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ON THE WORKS OF HIROSHI KUNITA IN THE SIXTIES

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Dedicated to the memory of Professor Hiroshi Kunita

Hiroshi Kunita was a master course graduate student of Kyoto University during the period 1959-60. I was one of his classmates and Shinzo Watanabe was a year senior to us. I have a vivid memory that Kunita was always calm and modest but exhibited a rare talent in mathematics from his student days.

In the first half of the 1960s, the Doob-Meyer decomposition theorem of submartingales was completed, while S. Watanabe and Minoru Motoo gave, for a general Markov process, a profound analysis on the structure of the space of additive functionals with mean zero and finite variances. These two works merged and developed into the joint paper [8] by Kunita and S.Watanabe, a magnificent extension of the stochastic integral and the transformation formula due to our supervisor Kiyosi Itô from the Brownian motion to general semi-martingales. Until now, the work [8] has kept to be a key ingredient in the so called *stochastic calculus*. The experience in [8] must be a constant source of Kunita's driving force in his unified and comprehensive approaches to stochastic controls and stochastic flows in later decades.

In the meantime, the study of the general boundary conditions for the one-dimensional diffusions initiated by William Feller in the 1950s was successfully completed by the joint works of K. Itô and Henry P. McKean, Jr. Thus many Japanese young probabilists in the 1960s were eager to investigate the so called *boundary problem for more general Markov processes* looking for analytic characterization of possible Markovian extensions of the transition semigroup of a given minimal Markov process and realization of those analytic extensions to be associated with genuine strong Markov processes with regular sample paths.

Kunita's paper [5] published as early as in 1962 on a construction of instantaneous return Markov process from the Martin boundary M of a given Markov process X with countable state space was the first work in Japan directly related to this problem. Kunita collaborated with Takesi Watanabe who was particularly concerned with the roles of the Martin boundary played in this problem. Kunita and T. Watanabe wrote several joint papers. Among them, [9] was relevant to the Ray-Knight compactification method on a regularization of the state space of a Markov process X to make X strong Markov with regular sample paths.

In [2], I also employed the Martin boundary M of the bounded Euclidean domain D to characterize all possible symmetric conservative Markovian extensions

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of the transition semigroup of the absorbed Brownian motion on D by means of the family of the symmetric Dirichlet forms over M dominating the Douglas integral.

In order to extend this result to a diffusion generated by an elliptic partial differential operator with measurable coefficients including drift ones treated by Guido Stampacchia in 1965, Kunita introduced in [6] the most general concept of a (non-symmetric) semi-Dirichlet form which is lower bounded (not necessarily non-negative definite).

A little later in [3], I constructed a Hunt process associated with a general regular symmetric Dirichlet form by establishing its strongly regular representation using the idea embodied in [9]. The result in [3] was later generalized to the non-symmetric Dirichlet form in Kunita's sense by Santiago Carrillo Menendez [1].

Non-symmetric Dirichlet forms are usually assumed to be non-negative definite. But Kunita's formulation is more useful. See my recent joint paper [4] with Toshihiro Uemura in this connection.

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