quad (11)$$

where $\delta(t)$ is a Dirac impulse and $N_p(z^{-1})$, $D_p(z^{-1})$ are coprime polynomials in z^{-1}, of degrees n_{N_p} and n_{D_p}, respectively. In the case of stationary disturbances the roots of $D_p(z^{-1})$ are on the unit circle. The energy of the disturbance is essentially represented by D_p. The contribution of the terms of N_p is weak compared to the effect of D_p, so one can neglect asymptotycally the effect of N_p.

A key point in the design of feedback control for attenuation of disturbances is the *Internal Model Principle*(Francis & Wonham, 1976).

Internal Model Principle: *The effect of the disturbance given in (11) upon the output:*

$$e(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \ , \quad (12)$$

where $D_p(z^{-1})$ *is a polynomial with roots on the unit circle and* $P(z^{-1})$ *is an asymptotically stable polynomial, converges asymptotically towards zero if and only if the polynomial* $S(z^{-1})$ *in the RS controller has the form:*

$$S(z^{-1}) = D_p(z^{-1})S'(z^{-1}) \ . \quad (13)$$

In other terms, the pre-specified part of $S(z^{-1})$ should be chosen as $H_S(z^{-1}) = D_p(z^{-1})$ and the controller is computed using eq. (8), where P, D_p, A, B, H_R and d are given

Obviously the model of the disturbance should be known in order to apply the IMP. Therefore, when the model of the disturbance is unknown and/or time varying in order to apply the IMP, one has to identify in real time the model of the disturbance and then solve in real-time a Bezout equation which in general is of high order (the orders of the polynomials A and B are large in ANC and AVC - see I. Landau et al. (2016) and Melendez et al. (2017). This lead to an *indirect adaptive regulation scheme*. One may ask however if it is possible to develop a direct adaptive regulation scheme. As it will be shown next this indeed is possible using Youla-Kucera parametrization of the controller.

3. Adaptive feedback regulation using Youla-Kucera paramametrization

3.1. Youla-Kucera parametrization for feedback regulation

Fig 3 gives the configuration of the YK parametrized feedback controller (to be compared with Figure 1). $q^{-d}B/A$ defines the model of the secondary path (called also

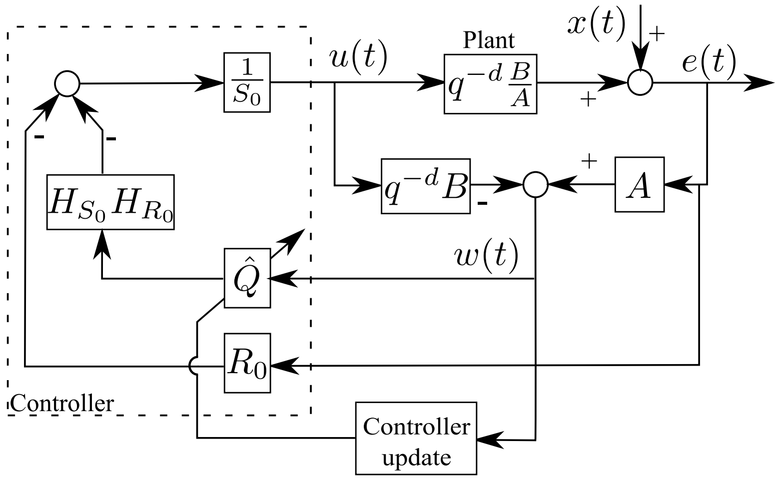

Figure 3. Youla-Kucera direct adaptive feedback regulation scheme.

plant), $\hat{Q}$ designates the so called *YK filter*. R_0 and S_0 defines what is called the *central controller.*

$$S_0 = 1 + s_1^0 z^{-1} + \ldots + s_{n_{S_0}}^0 z^{-n_{S_0}} = S_0'(z^{-1}) \cdot H_{S_0}(z^{-1}), \tag{14}$$

$$R_0 = r_0^0 + r_1^0 z^{-1} + \ldots + r_{n_{R_0}}^0 z^{-n_{R_0}} = R_0'(z^{-1}) \cdot H_{R_0}(z^{-1}), \tag{15}$$

where $H_{S_0}(q^{-1})$ and $H_{R_0}(q^{-1})$ represent pre-specified parts of the controller (used for example to incorporate the internal model of a known disturbance or to open the loop at certain frequencies) and $S_0'(q^{-1})$ and $R_0'(q^{-1})$ are computed using Pole Placement. The characteristic polynomial, which specifies the desired closed-loop poles of the system when using only the central controller is given by (see also I. Landau and Zito (2005)):[5]

$$P_0(z^{-1}) = A(z^{-1})S_0(z^{-1}) + z^{-d}B(z^{-1})R_0(z^{-1}), \tag{16}$$

Introducing the expressions of S_0 and R_0 given in Eqs. (14) and (15), R_0' and S_0' are solutions of:

$$P_0(z^{-1}) = A(z^{-1})S_0'(z^{-1})H_{S_0}(q^{-1}) + z^{-d}B(z^{-1})R_0'(z^{-1})H_{R_0}(q^{-1}), \tag{17}$$

In what follows the Youla–Kučera parametrization (Anderson (1998); Tsypkin (1997)) is used. Nevertheless, the Youla–Kučera parametrization is not unique. It depends on the right coprime factorization selected $G = ND^{-1}$. Four factorization are mostly used

[5] It is assumed that a reliable model identification is achieved and therefore the estimated model is assumed to be equal to the true model.

I. Landau, Castellanos Silva, Airimitoaie, Buche, and Noé (2013):

$$N = G; \quad D = I. \tag{18}$$

$$N = z^{-m}; \quad D = P_m \quad \text{with} \quad G \approx z^{-m} P_m^{-1}. \tag{19}$$

$$N = q^{-d} B; \quad D = A \quad \text{with} \quad G = q^{-d} \frac{B}{A}. \tag{20}$$

$$N = q^{-d} BF; D = AF \quad \text{with} \quad G = q^{-d} \frac{B}{A}; F = \frac{F_N}{F_D}, \tag{21}$$

with F and F^{-1} asymptotically stable [6]. More details can be found in I. Landau et al. (2013). Subsequently the parametrization (20) will be used.

