On Timing Offset Estimation for OFDM Systems

H. Minn, M. Zeng, and V. K. Bhargava, *Fellow, IEEE*

***Abstract*—Two timing offset estimation methods for orthogonal frequency division multiplexing (OFDM) systems as modifications to Schmidl and Cox's method [6] are presented. The performances of the timing offset estimators in additive white Gaussian noise channel and intersymbol interference channel are compared in terms of estimator variance obtained by simulation. Both proposed methods have significantly smaller estimator variance in both channel conditions.**

***Index Terms*—OFDM, sliding window, timing offset estimation, timing synchronization, training symbol.**

I. Introduction

OFDM SYSTEMS are much more sensitive to synchronization errors than single carrier systems [1], [2]. Several approaches have been proposed to estimate time and frequency offset either jointly or individually. In [3] and [4], the redundancy of the signal due to cyclic prefix is used. Since guard interval is usually affected by ISI, the result of estimation depends on *á priori* assumption about the channel. The method in [5] uses a longer guard interval where ISI free part of the guard interval is used for timing offset estimation, and fails under some channel conditions. To avoid these problems in timing estimation, Schmidl and Cox [6] use a training symbol containing two identical halves. But, the timing metric plateau inherent in this method causes large variance of the timing estimate. Alternatively, in [7] cyclic prefix and pilot symbols used for channel estimation are exploited for timing estimation. In this letter, we present two methods as modifications to [6] in an attempt to avoid timing metric plateau. Section II briefly describes the timing estimation method of [6] and Section III presents the two proposed methods. In Section IV, the performance of the proposed methods and [6], [7] are compared in terms of estimator variance obtained by simulation.

II. System Description

The samples of transmitted baseband OFDM signal can be given by

$$s(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N_u-1} c_n \exp(j2\pi kn/N) - N_g \le k \le N-1 \tag{1}$$

where c_n is modulated data on the nth subcarrier, N is the number of inverse fast Fourier transform (IFFT) points, $N_u(\le N)$ is the number of subcarriers, N_g is the number of guard samples, $j = \sqrt{-1}$, and the sampling period is T_u/N with $1/T_u$ being subcarrier spacing. The samples at the receiver, if assuming *Nyquist* channel and perfect timing, is

$$x(k) = \exp(j2\pi vk/N)s(k) + n(k) \tag{2}$$

where v is the carrier frequency offset normalized to subcarrier spacing, $n(k)$ is the sample of zero mean complex AWGN process. Including a timing offset l, the received sample is given by $r(k) = x(k-l)$. The symbol timing estimator finds the start of the OFDM symbol. Let the training symbol (excluding cyclic prefix) contain two identical halves in time domain each having $L = N/2$ samples. At the receiver there will be a phase difference between the samples in the first half and their replica in the second half caused by the carrier frequency offset. Training data is usually a PN sequence. Then the Schmidl and Cox's timing estimator takes as the start of the symbol the maximum point of the timing metric given by

$$M(d) = \frac{|P(d)|^2}{R^2(d)} \tag{3}$$

where d is a time index corresponding to the first sample in a window of $2L$ samples and

$$P(d) = \sum_{m=0}^{L-1} r^*(d+m) \cdot r(d+m+L) \tag{4}$$

and

$$R(d) = \sum_{m=0}^{L-1} |r(d+m+L)|^2 \tag{5}$$

The timing metric reaches a plateau (see Fig. 1) which leads to some uncertainty as to the start of the frame. To alleviate this, Schmidl and Cox proposes an averaging method where the maximum point is first found and then two points with 90% of the maximum value, one to the left and the other to the right of the maximum point, are found. The timing estimate is taken as the average of the two 90% points.

III. Proposed Methods

In this section, we present two methods to reduce the uncertainty due to the timing metric plateau and thus improve the timing offset estimation scheme proposed by Schmidl and Cox.

A. Sliding Window Method

Firstly, in calculation of the half symbol energy $R(d)$, all samples over one symbol period (excluding guard interval) is used instead of over the second half symbol period. Secondly, instead

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The authors are with the Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC., Canada V8W 3P6.

