PROCEEDINGS OF

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Section A.—Mathematical and Physical Sciences.

On the Viscous Flow in Metals, and Allied Phenomena.

By E. N. da C. Andrade, B.Sc., Physics Research Scholar, University College, London.

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It has been observed* that for a lead wire, loaded well beyond the elastic limit, the extension after some time becomes proportional to the time, or the flow becomes viscous in character. The rate of this viscous flow varies with the load, and the following work was undertaken to investigate the law of this variation, and the phenomenon in general.

Methods of Experiment. The Hyperbolic Weight.

The experiments were all done on wires of the metal, and the preliminary observations were all made on lead. To suspend the wires, they were soldered into stout brass hooks. In the first rough experiments the extension registered itself on a clockwork-driven drum; these experiments showed that the rate of extension for a given load became constant for a time, but finally increased, and continued increasing. This is due to two causes: (1) As the wire stretches, the length of wire being experimented on at any moment increases. (2) As the wire stretches, the cross-section diminishes, and thus the load per unit area or stress increases. This is by far the more disturbing cause, since, as shown later, the rate increases much more rapidly with the stress than would be given by a linear law. To

* *E.g.*, Trouton and Rankine, "On the Stretching and Torsion of a Lead Wire beyond the Elastic Limit," 'Phil. Mag.,' October, 1904.

obviate this difficulty an automatic method of keeping the stress constant was devised.

Constant stress was obtained by letting the weight producing the stress sink into a liquid (fig. 1) as the wire stretched, the form of the weight being so chosen that the upthrust at any moment was such that the effective load was inversely as the length of the wire at that moment, and thus was directly as the cross-sectional area. The required shape is easily shown to be given by a hyperbola of revolution,

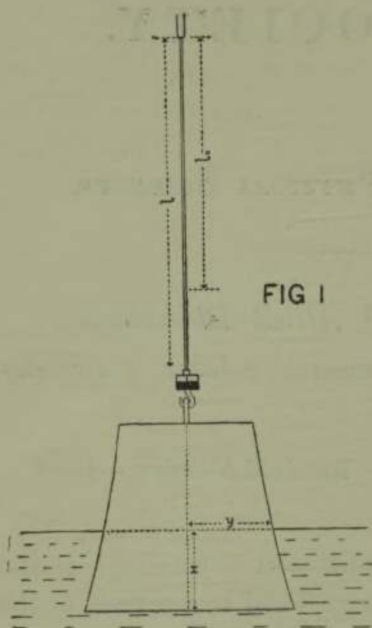


FIG 1

$$y = \sqrt{\frac{Ml_0}{\rho\pi}} \cdot \frac{1}{l_0+x},$$

where M is the mass of the load, l_0 the initial length of the wire, and ρ the density of the liquid. As water was always used as the liquid, ρ is fixed. Having chosen a particular M and l_0, the exact size of the weight is given. From the equation we see that a weight, once constructed, is only exactly efficient for one particular initial load, but with reasonable approximation the same weight can be used over a limited range of loads.

An additional advantage accruing from the device of the hyperbolic weight was soon discovered, viz., that stresses could be freely applied much greater than can possibly be used with constant loading. The possibility of working at much greater known stresses than is otherwise feasible opens up a new region for experiment.

Description of Experiments.

Experiments were made on three kinds of wires :—

(1) Approximately pure lead.
(2) "Fuse wire." An alloy of lead and tin.
(3) Approximately pure copper.

Fuse wire was used, because it shows the viscous flow to a remarkable extent; copper as being a metal usually supposed not to show a viscous flow.