Selecting a FIR structure for the Q filter associated to the Youla–Kučera parametrization, the controller's polynomials become:

$$R = R_0 + AQH_{S_0} H_{R_0}, \tag{22}$$
$$S = S_0 - z^{-d} BQH_{S_0} H_{R_0}, \tag{23}$$

where R_0 and S_0 define the central controller which verifies the desired specifications in the absence of the disturbance. The characteristic polynomial of the closed-loop is still given by eq. (17) (can be verified by simple calculations). The output and input sensitivity functions are still given by eq.(6) and (7) with P replaced by P_0 (the denominator of the sensitivity functions remains unchanged whatever FIR Q polynomial is used).

It is important to remark that the signal $w(t)$ in Fig. 3 is an image of the disturbance. Its expression (when plant parameters are perfectly known) is:

$$w(t) = A(q^{-1}) x(t) \tag{24}$$

As such the YK parametrization can be interpreted also as a "disturbance observer" i.e. it also belong to the class of solutions called DOB (disturbance observer based) control Li, Qiu, Ji, Zhu, and Li (2011).
Applying the internal model principle (IMP), the pre-specified part of $S(z^{-1})$ (denoted H_S see Eq. (5)) should incorporate the denominator of the model of the disturbance D_p, i.e.

$$H_S(z^{-1}) = D_p(z^{-1}) H_{S_0}(z^{-1}).$$

The controller is computed solving

$$P = AD_p H_{S_0} S' + z^{-d} BH_{R_0} R', \tag{25}$$

[6]As a consequence of the presence of the filter F both in N and in D, strictly speaking N and D will no more be coprime. This factorization is used in (de Callafon & Fang, 2013)

where P, D_p, A, B, H_{R_0}, H_{S_0} and d are given.[7] In the context of the Youla–Kučera controller parametrization using a FIR Q filter,

$$Q(z^{-1}) = q_0 + q_1 z^{-1} + \ldots + q_{n_Q} z^{-n_Q}. \tag{26}$$

application of the internal model principle leads to the problem of finding Q such that:

$$S = S_0' H_{S_0} - z^{-d} B Q H_{S_0} H_{R_0} = D_p H_{S_0} S' \tag{27}$$

So in order to compute the corresponding Q polynomial one has to solve the diophantine equation

$$S' D_p + z^{-d} B H_{R_0} Q = S_0', \tag{28}$$

where D_p, d, B, S_0', and H_{R_0} are known and S' and Q are unknown. The Bezout type Eq. (28) has a unique solution for S' and Q with: $n_{S_0'} \leq n_{D_p} + n_B + d + n_{H_{R_0}} - 1$, $n_{S'} = n_B + d + n_{H_{R_0}} - 1$, $n_Q = n_{D_p} - 1$, (I. Landau & Zito, 2005). One sees that the order n_Q of the polynomial Q depends upon the structure of the disturbance model and not upon the structure of the plant model.

This approach leads to indirect adaptive regulation scheme when one would like to handle unknown and time varying disturbances. However the size of the Bezout equation which has to be solved in real-time has been drastically reduced (compare eq. (28) with eq. (8)).

3.2. Direct adaptive regulation for disturbance attenuation

The objective is to find an estimation algorithm which will directly estimate the parameters of the internal model in the controller in the presence of an unknown disturbance (but of known structure) without modifying the closed loop poles. Clearly, the Q-parametrization is a potential option since modifications of the Q polynomial will not affect the closed loop poles. In order to build an estimation algorithm it is necessary to define an *error equation* which will reflect the difference between the optimal Q polynomial and its current value.

In Tsypkin (1997), such an error equation is provided and it can be used for developing a direct adaptive control scheme. This idea has been used in Ben Amara, Kabamba, and Ulsoy (1999a, 1999b); I. Landau, Constantinescu, and Rey (2005); Valentinotti (2001). Using the Q-parametrization, the output of the system in the presence of a disturbance can be expressed as:

$$e(t) = \frac{A[S_0 - q^{-d} B H_{S_0} H_{R_0} Q]}{P} \cdot \frac{N_p}{D_p} \cdot \delta(t) = \frac{S_0 - q^{-d} B H_{S_0} H_{R_0} Q}{P} \cdot w(t), \tag{29}$$

where $w(t)$ (see also Fig. 3) is given by (using Eqs. (24) and (11)):

$$w(t) = \frac{A N_p}{D_p} \cdot \delta(t) = A \cdot y(t) - q^{-d} B \cdot u(t). \tag{30}$$

Taking into consideration that the adaptation of Q is done in order to obtain an output $e(t)$ which tends asymptotically to zero, one can define $\varepsilon^0(t+1)$ as the value of $e(t+1)$

[7] Of course, it is assumed that D_p and B do not have common factors.

obtained with $\hat{Q}(t, q^{-1})$ (the estimate of Q at time t, written also $\hat{Q}(t)$)

$$\varepsilon^{\circ}(t+1) = \frac{S_0}{P} \cdot w(t+1) - \hat{Q}(t)\frac{q^{-d}B^* H_{S_0} H_{R_0}}{P} \cdot w(t). \tag{31}$$

Similarly, the *a posteriori* error becomes (using $\hat{Q}(t+1)$) as:[8]

$$\varepsilon(t+1) = \frac{S_0}{P} \cdot w(t+1) - \hat{Q}(t+1)\frac{q^{-d}B^* H_{S_0} H_{R_0}}{P} \cdot w(t). \tag{32}$$

Replacing S_0 from the last equation using the expression given in (14) ($S_0 = S_0' H_{S_0}$) and using (28) for S_0', one obtains:

$$\varepsilon(t+1) = [Q - \hat{Q}(t+1)] \cdot \frac{q^{-d}B^* H_{S_0} H_{R_0}}{P} \cdot w(t) + \eta(t+1), \tag{33}$$

where

$$\eta(t) = \frac{S' D_p H_{S_0}}{P} \cdot w(t) = \frac{S' H_{S_0} A N_p}{P} \cdot \delta(t) \tag{34}$$

is a signal which tends asymptotically towards zero since P is an asymptotically stable polynomial.

