BIT 23 (1983), 456–471

ON THE X-Y CONVEX HULL OF A SET OF X-Y POLYGONS

T. M. NICHOLL[1], D. T. LEE[1]*, Y. Z. LIAO[2] and C K. WONG[2]

1. *Department of Electrical Engineering and Computer Science, Northwestern University, Evanston, IL 60201, U.S.A.*

2. *IBM T. J. Watson Research Center, P.O. Box 218, Yorktown Heights, NY 10598, U.S.A.*

Abstract.

We study the class of rectilinear polygons, called $X-Y$ polygons, with horizontal and vertical edges, which are frequently used as building blocks for very large-scale integrated (VLSI) circuit layout and wiring. In the paper we introduce the notion of convexity within the class of $X-Y$ polygons and present efficient algorithms for computing the $X-Y$ convex hulls of an $X-Y$ polygon and of a set of $X-Y$ polygons under various conditions. Unlike convex hulls in the Euclidean plane, the $X-Y$ convex hull of a set of $X-Y$ polygons may not exist. The condition under which the $X-Y$ convex hull exists is given and an algorithm for testing if the given set of $X-Y$ polygons satisfies the condition is also presented.

Keywords. Analysis of algorithms, convexity, rectilinear polygons.

1. Introduction.

Rectilinear polygons are frequently used as building blocks for VLSI layout and wire routing. They are also used in image processing to describe images on rectangular grids [1, 8]. In automating VLSI design, a problem generally felt is how to store the information of the hundreds of thousands of rectilinear polygons on a chip without imposing an excessive demand on the amount of memory. For example, in a hierarchical VLSI design where compactions are done phase by phase, we can use as bases the convex hulls of these rectilinear polygons that represent functional blocks into which various components have been compacted. The convex hulls can both provide compact representations of functional blocks and be used as basic units for further compaction. For other compaction issues see, for example, [2, 6, 7, 9, 10]. In this paper we study the class of rectilinear polygons, called $X-Y$ polygons and introduce the notion of convexity within this class of polygons. Efficient algorithms to find the convex hulls of an $X-Y$ polygon and of a set of $X-Y$ polygons under various conditions are presented.

* This author's research was supported in part by the National Science Foundation under Grants MCS-8202359 and ECS-8121741.

Received April 30, 1983. Revised June 30, 1983.

We will state some definitions of the shapes that interest us and describe our motive to study them.

DEFINITION 1: An $X-Y$ polygon is a simple polygon with only vertical and horizontal edges.

DEFINITION 2: An $X-Y$ convex polygon is an $X-Y$ polygon, such that within the polygon, for any two points lying on the same horizontal or vertical line, the straight line segment between them lies completely within the polygon.

DEFINITION 3: The $X-Y$ convex hull of a set of $X-Y$ polygons is, if it exists, an $X-Y$ convex polygon that contains the set of $X-Y$ polygons and has minimal area.

Fig. 1 shows a set of $X-Y$ polygons and the $X-Y$ convex hull of the set. It is clear from the figure that if only the outer boundary of the set is needed, then to store only the vertices (or edges) of the $X-Y$ convex hull rather than those of all the $X-Y$ polygons in the set will reduce the amount of memory needed.

However, theoretically, the $X-Y$ convex hull of an arbitrary set of $X-Y$ polygons, may not exist. Fortunately, this arises mostly in cases where the set of $X-Y$ polygons needs further compaction before it is practically worthwhile to compute the $X-Y$ convex hull for it. Later, we will discuss this problem in greater detail, and see under what circumstances it occurs and how it is detected.

Note that the perimeter of the $X-Y$ convex hull in Fig. 1 consists of four staircases. By a staircase we mean a sequence of consecutive edges which forms the side view of a staircase in ordinary life, except that the height and width of each step may not be regular. Fig. 2 shows an example of a staircase.

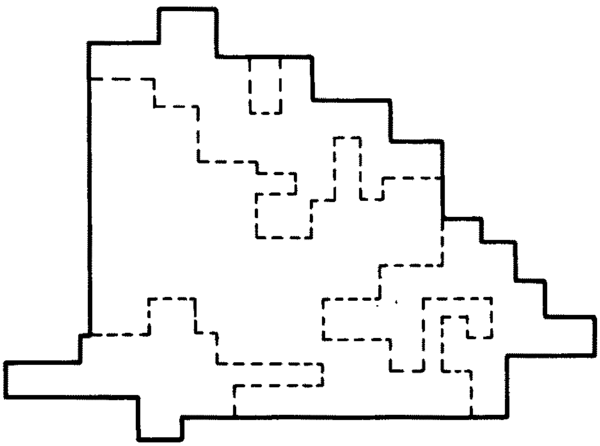

Fig. 1. The $X-Y$ convex hull of a set of polygons.

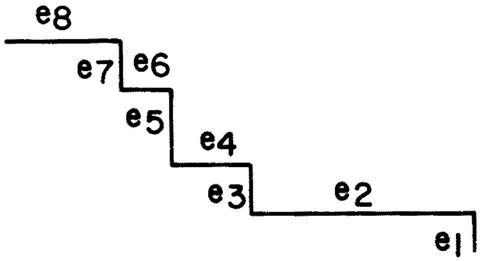

Fig. 2. A staircase $e_1, \ldots, e_8$ with horizontal edges having increasing y-coordinate and vertical edges having decreasing x-coordinate.

A staircase is formally defined as follows:

DEFINITION 4: A *staircase* is a sequence of alternately vertical and horizontal edges such that:
i) All the horizontal edges have ordered y-coordinates,
ii) All the vertical edges have ordered x-coordinates, and
iii) The edge sequence forms a continuous curve.

If the reader examines Definition 3 again or tries to construct an $X-Y$ convex polygon, it will become obvious that not just the perimeter of the particular $X-Y$ convex hull consists of four staircases, but every $X-Y$ convex polygon shares the same property. The four staircases that compose the perimeter of an $X-Y$ convex polygon are the four sequences of edges between the mid-points of the edges with extremum x- or y-coordinate values. Fig. 3 shows the four staircases of an $X-Y$ convex polygon.

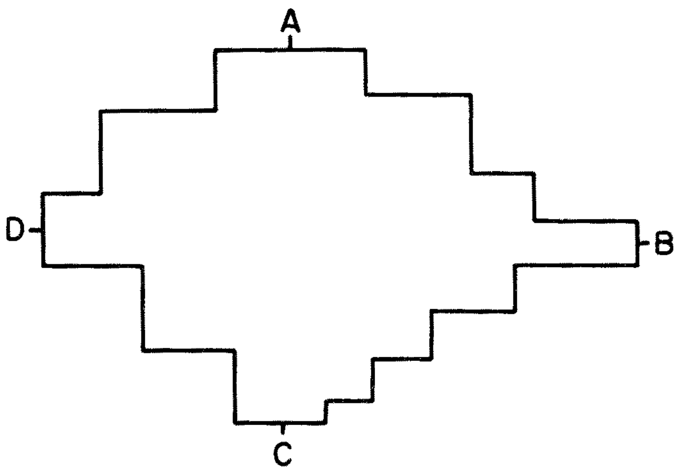

Fig. 3. An $X-Y$ convex polygon. Its four staircases are the sequences of edges from A to B, from B to C, from C to D and from D to A.