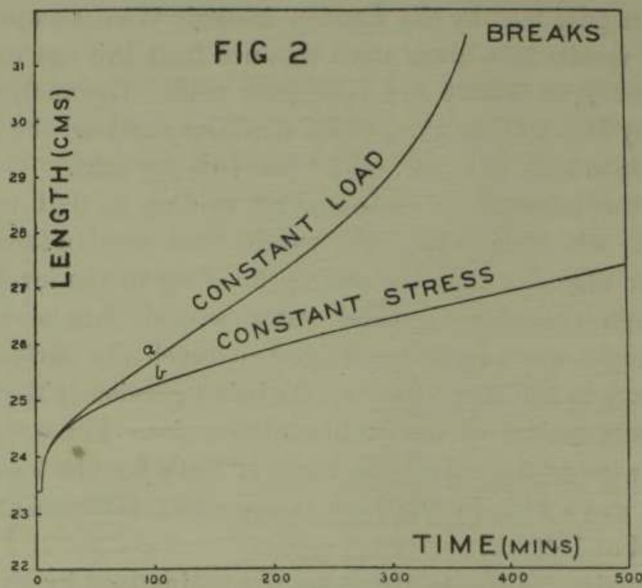
The first experiments were made on lead. The preliminary tests were made on ordinary commercial wire; the last specimen, here called lead D, was supplied as pure lead by the London Electric Wire Company. Most of the numerical results here given were obtained from this specimen, which on analysis was found to be 99·8 per cent. pure lead. The analysis was kindly done for me by Mr. I. G. Rankin, B.Sc., who also analysed the fuse wire, and found it contained lead, 21·5 per. cent.; tin, 78·5 per cent.

To observe the extension a cathetometer reading to 0·01 mm. was used. The water into which the hyperbolic weight sank was contained in a vessel of much larger diameter than the weight, filled up to the level of a hole in the side, through which the displaced water escaped, thus securing constant level. The weight was adjusted so as just to touch the surface by observation of the image in the water, the weight being previously dry; this was to avoid uncertainty caused by the surface-tension rise. Three different hyperbolic weights were used to cover the range of loads for the lead, a fourth for the fuse wire, and a fifth in the high temperature experiments, which were done with lead at 162° C.

In these experiments the temperature was maintained by an oil bath kept at constant temperature by a Riechert regulator. The wire was pulled upwards, the lower hook being hooked on to a horizontal bar running across the bottom of the bath. Short lengths of lead wire were used, about 10 cm., so that the temperature throughout might be constant, and the load was applied by a fine steel wire running over two pulleys, to which was attached a hyperbolic weight. The friction of the steel wire on the pulleys was less than 10 grammes weight.

Experimental Results.—The general nature of the stretch in a lead wire loaded beyond the elastic limit has been noted by previous workers.* An immediate extension is followed by a continuous stretch which decreases in rapidity to a final rate, constant if the extension is sufficiently small to make negligible the increase in stress due to thinning. The first result from the use of the hyperbolic weight was that even for large extensions the rate settles down to a constant value, if the stress is kept constant. In fig. 2 the extensions are shown in the two cases of constant load and constant stress for the same initial length, and the same initial load. It is to be noted that even when the stress is kept constant we should expect the rate of flow to increase again, owing to the length flowing at any moment increasing with the time; this effect, however, would not be very noticeable in the case shown, as the rate of flow at the constant stage is not very rapid. In the case of fuse wire, however, where the viscous rate is large, this effect is marked. It was found that taking the rate at different times in the viscous flow, and dividing by the mean length pertaining to each rate,

* Trouton and Rankine, *loc. cit.*



a rate per unit length was obtained, which was constant at different stages of the flow. It was thus established that *under constant stress* the rate per unit length, once the initial effect has died out, is constant right up to breaking.*

A series of results for lead under constant stress at various stresses was obtained. The experiments at high temperatures gave similar extension curves, but the initial effect was much less marked. In the extension curves for fuse wire the initial effect was smaller still; the increase in viscous rate, owing to increasing length, can be easily seen in fig. 3.

I was led to dividing the extension into three parts:—

(1) The immediate extension on loading.

(2) An initial flow which gradually disappears.