Define the estimated polynomial $\hat{Q}(t, q^{-1}) = \hat{q}_0(t) + \hat{q}_1(t)q^{-1} + \ldots + \hat{q}_{n_Q}(t)q^{-n_Q}$ and the associated estimated parameter vector $\hat{\theta}(t) = [\hat{q}_0(t)\ \hat{q}_1(t)\ \ldots\ \hat{q}_{n_Q}(t)]^T$. Define the fixed parameter vector corresponding to the optimal value of the polynomial Q as: $\theta = [q_0\ q_1\ \ldots\ q_{n_Q}]^T$.

Denote

$$w_2(t) = \frac{q^{-d}B^* H_{S_0} H_{R_0}}{P} \cdot w(t) \tag{35}$$

and define the following observation vector:

$$\phi^T(t) = [w_2(t) \quad w_2(t-1) \ldots w_2(t-n_Q)] \ . \tag{36}$$

Equation (33) becomes

$$\varepsilon(t+1) = [\theta^T - \hat{\theta}^T(t+1)] \cdot \phi(t) + \eta(t+1) \ . \tag{37}$$

One can remark that $\varepsilon(t)$ corresponds to an adaptation error (I. D. Landau, Lozano, et al. (2011)).

From equation (31) one obtains the *a priori* adaptation error:

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t) \ ,$$

[8]In adaptive control and estimation the predicted output at $t+1$ can be computed either on the basis of the previous parameter estimates (*a priori*, time t) or on the basis of the current parameter estimates (*a posteriori*, time $t+1$).

with

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) \; ; \tag{38}$$

$$w(t+1) = A(q^{-1}) \cdot y(t+1) - q^{-d} B^*(q^{-1}) \cdot u(t) \; , \tag{39}$$

where $B(q^{-1})u(t+1) = B^*(q^{-1})u(t)$.

The *a posteriori* adaptation error is obtained from (32):

$$\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t) \; .$$

Taking into account that $\eta(t)$ tends asymptotically to *zero* eq. (37) has the standard form of an adaptation error equation leading to the following algorithm for the estimation of the parameters of $\hat{Q}(t, q^{-1})$ (I. D. Landau, Lozano, et al. (2011)):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1) \; ; \tag{40}$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \phi^T(t)F(t)\phi(t)} \; ; \tag{41}$$

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t) \; ; \tag{42}$$

$$F(t+1) = \frac{1}{\lambda_1(t)} \left[F(t) - \frac{F(t)\phi(t)\phi^T(t)F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \phi^T(t)F(t)\phi(t)} \right] \; . \tag{43}$$

$$1 \geq \lambda_1(t) > 0; 0 \leq \lambda_2(t) < 2 \tag{44}$$

where $\lambda_1(t), \lambda_2(t)$ allow to obtain various profiles for the evolution of the adaption gain $F(t)$ (for details see I. Landau and Zito (2005); I. D. Landau, Lozano, et al. (2011)).

In order to implement this methodology for disturbance rejection (see fig. 3), it is supposed that the plant model $\frac{z^{-d}B(z^{-1})}{A(z^{-1})}$ is known (identified) and that it exists a controller $[R_0(z^{-1}), S_0(z^{-1})]$ which verifies the desired specifications in the absence of the disturbance. One also supposes that the degree n_Q of the polynomial $Q(z^{-1})$ is fixed, $n_Q = n_{D_p} - 1$, i.e. the structure of the disturbance is known. A stability analysis is provided in I. Landau et al. (2005). It does not require a SPR condition. If the disturbance is a sum of n narrow band disturbances, the order of the polynomial D_p is $2n$. It is also shown in I. Landau et al. (2005) that if $n_Q = 2n - 1$ and the disturbance contains n distinct sinusoids (richness condition), asymptotic convergence of the Q parameters towards their optimal values is obtained,

3.3. Design of the central controller

As it is well known, the introduction of the internal model for the perfect rejection of the disturbance (asymptotically) may have as effect to raise the maximum value of the modulus of the output sensitivity function in the vicinity of the attenuation zone (waterbed effect). This may lead to unacceptable values for the modulus margin and the delay margins if the design of the central controller is not appropriately done (see I. Landau and Zito (2005)). As a consequence, a robust control design should be considered assuming that the model of the disturbance is known as well as its domain of variations in the frequency domain. The objective is that for all situations an acceptable modulus margin and delay margin are obtained.

On the other hand at the frequencies where perfect rejection of the disturbance is achieved one has $S_{yp}(e^{-j\omega}) = 0$ and

$$\left|S_{up}(e^{-j\omega})\right| = \left|\frac{A(e^{-j\omega})}{B(e^{-j\omega})}\right| . \tag{45}$$

Equation (45) corresponds to the inverse of the gain of the system to be controlled. The implication of equation (45) is that cancellation (or in general an important attenuation) of disturbances on the output should be done only in frequency regions where the system gain is large enough. If the gain of the controlled system is too low, $|S_{up}|$ will be large at these frequencies. Therefore, the robustness vs additive plant model uncertainties will be reduced and the stress on the actuator will become important I. Landau and Zito (2005). Equation (45) also implies that serious problems will occur if $B(z^{-1})$ has complex zeros close to the unit circle (stable or unstable zeros) at frequencies where an important attenuation of disturbances is required since at these frequencies the modulus of $B(e^{-j\omega})$ will be very low. It is mandatory to avoid attenuation of disturbances at these frequencies.