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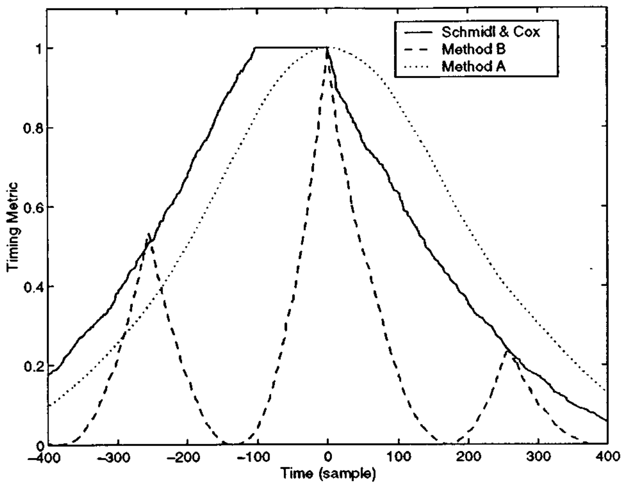

Fig. 1. Timing metric under no noise and distortion condition

of 90% points averaging approach, the timing metric is simply averaged over a window of length $N_g + 1$ samples. Then the timing metric is given by

$$M_1(d) = \frac{1}{N_g + 1} \sum_{k=-N_g}^{0} M_f(d+k) \tag{6}$$

where $M_f(d)$ can be calculated as

$$M_f(d) = \frac{|P(d)|^2}{R_f^2(d)} \tag{7}$$

and

$$R_f(d) = \tfrac{1}{2} \sum_{m=0}^{N-1} |r(d+m)|^2 \tag{8}$$

and $P(d)$ is given by (4).

B. Training Symbol Method

The samples of the training symbol (excluding cyclic prefix) are designed to be of the form

$$s = [A \;\; A \;\; -A \;\; -A] \tag{9}$$

where A represents samples of length $L = N/4$ generated by $N/4$ point IFFT of $N_u/4$ length modulated data of a PN sequence. The abrupt amplitude change due to sign conversion in the training symbol can easily be avoided by modifying the PN sequence such that the sum of the corresponding modulated data equals zero. Then the timing metric is given by

$$M_2(d) = \frac{|P_2(d)|^2}{R_2^2(d)} \tag{10}$$

where

$$P_2(d) = \sum_{k=0}^{1} \sum_{m=0}^{L-1} r^*(d + 2Lk + m) \cdot r(d + 2Lk + m + L) \tag{11}$$

and

$$R_2(d) = \sum_{k=0}^{1} \sum_{m=0}^{L-1} |r(d + 2Lk + m + L)|^2 \tag{12}$$

In both methods, $P_2(d)$ and $R_2(d)$ or $P(d)$ and $R_f(d)$ can be calculated iteratively. The timing metrics of the Schmidl and Cox's method [see (3)], and the proposed methods [see (6) and (10)] are shown in Fig. 1 under no noise and distortion condition with the system parameters given in Section IV. The correct timing point (index 0 in the figure) is taken as the start of the useful part of training symbol (after cyclic prefix).

IV. Simulation Results and Discussion

Performance of the timing offset estimators have been investigated by computer simulation for six cases: I) Schmidl and Cox method (see (3) with 90% maximum points averaging); II) Schmidl and Cox method with modified $R(d)$ (same as I, except that $R(d)$ is replaced by $R_f(d)$), III) proposed Method A [see (6)], IV) proposed Method B [see (10)], V) proposed Method B with $R_2(d)$ replaced by $R_f(d)$, and VI) the method from [7]. The system used is 1000 subcarriers OFDM system with 1024 point FFT, 10% guard interval (102 samples), carrier frequency offset of 12.4 subcarrier spacing, QPSK modulation and 10 000 simulation runs. Two channel conditions are considered: 1) An AWGN channel with no inter-symbol interference (ISI) (it will be called AWGN channel) and 2) An AWGN channel with ISI (it will be called ISI channel). The ISI channel is modeled as 16 paths with path delays τ_i of $0, 4, 8, \cdots, 60$ samples and path gains given by

$$h_i = \frac{\exp(-\tau_i/60)}{\sqrt{\sum_{k=1}^{16} \exp(-\tau_k/30)}}, \qquad i = 1, 2, \cdots, 16. \tag{13}$$

For the timing estimator of [7], the additional parameters are: one pilot every 40th subcarrier and dummy SNR value $\tilde{\mathrm{SNR}} = 5$ dB.