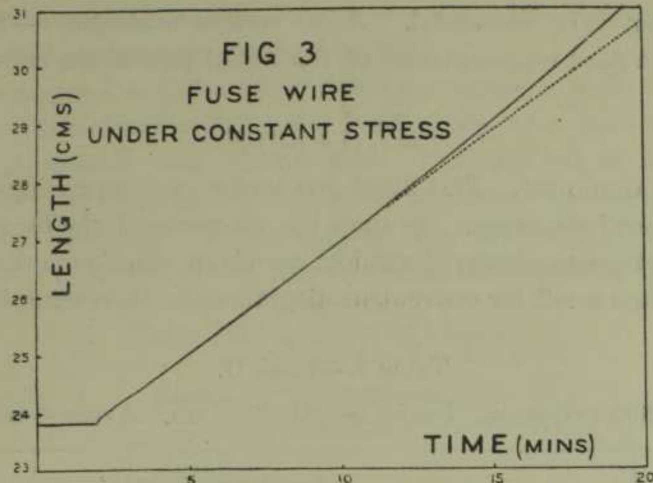
(3) A constant flow, taking place throughout the extension.

I shall call (2) the β-flow, as a measure of it is given by the constant β in an equation to be dealt with later. The constant flow per unit length has already been mentioned as viscous flow. A discussion of a conception of the mechanism of these flows is given in the theoretical section.

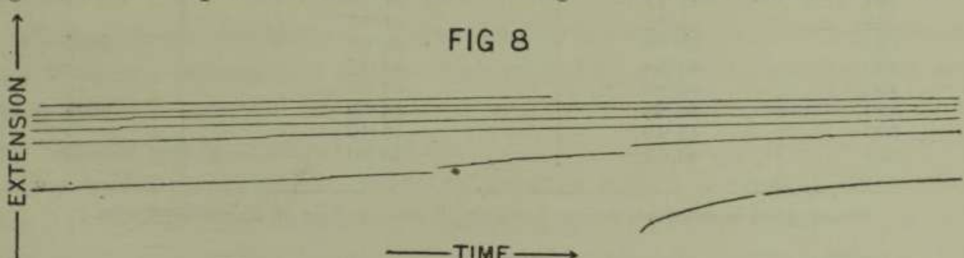
A few experiments were done on copper with the automatic recorder before mentioned. P. Phillips† states that "the copper wires made no attempt

* Breaking in such a case is merely due to a cumulative local thinning at some initial irregularity; in the case of fuse wire an extension of more than 50 per cent. of the original length was frequently obtained.

† "The Slow Stretch in Indiarubber, Glass, and Metal Wires, when subject to a Constant Pull," P. Phillips, M.Sc., 'Phil. Mag.,' April, 1905.



towards approximation to a linear function"; I found, however, that *when loaded near the breaking stress,* annealed copper wires gave a linear flow which was still continuing after seven days. A peculiarity of copper wire was that it showed sudden slips at irregular intervals, which, perhaps, may be called "copper quakes," as being analogous to the geological slipping supposed to result in earthquakes. These were not due to the recording apparatus. In fig. 8, which represents one of the tracings from the drum, both these



slips and the existence of a final viscous flow are evident. In copper, the viscous flow is very small, the immediate extension very large in comparison to a metal such as lead.

Before considering variations with the stress, it was thought advisable to obtain an equation to the curve of length against time at constant stress, to see how to divide up the effect into initial flow and viscous flow. For large values of the time, the length l is given by $l = Ce^{kt}$, where C is approximately constant, since, experimentally, $\frac{1}{l}\frac{dl}{dt} =$ constant.

Since Phillips gives $l = a + b \log t$ for the case when there is no viscous flow, *i.e.* when $k = 0$, I tried

$$l = A(a + b \log t) e^{kt}.$$

This was not very successful. After various attempts, $l - l_0 = At^{\frac{1}{3}}$ was found to give a good representation of the initial part of the curve; the form of equation

$$l = l_0(1+\beta t^{\frac{1}{3}})e^{kt}$$

was therefore attempted. This fitted our lead curves surprisingly accurately throughout the whole range. To show the closeness of the fit, the following tables for two stresses chosen at random are given, since such divergencies as there are, are too small for convenient diagrammatic representation :—

Table I.—Lead D.