Since on one hand we would not like to react to very high frequency disturbances and on the other hand we would like to have a good robustness it is often wise to open the loop at $0.5f_s$ (f_s is the sampling frequency) by introducing a fixed part in the controller $H_{R_0}(q^{-1}) = 1 + q^{-1}$ (for details see I. Landau and Zito (2005) and I. Landau et al. (2016).

3.4. Use of IIR Youla Kucera parametrization

Consider now the case of the disturbance model given in eq. (11) and lets assume that we are in a stochastic context and that we deal with a narrow band disturbances. In this case $x(t)$ has the expression:

$$x(t) = \frac{D_p(\rho z^{-1})}{D_p(z^{-1})} v(t) \tag{46}$$

where $v(t)$ is a zero mean discret time white Gaussian noise sequence and

$$D_p(z^{-1}) = 1 + \alpha z^{-1} + z^{-2}, \tag{47}$$

is a polynomial with roots on the unit circle.[9] In (47), $\alpha = -2\cos(2\pi\omega_1 T_s)$, ω_1 is the frequency of the disturbance in Hz, and T_s is the sampling time. $D_p(\rho z^{-1}))$ is given by:

$$D_p(\rho z^{-1}) = 1 + \rho\alpha z^{-1} + \rho^2 z^{-2}, \tag{48}$$

with $0 < \rho < 1$. The roots of $D_p(\rho z^{-1})$ are in the same radial line as those of $D_p(z^{-1})$ but inside of the unitary circle, and therefore stable Nehorai (1985).

[9] Its structure in a mirror symmetric form guarantees that the roots are always on the unit circle.

Using the output sensitivity function, the output of the plant in the presence of the disturbance can be expressed as

$$e(t) = \frac{AS'}{P_0} \frac{H_S}{P_{aux}} \frac{D_p(\rho q^{-1})}{D_p(q^{-1})} v(t) \tag{49}$$

where $P = P_0 P_{aux}$ (see eq. (9). One can rewrite eq. (49) as:

$$e(t) = \frac{AS'}{P_0} \beta(t) \tag{50}$$

where

$$\beta(t) = \frac{H_S}{P_{aux}} \frac{D_p(\rho q^{-1})}{D_p(q^{-1})} v(t) \tag{51}$$

In order to minimize the effect of the disturbance upon $y(t)$, one should minimize the variance of $\beta(t)$. One has two tuning devices: H_S and P_{aux}. Minimization of the variance of $\beta(t)$ is equivalent of searching H_S and P_{aux} such that $\beta(t)$ becomes a white noise Astrom and Wittenmark (1984); I. Landau and Zito (2005). The obvious choices are $H_S = D_p$ (which corresponds to the IMP) and $P_{aux} = D_p(\rho q^{-1})$. Of course this development can be generalized for the case of multiple narrow-band disturbances. Therefore in order to optimally reject a single or multiple narrow band disturbance one needs to introduce in addition to the internal model (the denominator of the disturbance model) auxiliary tuned closed loop poles. This can be achieved by using an IIR YK parametrization.
Consider now the case of a Q filter as ratio of rational polynomials (IIR filter) with an asymptotically stable denominator:

$$Q(z^{-1}) = \frac{B_Q(z^{-1})}{A_Q(z^{-1})} \tag{52}$$

The YK controller will have the structure:

$$R(z^{-1}) = A_Q(z^{-1})R_0(z^{-1}) + A(z^{-1})B_Q(z^{-1}) \tag{53}$$

$$S(z^{-1}) = A_Q(z^{-1})S_0(z^{-1}) - z^{-d}B(z^{-1})B_Q(z^{-1}) \tag{54}$$

but in this case the poles of the closed-loop will be given by

$$P(z^{-1})_{QIIR} = P(z^{-1})A_Q(z^{-1}) \tag{55}$$

In the case of IIR Q filters, the poles of the denominator of Q will appears as additional poles of the closed-loop. Therefore the extension of this approach to the adaptive case lead to the need of adapting both the parameters of the numerator and denominator of the IIR Q filter. While an adaptation algorithm can be derived (see (I. Landau et al., 2016)) there is not enough richness in the signal to adjust the parameters of both $B_Q$and A_Q. A solution has been proposed and successfully tested in Castellanos-Silva, Landau, Dugard, and Chen (2016) where a direct adaptation of the parameters of B_Q is done and the parameters of the A_Q are cleverly computed from the estimation of the parameters of the disturbance (see also Chen and Tomizuka (2013)).

3.5. Overparametrization

When the objective is to reject disturbances using the IMP, the order of the Q FIR polynomial results directly from the supposed order of D_P (the denominator of the disturbance model). The idea of using over parametrized Q FIR filters emerged as a possibility for handling uncertainties in the plant model Valentinotti (2001). This will require to formalize the plant uncertainties in a proper way. Work remains to be done in this direction even if some tests have been done. A reference dealing with the stability of the adaptive schemes in the presence of model errors is Mullhaupt and Bonvin (2012). The use of over parametrized Q filters was also considered in Jafari, Ioannou, Fitzpatrick, and Wang (2013) as a possibility of improving the robustness and performances of the scheme. The paper Castellanos-Silva, Landau, and Ioannou (2015) evaluates this approach and compares it with a procedure for designing the central controller in order to improve the performances. The basic conclusion is that small increase of the order of the Q filter with respect to the minimal order is beneficial in practice and this can be combined with an improved design of the central controller. The explanation is related to the presence of some low level disturbances in addition to the main narrow band disturbances to be canceled. These secondary disturbances will be attenuated by the additional Q parameters.

3.6. Indirect adaptive feedback attenuation

In indirect adaptive feedback regulation, a disturbance observer is built which allows to identify the model of the disturbance used for the design of the controller. The interest of the indirect approach is related to the assignment of different attenuation levels in the frequency domain (instead of total rejection when using IMP). This is achieved by shaping in real time the output sensitivity function using band stop filters assuring a desired amount of attenuation for each narow band disturbance. Details can be found in Airimitoaie and Landau (2014). Using a YK parametrization of the band stop filters, a drastic reduction of the computation load is obtained since there is a significant reduction in the size of the Bezout equation which has to be solved.

4. Adaptive feedforward disturbance attenuation using Youla-Kucera parametrization

When a correlated measurement with the disturbance is available, adaptive feedforward compensation of broadband vibrations or noise can be considered Elliott and Nelson (1994); Jacobson, Johnson, McCormick, and Sethares (2001); Kuo and Morgan (1996); Zeng and de Callafon (2006). However in many AVC (Active Vibration Control) or ANC (Active Noise Control) systems there is a "positive" feedback coupling between the compensator system and the correlated measurement of the disturbance which serves as reference Hu and Linn (2000); Jacobson et al. (2001); Zeng and de Callafon (2006). The positive feedback may destabilize the system.
The disturbance is assumed to be unknown and with variable spectral characteristics, but the dynamic models of the AVC and ANC systems are supposed to be constant and known (these models can be identified).
In Jacobson et al. (2001) and I. Landau, Alma, and Airimitoaie (2011), algorithms for adapting an IIR feedforward compensator in real time taking into account the presence of the internal positive feedback have been proposed, analyzed and evaluated. In

Zeng and de Callafon (2006), the idea of using a Youla-Kucera parametrization of the feedforward compensator is illustrated in the context of active noise control. Based on the identification of the system, a stabilizing YK controller is designed. The YK parameters are then updated by using a two time scale indirect procedure: (1) estimation of the Q-filter's parameters over a certain horizon, (2) updating of the controller. However direct adaptation of the Q parameters is possible. Direct adaptive schemes using FIR YK filters has been proposed and analyzed in (I. Landau, Airimitoaie, & Alma, 2012) Use of IIR YK filters has been considered in (I. D. Landau, Airimitoaie, & Alma, 2013). Both references provide also experimental results and relevant comparisons. As indicated in the introduction, the basic motivation of using a YK structure for the feedforward compensator is that the problem of the stabilization of the internal positive feedback loop can be dissociated from the problem of the optimization of the parameters of the feedforward compensator in order to minimize the residual noise or acceleration. In what follows the basic ideas for the development of such algorithms will be presented.

4.1. Basic configuration

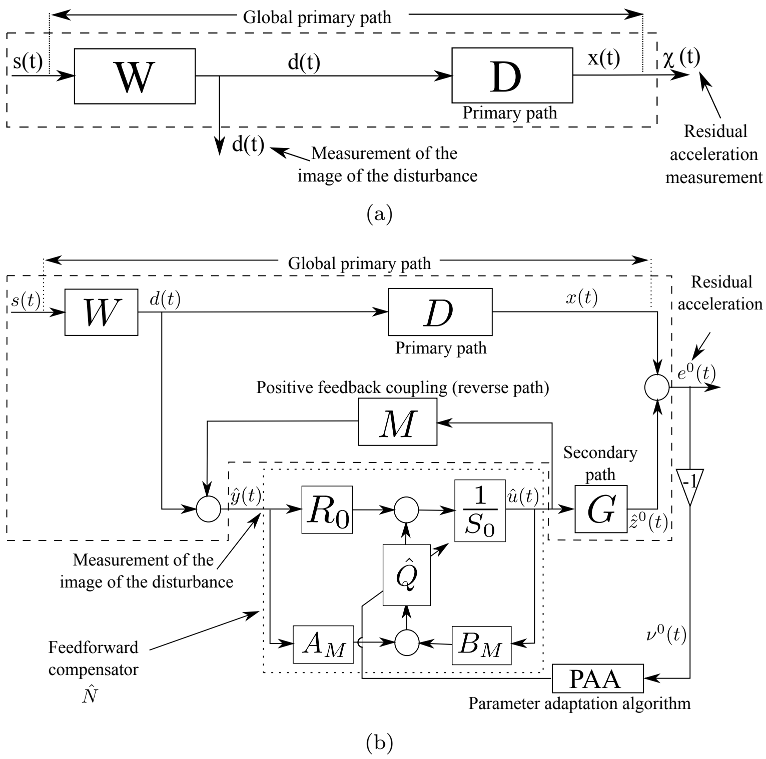

Figure 4. Feedforward AVC: in open loop (a) and with adaptive feedforward compensator (b)

The block diagrams of adaptive feedforward compensation associated with an AVC or an ANC system when an image of the disturbance is available, are shown in fig. 4. The open loop configuration is shown in fig. (4(a)). The case when the Youla-Kucera compensator is active is shown in fig. (4(b)). For adaptive IIR feedforward compensators see I. Landau et al. (2011). s(t) is the disturbance and d(t) is the correlated measurement with the disturbance (the image of the disturbance). The primary (D), secondary (G) and reverse (positive coupling) (M) paths represented in (4(b)) are respectively characterized by the asymptotically stable transfer operators:

$$D(q^{-1}) = \frac{B_D(q^{-1})}{A_D(q^{-1})} = \frac{b_1^D q^{-1} + ... + b_{n_{B_D}}^D q^{-n_{B_D}}}{1 + a_1^D q^{-1} + ... + a_{n_{A_D}}^D q^{-n_{A_D}}}, \tag{56}$$

$$G(q^{-1}) = \frac{B_G(q^{-1})}{A_G(q^{-1})} = \frac{b_1^G q^{-1} + ... + b_{n_{B_G}}^G q^{-n_{B_G}}}{1 + a_1^G q^{-1} + ... + a_{n_{A_G}}^G q^{-n_{A_G}}}, \tag{57}$$