Initial load, 3600 grammes. Initial length, 38·8 cm. Average temp., 14°·5 C.

Time (in minutes).	Length observed.	Length from formula $l = (39 \cdot 13 + 0 \cdot 57 \sqrt[3]{t})\, e^{0 \cdot 0000695t}$.	Stress.
0	38·8 +	39·13	3600
1	39·73	39·70	
2	39·87	39·85	
4	40·04	40·04	
5	40·12	40·12	
15	40·56	40·57	
25	40·85	40·86	3597
35	41·07	41·09	
40	41·17	41·18	
80	41·81	41·81	
130	42·38	42·39	3597
250	43·48	43·47	
300	43·86	43·85	
420	44·68	44·69	3601
500	45·19	45·19	
550	45·51	45·50	3602

Stress given in grammes weight per area of cross-section of unstretched wire.

Table II.—Lead D.

Initial load, 1800 grammes. Initial length, 9·9 cm. Average temp., 162° C.

Time (in minutes).	Length observed.	Length from formula $l = (9 \cdot 94 + 0 \cdot 43 \sqrt[3]{t})\, e^{0 \cdot 0106t}$.	Stress.
0	9·9 +	9·94	1800
1	10·46	10·48	
2	10·70	10·71	
3	10·90	10·90	
5	11·27	11·26	1800
8	11·76	11·76	
10	12·08	12·08	
12	12·40	12·40	
14	12·73	12·73	
15	12·90	12·90	1800

The extension curves for fuse wire, when the stress was truly constant, were equally well represented by the formula.

Since the equation $l = l_0(1+\beta t^{\frac{1}{3}})e^{kt}$ fitted my results so well, it was thought well to try the equation $l = l_0(1+\beta t^{\frac{1}{3}})$ for the case of a pure β-flow, when there is no appreciable final viscous flow. Taking a set of readings from Phillips' paper,* the equation $l = l_0(1+\beta t^{\frac{1}{3}})$ was found to fit quite as well as his logarithmic equation. To investigate this a little further, I did one experiment on β-flow in copper. The following table exhibits the result:—

Table III.—β-Flow in Copper.

Time (in minutes).	Reading.	$3\cdot089-\text{reading} = l-l_0$.	$0\cdot131\sqrt[3]{t}$.
0	Loaded		
2	2·924	0·165	0·165
3	2·900	0·189	0·189
4	2·881	0·208	0·208
5	2·863	0·226	0·224
6	2·850	0·239	0·239
7	2·840	0·249	0·250
8	2·827	0·262	0·262
9	2·814	0·275	0·273
10	2·807	0·282	0·282
15	2·767	0·322	0·322
20	2·740	0·349	0·355

Phillips states that there were considerable deviations from the logarithmic law when the stretch had become very slow, but, unfortunately, he does not state in which direction the deviations were, so it cannot be surmised whether my formula would afford a better fit.