$$M(q^{-1}) = \frac{B_M(q^{-1})}{A_M(q^{-1})} = \frac{b_1^M q^{-1} + ... + b_{n_{B_M}}^M q^{-n_{B_M}}}{1 + a_1^M q^{-1} + ... + a_{n_{A_M}}^M q^{-n_{A_M}}}, \tag{58}$$

with $B_X = q^{-1}B_X^*$ for any $x \in \{D, G, M\}$. $\hat{G}$ and $\hat{M}$ denote the identified (estimated) models of G and M[10]. The optimal IIR feedforward compensator which will minimize the residual acceleration can be written, using the Youla-Kucera parametrization (Q-parametrization), as

$$N_F(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} = \frac{R_0(q^{-1}) - A_M(q^{-1})Q(q^{-1})}{S_0(q^{-1}) - B_M(q^{-1})Q(q^{-1})} \tag{59}$$

where the optimal polynomial $Q(q^{-1})$ has a FIR structure:

$$Q(q^{-1}) = q_0 + q_1 q^{-1} + ... + q_{n_Q} q^{-n_Q}. \tag{60}$$

and $R_0(q^{-1})$, $S_0(q^{-1}) = 1 + q^{-1}S_0^*(q^{-1})$ are the polynomials of the central (stabilizing) filter and $A_M(q^{-1})$, $B_M(q^{-1})$ are given in (58).

The estimated Q polynomial is denoted by $\hat{Q}(q^{-1})$ or $\hat{Q}(\hat{\theta}, q^{-1})$ when it is a linear filter with constant coefficients or $\hat{Q}(t, q^{-1})$ during estimation (adaptation).

The input of the feedforward filter is denoted by $\hat{y}(t)$ and it corresponds to the measurement provided by the primary transducer (force or acceleration transducer in AVC or a microphone in ANC) in the absence of the compensation loop (in open loop operation $\hat{y}(t) = d(t)$). The output of the feedforward filter (which is the control signal applied to the secondary path) is denoted by $\hat{u}(t) = \hat{u}(t+1|\hat{\theta}(t+1))$ (a posteriori output). The a priori output $\hat{u}^0(t+1) = \hat{u}(t+1|\hat{\theta}(t))$ is given by:

$$\hat{u}^0(t+1) = -S_0^*\hat{u}(t) + R_0\hat{y}(t+1) + \hat{Q}(t, q^{-1})[B_M^*\hat{u}(t) - A_M\hat{y}(t+1)], \tag{61}$$

where $\hat{u}(t)$, $\hat{u}(t-1)$, ... are the "a posteriori" outputs of the feedforward filter generated by

$$\hat{u}(t+1) = -S_0^*\hat{u}(t) + R_0\hat{y}(t+1) + \hat{Q}(t+1, q^{-1})[B_M^*\hat{u}(t) - A_M\hat{y}(t+1)]. \tag{62}$$

The measured input to the feedforward filter satisfies the following equation (when feedforward compensation is active)

$$\hat{y}(t+1) = d(t+1) + \frac{B_M^*(q^{-1})}{A_M(q^{-1})}\hat{u}(t). \tag{63}$$

[10]Like for the feedback compensation it is assumed that a reliable model identification is achieved and therefore the estimated models are considered to be equal to the real models

The unmeasurable value of the output of the primary path is denoted $x(t)$. The unmeasurable "a priori" output of the secondary path will be denoted $\hat{z}^0(t+1)$.

$$\hat{z}^0(t+1) = \hat{z}(t+1|\hat{\theta}(t)) = \frac{B_G^*(q^{-1})}{A_G(q^{-1})}\hat{u}(t) \tag{64}$$

The "a posteriori" unmeasurable value of the output of the secondary path is denoted by:

$$\hat{z}(t+1) = \hat{z}(t+1|\hat{\theta}(t+1)) \tag{65}$$

The a priori adaptation error is defined as:

$$\nu^0(t+1) = \nu(t+1|\hat{\theta}(t)) = -e^0(t+1) = -x(t+1) - \hat{z}^0(t+1) \tag{66}$$

where $e^0(t+1)$ is the measured residual noise or acceleration (or force). The "a posteriori" adaptation error (computed) will be given by:

$$\nu(t+1) = \nu(t+1|\hat{\theta}(t+1)) = -x(t+1) - \hat{z}(t+1). \tag{67}$$

When using an estimated filter $\hat{N}_F$ with constant parameters: $\hat{u}^0(t) = \hat{u}(t)$, $\hat{z}^0(t) = \hat{z}(t)$ and $\nu^0(t) = \nu(t)$.

The objective is to develop stable recursive algorithms for adaptation of the parameters of the Q filter such that the measured residual error (acceleration or force in AVC, noise in ANC) be minimized in the sense of a certain criterion. This has been done for broadband disturbances $d(t)$ (or $s(t)$) with unknown and variable spectral characteristics and an unknown primary path model.

4.2. Parameter adaptation algorithm

The algorithm for adaptive feedforward compensation has been developed under the following hypotheses:

(1) The signal $d(t)$ is bounded (which is equivalent to say that $s(t)$ is bounded and $W(q^{-1})$ in figure 4 is asymptotically stable).

(2) It exists a central feedforward compensator N_F^0 $(R_0,\ S_0)$ which stabilizes the inner positive feedback loop formed by N_F^0 and M such that its characteristic polynomial [11]

$$P_0(z^{-1}) = A_M(z^{-1})S_0(z^{-1}) - B_M(z^{-1})R_0(z^{-1}) \tag{68}$$

is a Hurwitz polynomial.

(3) (Perfect matching condition) It exists values of the Q filter parameters such that

$$\frac{G \cdot A_M(R_0 - A_M Q)}{A_M S_0 - B_M R_0} = -D. \tag{69}$$

[11] The parenthesis (q^{-1}) will be omitted in some of the following equations to make them more compact.