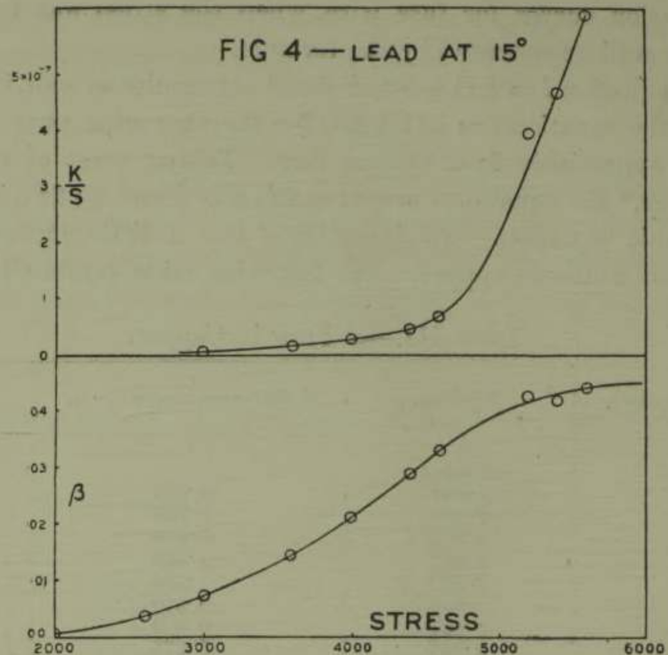
The tables given, which are representative, suffice to show, I think, that $l = l_0(1+\beta t^{\frac{1}{3}})e^{kt}$ is successful in representing the observed facts. l_0 is not the exact unstretched length, but a number slightly larger, probably the immediate length on loading, which could not be measured. β being fairly small compared to 1, $1+\beta t^{\frac{1}{3}}$ becomes nearly constant for large values of t, as $t^{\frac{1}{3}}$ then increases very slowly. We have for convenience called that part of the flow represented by the coefficient β the β-flow; k is a measure of the purely viscous flow.

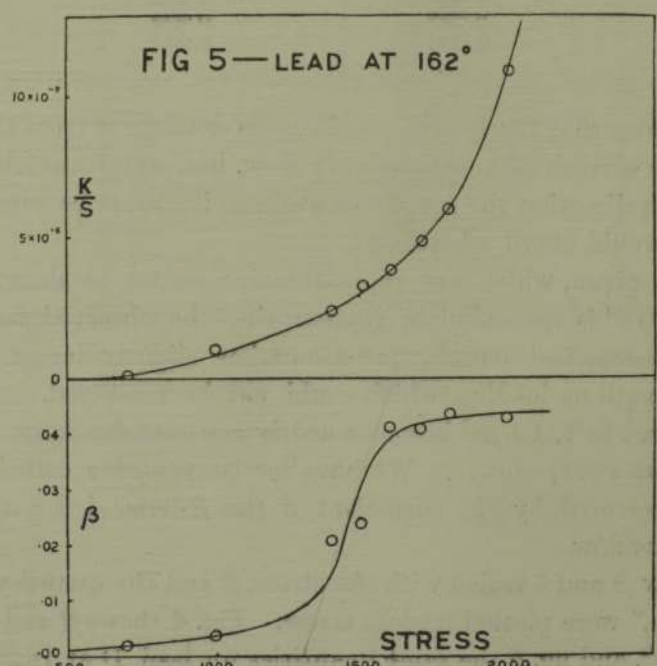
To see how β and k varied with the stress, β and the quantity $k/$ (stress), or the "fluidity," were plotted against stress. Fig. 4 shows β and $k/$ (stress) for lead D at 15°, and fig. 5 the same quantities for lead D at 162°. It is to be especially noted that β tends to become constant for large values of the stress.

A form of curve similar to our β-curves was found by Philiips† for th

* *Loc. cit.*, p. 525.

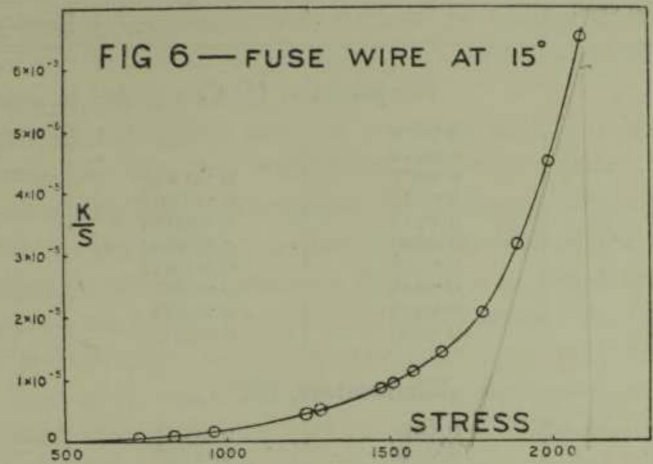
† *Loc. cit.*, p. 526. Curves III *a*, *b*, *c*, *d*.