(4) The effect of the measurement noise upon the measurement of the residual acceleration is neglected (deterministic context).

Then resulting algorithms have been analyzed when hypoteses 3 and 4 have been removed in I. Landau et al. (2012).

A first step in the development of the algorithms is to establish for a fixed estimated compensator a relation between the error on the Q-parameters (with respect to the optimal values) and the adaptation error ν (i.e. the residual acceleration or noise). This equation has the form (I. Landau et al. (2012)):

$$\nu(t+1/\hat{\theta}) = \frac{A_M(q^{-1})G(q^{-1})}{P_0(q^{-1})}[\theta - \hat{\theta}]^T \phi(t), \tag{70}$$

where θ, $\hat{\theta}$ and ϕ are given respectively by:

$$\theta^T = [q_0, q_1, q_2, \ldots, q_{n_Q}] \tag{71a}$$
$$\hat{\theta}^T = [\hat{q}_0, \hat{q}_1, \hat{q}_2, \ldots, \hat{q}_{n_Q}] \tag{71b}$$
$$\phi^T(t) = [\alpha(t+1), \alpha(t), \ldots, \alpha(t-n_Q+1)]. \tag{71c}$$
$$\begin{aligned}\alpha(t+1) &= B_M \hat{y}(t+1) - A_M \hat{u}(t+1) \\ &= B_M^* \hat{y}(t) - A_M \hat{u}(t+1)\end{aligned} \tag{71d}$$

where q_i are the coefficients of the optimal Q-filter and $\hat{q}_i$ are the coefficients of the fixed estimated $\hat{Q}$-filter.

Filtering the vector ϕ by an asymptotically stable filter $L(q^{-1})$, eq. (70) becomes

$$\nu(t+1/\hat{\theta}) = \frac{A_M(q^{-1})G(q^{-1})}{P_0(q^{-1})L(q^{-1})}[\theta - \hat{\theta}]^T \phi_f(t) \tag{72}$$

with

$$\begin{aligned}\phi_f(t) &= L(q^{-1})\phi(t) \\ &= [\alpha_f(t+1), \alpha_f(t), \ldots, \alpha_f(t-n_{Q+1})],\end{aligned} \tag{73}$$

where

$$\alpha_f(t+1) = L(q^{-1})\alpha(t+1). \tag{74}$$

Eq. (72) will be used to develop the adaptation algorithms.

When the parameters of $\hat{Q}$ evolve over time and neglecting the non-commutativity of the time varying operators (which implies slow adaptation Anderson et al. (1986) i.e., a limited value for the adaptation gain), equation (72) transforms into[12]

$$\nu(t+1/\hat{\theta}(t+1)) = \frac{A_M(q^{-1})G(q^{-1})}{P_0(q^{-1})L(q^{-1})}[\theta - \hat{\theta}(t+1)]^T \phi_f(t). \tag{75}$$

Eq. (75) has the standard form of an "a posteriori adaptation error equation"

[12]However, exact algorithms can be developed taking into account the non-commutativity of the time varying operators - see I. D. Landau, Lozano, et al. (2011)

I. D. Landau, Lozano, et al. (2011), which immediately suggests to use the following parameter adaptation algorithm:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\psi(t)\nu(t+1) \tag{76a}$$

$$\nu(t+1) = \frac{\nu^0(t+1)}{1+\psi^T(t)F(t)\psi(t)} \tag{76b}$$

$$F(t+1) = \frac{1}{\lambda_1(t)}\left[F(t) - \frac{F(t)\psi(t)\psi^T(t)F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \psi^T(t)F(t)\psi(t)}\right] \tag{76c}$$

$$1 \geq \lambda_1(t) > 0; 0 \leq \lambda_2(t) < 2; F(0) = \alpha I; \alpha_{max} > \alpha > 0 \tag{76d}$$

$$\psi(t) = \phi_f(t) \tag{76e}$$

where $\lambda_1(t)$ and $\lambda_2(t)$ allow to obtain various profiles for the adaptation gain $F(t)$ (see I. D. Landau, Lozano, et al. (2011)).

Two choices for the filter L will be considered:

Algorithm I $\quad L = \hat{G}$

Algorithm II

$$L = \frac{\hat{A}_M}{\hat{P}_0}\hat{G} \tag{77}$$

where

$$\hat{P}_0 = \hat{A}_M S_0 - \hat{B}_M R_0. \tag{78}$$

Equation (75) for the a posteriori adaptation error has the form:

$$\nu(t+1) = H(q^{-1})[\theta - \hat{\theta}(t+1)]^T\psi(t), \tag{79}$$

where

$$H(q^{-1}) = \frac{A_M(q^{-1})G(q^{-1})}{P_0(q^{-1})L(q^{-1})}, \quad \psi = \phi_f. \tag{80}$$

Using the results of I. D. Landau, Lozano, et al. (2011) asymptotic stability will be assured provided that the transfer funtion

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda}{2}, \ \max_t[\lambda_2(t)] \leq \lambda < 2 \tag{81}$$

is a strictly positive real (SPR) transfer function.

Remark 1: For algorithm II, the stability condition (81) for $\lambda_2 = 1$ can be transformed into Ljung and Söderström (1983)

$$\left|\left(\frac{A_M(e^{-j\omega})}{\hat{A}_M(e^{-j\omega})} \cdot \frac{\hat{P}_0(e^{-j\omega})}{P_0(e^{-j\omega})} \cdot \frac{G(e^{-j\omega})}{\hat{G}(e^{-j\omega})}\right)^{-1} - 1\right| < 1 \tag{82}$$

for all ω, which is always true provided that the estimates of M and G are close to the true values (the differences between P_0 and $\hat{P}_0$ depend only upon the estimation errors of $\hat{M}$).