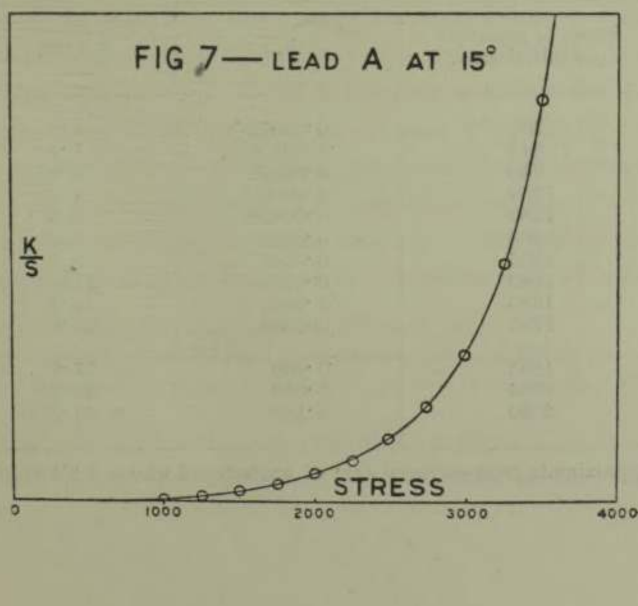
variations of his coefficient b with the stress in his experiments on copper, where he was dealing with a case involving no sensible viscous flow. (Owing to the similarity of our equations, b and β are measures of approximately the same effect.) It is remarkable that β tends to the same limit in the experiments at 162° as in those at 15°, though the corresponding values of k are very much greater in the former case.

Fig. 6 shows the variations of k/s (where s is the stress) for fuse wire. It is of exactly the same character as in lead. β is not here shown, as it is so



small in the case of fuse wire that satisfactory determinations were not made. The curves of k/s against s can be approximately fitted by hyperbolæ having one asymptote parallel to the axis of s, and the other asymptote inclined at an acute angle to this axis.

Fig. 7 shows k/s for an early specimen of lead, A. k was computed from the later readings of the extension, it being assumed that the β-flow had died out. The values are therefore only approximate; the figure is given as a



confirmation of the character of the fluidity curve, the specimen being a softer lead than D.

Table IV.—Lead D.

Stress = s in grammes/initial area.	β.	k.	$\frac{k}{s} \times 10^6$.
	Temperature, 15° C.		
2000	0·0008		
2600	0·0044		
3000	0·0077	0·000035	1·16
3600	0·0145	0·000069	1·92
4000	0·0210	0·000110	2·75
4400	0·0290	0·000202	4·60
4600	0·0334	0·000322	7·00
5200	0·0424	0·00207	40·0
5400	0·0420	0·00253	47·0
5600	0·0440	0·00345	61·6
	Temperature, 162° C.		
700	0·002	0·0001	14
1010	0·0045	0·00095	94
1400	0·021	0·0032	230
1500	0·024	0·0048	320
1600	0·041	0·0060	375
1700	0·041	0·0083	488
1800	0·043	0·0106	588
2000	0·043	0·0216	1080

Approximate cross-sectional area of unstretched wire = 2·16 sq. mm.

Table V.—Fuse Wire. Temperature, 15° C.

Stress in grammes per initial area.	k.	$\frac{k}{s} \times 10^6$.
730	0·00072	1·0
840	0·00116	1·4
960	0·00197	2·05
1250	0·00590	4·7
1280	0·00628	4·9
1480	0·0132	8·9
1505	0·0146	9·7
1580	0·0174	11·0
1680	0·0241	14·3
1790	0·0369	20·6
1885	0·060	31·8
1985	0·090	45·0
2090	0·136	65·0

Approximate cross-sectional area of unstretched wire = 0·64 sq. mm.

Theoretical Considerations.

There appears to have been but little previously written on the flow in metals which will aid in formulating a theory to accord with these results. Boltzmann's well-known paper on "Elastische Nachwirkung"* offers no help. It is interesting, however, to consider the phenomena from the point of view suggested by Ewing in his Presidential Address to Section G of the British Association on the structure of metals.† In this he insists upon the crystalline nature of the structure.

We may look at the initial flow as resulting from the addition to a permanent viscous flow of a flow which dies out with the time. This latter may be taken as arising from a rearrangement or rotation of small parts of a crystalline nature in the material, much after the fashion of the rotation seen in Ewing's magnetic model illustrating hysteresis in iron; the β-curve is of the same form as the curve of magnetisation. I assume the rotation of the crystals to be opposed by some kind of elastic force proportional to the displacement, but, as in the magnetic model, a succession of equilibrium positions is passed, through which return is not spontaneously possible on the removal of the stress. The viscous flow may be regarded as taking place in a viscous matrix, in which the crystals are embedded.

If we take the formula $l = l_0(1+\beta t^{\frac{1}{3}})\,e^{kt}$, we may write

$$\frac{dl}{dt} = kl + \tfrac{1}{3}l_0\beta t^{-\frac{2}{3}} + \tfrac{1}{3}k\int_0^t \tfrac{1}{3}l_0\beta t^{-\frac{2}{3}}\,dt,$$

since β and kt are small within the range.‡ The first term represents a purely viscous flow, the second the flow due to rearrangement or rotations, if we assume that the number of crystals rotating per unit time diminishes as $t^{-\frac{2}{3}}$. The third term, which is small, may be regarded as a small correction to the permanent flow arising from the rotations increasing the amount of material which at any moment is in a suitable attitude for purely viscous flow: it is proportional to the number of crystals which have rotated. (The formula is not presumed to do more than represent the phenomena throughout any convenient experimental range; for very large values of t it is incorrect, as it would give the number of crystals rotating in an infinite time as being infinite. $t^{\frac{1}{3}}$ should probably be replaced by a function of t which approximates

* L. Boltzmann, 'Wiener Sitzungsberichte,' October, 1874.

† J. A. Ewing, 'Report of the British Association,' 1906, p. 657.

‡ For example, take lead for stress 4600 grammes per initial area. Here the largest value of kt was 0·079,

$$dl/dt = 0{\cdot}0065 + 0{\cdot}0054\,(1 + 0{\cdot}079 + \tfrac{1}{2}(0{\cdot}079)^2 + \ldots),$$

and the omission of squares and higher powers of kt makes an error of about 0·15 per cent. in the flow.

closely to $t^{\frac{1}{3}}$ for such values as are here considered, but which as t increases ultimately becomes constant.)

The fact that β tends to the same limit at both 162° and 15° is significant, as suggesting that the division of the flow into β-flow and viscous flow is valid physically, β-flow being dependent on a purely geometrical arrangement in the structure of the metal. Such a process as the rotations suggested would give the experimentally observed constancy of β for large stresses.

The following is a brief summary of the paper:—

(1) Establishment of the existence of a viscous flow in lead, in a lead-tin alloy, and in copper.

(2) Realisation of the necessity of working at constant stress, and the device of the "hyperbolic weight" for this purpose.

(3) Demonstration that at constant stress the flow is purely viscous right up to breaking for the metals used.

(4) The empirical formula $l = l_0(1+\beta t^{\frac{1}{3}})e^{kt}$ is shown to fit very closely all the extension-curves. The flow can be divided into β-flow and viscous flow.

(5) The β-flow is experimentally shown to tend to a limit for large stresses.

(6) The β-flow is shown to tend to the same limit at 162° as at 15° C.

(7) The curve of fluidity against stress is shown to be roughly a hyperbola, with one asymptote parallel to, and the other steeply inclined to, the stress axis.

It is a great pleasure to me to offer my thanks to Prof. Trouton, to whose unremitting kindness and encouragement whatever merit there may be in this work is largely due. I also owe my thanks to Assistant-Professor A. W. Porter for many suggestions, and for reading through the paper.