4.3. Comparison with IIR adaptive feedforward compensators

Lets focus now on the differences between the IIR adaptive compensator given in I. Landau et al. (2011) and the YK adaptive compensator. Without going into the details, using a standard adaptive feedforward compensator there is also a strictly positive real condition to be satisfied in order to guarantee asymptotic stability. This condition has the form of eq. (81) where H will be given by:

$$H(q^{-1}) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})L(q^{-1})}, \quad \psi = \phi_f. \tag{83}$$

where P is given by

$$P = \hat{A}_M S - \hat{B}_M R. \tag{84}$$

and R and S are the numerator and denominator of the IIR compensator. Therefore a filter L similar to the one used above will be considered, but it will depend on the estimations of R and S.

Remark 1: For YK FIR the filter L depends on R_0 and S_0 which are known a priori while for IIR feedforward compensators the filter L depends on estimates of R and S requiring a specific procedure for starting the algorithm (initialization horizon) I. Landau et al. (2011).

Remark 2: For IIR adaptive compensators, provided that the SPR condition is satisfied, the poles of the internal "positive" loop will be asymptotically stable but they can be very close to the unit circle (they can be inside of a circle of radius 0.99999..). This may induce some numerical problems in practice (when using truncation or fixed point arithmetic).

Remark 3: The central YK controller allows to assign the poles of the internal closed loop. Therefore one can impose that all the poles of the internal loop be inside of a circle of radius $1 - \delta, \delta > 0$ (δ takes care of the numerical approximations).

Remark 4: If a model based initial IIR compensator is available, it can not in general be used to initialize the parameters of the IIR adaptive compensator since often the number of parameters of the fixed compensator is higher than the number of parameters of the adaptive IIR compensator. The situation is different for YK adaptive compensator where any initial stabilizing compensator can be used whatever its complexity is.

4.4. Adaptive feedforward compensation using IIR YK compensator

Adaptive feedforward compensators using an IIR YK parametrization have been also developed and implemented. See I. D. Landau et al. (2013). The important questions to be answered are related to the relative advantages of using FIR or IIR.

The number of adjustable parameters

The main advantage of the IIRYK adaptive feedforward compensators compared with FIRYK adaptive compensators is that they require a significantly lower number of

adjustable parameters for a given level of performance (a reduction by a factor of 2 in the application reported in I. D. Landau et al. (2013)). This is without doubt a major practical advantage in terms of implementation complexity.
The poles of the internal positive closed-loop
For FIRYK, the poles of the internal positive feedback loop are assigned by the central stabilizing controller and they remain unchanged under the effect of adaptation. For IIRYK, part of the poles of the internal positive feedback loop are assigned by the central stabilizing controller but there are additional poles corresponding to the denominator of the IIR YK filter. These poles will be asymptotically inside the unit circle if the positive real condition for stability is satisfied.

Implementation of the filter L
For IIRYK adaptive compensators a filter L is also used, but it will depend on an estimation of the denominator of the IIR YK filter, therefore an initialization procedure is necessary (which is not the case for FIR YK adaptive compensators).

Influence of the initial stabilizing controller
The performances of IIRYK adaptive compensator are less sensitive than those of FIRYK adaptive compensator with respect to the performances of the initial model based stabilizing controller.

4.5. Combing adaptive feeforward and adaptive feedback compensation

Combining adaptive feedforward compensation with fixed feedback compensation Alma, Landau, and Airimitoaie (2012) or with adaptive feedback compensation Airimitoaie and Landau (2018) lead to the improvement of the performances. However there is a strong interaction between the feedback compensator and the adaptive feedforward compensator. The filters used in the implementation of the parameter adaptation algorithm for feedforward compensation will depend upon the current value of the feedback controller parameters.

5. Experimental results

5.1. Feedback compensation of narrow band disturbances

There are many papers reporting results in using Youla Kucera parametrization for rejection of narrow band disturbances in active vibration control and in adaptive active noise control. See Ben Amara et al. (1999b)Ben Amara et al. (1999a),I. D. Landau, Alma, et al. (2011)Ficocelli and Ben Amara (2009)I. Landau et al. (2005),Martinez and Alma (2012),I. Landau et al. (2016), Airimitoaie and Landau (2016), I. Landau, Airimitoaie, and Castellanos Silva (2015) among other references.
To validate comparatively the various approaches to adaptive rejection of multiple narrow band disturbances in active vibration control, a benchmark is available. The synthesis of the results of the benchmarking can be found in I. Landau et al. (2013) as well as in the references of the contributors: Aranovskiy and Freidovich (2013),Airimitoaie, Castellanos Silva, and Landau (2013),Castellanos-Silva, Landau, and Airimitoaie (2013),Castellanos-Silva et al. (2016),de Callafon and Fang (2013),Chen and Tomizuka (2013),Karimi and Emedi (2013),Wu and Ben Amara (2013).
A recent application paper using adpative Youla Kucera feedback compensation is Wu, Zhang, Chen, and Wang (2018)

5.2. Adaptive feedforward compensation of broad band disturbances

The use of Youla - Kucera parametrization in adaptive feedforward control being more recent, there are less references available for experimental results. See Zeng and de Callafon (2006),I. Landau et al. (2012),I. D. Landau et al. (2013).

6. Conclusions

The present paper has tried to survey the use of the Youla-Kucera parametrization in active vibration and noise control. The advantages of using this parametrization both in adaptive feedback configurations as well as in adaptive feedforward compensation schemes have been enhanced. The developments and applications considered have assumed that the compensatory path is characterized by a model with constant parameters with small uncertainties. System identification allows to identify accurate models which are used for design.
The challenging issue for some new potential applications is to explore the use of the Youla-Kucera parametrization in the presence of large variations of the model of the compensatory path. This involve as an intermediate step to assess the effect of plant parameter uncertainties upon the current used schemes (see for example Mullhaupt and Bonvin (2012)).

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