Figs. 2 and 3 show the means and variances of the timing offset estimates in AWGN channel and ISI channel respectively. For AWGN channel, the means of the cases I and II are about the middle of timing metric plateau (within the cyclic prefix) while the means of the cases III–VI are about the correct timing point. For ISI channel, the means are observed to be shifted to the right in time axis (i.e., delayed) by some amount depending on the shapes of the timing metric and ISI channel. Note that if the timing estimate is desired to lie within guard interval, then the means of the cases III–VI can easily be shifted to the left (i.e., advanced) by some appropriate design amount. As for the variances, the following are observed:

- In AWGN channel, cases I, II, and VI show floor in estimator variance curves while the proposed methods (cases III–V) do not. In ISI channel, all methods have estimator variance floor.
- Performance of case II is better than case I for all conditions considered, at the expense of some additional complexity. The smaller variance of case II is due to using

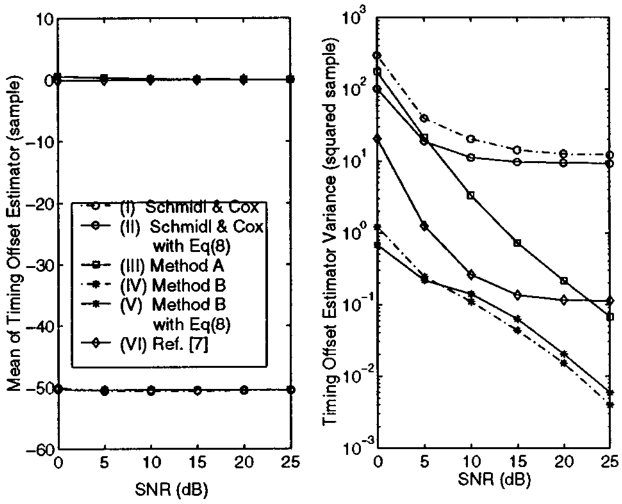

Fig. 2. Mean and variance of timing offset estimators in AWGN channel

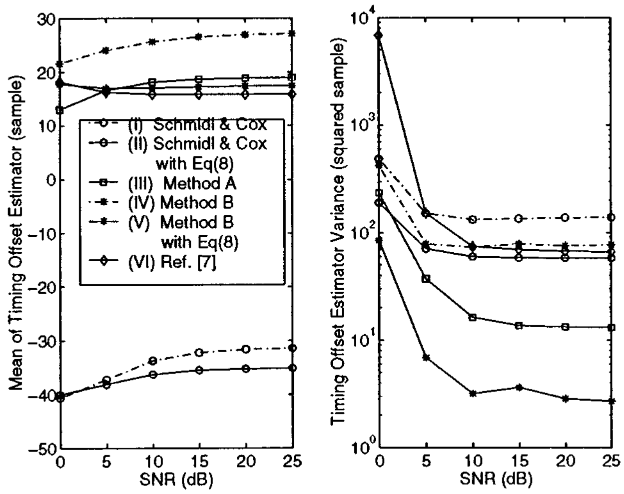

Fig. 3. Mean and variance of timing offset estimators in ISI channel

more samples to calculate half symbol energy used in timing metric.

- Proposed methods (cases III, IV, and V) have significantly smaller estimator variance than Schmidl and Cox's method (case I) for all conditions considered. The better performance of the proposed methods is generally due to the absence of timing metric plateau.
- As comparison of proposed methods to case VI, in AWGN channel, cases IV and V have significantly smaller estimator variance but case III has larger variance than case VI for SNR values less than about 23 dB; while in ISI channel, case IV has slightly larger variance for SNR values greater than 10 dB but cases III and V have significantly smaller variances.
- In AWGN channel, the slope of the timing metric off the correct timing point determines the estimator variance (see cases I and II versus cases III– V in Fig. 2).
- In ISI channel, using all the samples over one symbol period (excluding cyclic prefix) to calculate the half symbol energy (i.e. using R_f) has more effect on reducing the estimator variance than the slope of the timing metric (see case IV versus cases II, III, and V in Fig. 3).
- As an overall evaluation for both channel conditions, case V (proposed method B using R_f) performs the best.

The cases II, III, and V have some additional complexity due to using R_f as compared to case I. However, cases IV and V do not need averaging of timing metric as required in cases I–III. Hence case IV has even smaller complexity than case I.

V. Conclusions

Two timing offset estimation methods for OFDM systems are presented as modifications to Schmidl and Cox's method [6] where a training symbol containing two identical halves is used. The first method uses two modifications: 1) all samples over one symbol period (excluding guard interval), instead of over half symbol period, are used in calculation of half symbol energy required in timing metric and 2) averaging of timing metrics over a window of guard interval length is used instead of 90% maximum points averaging. The second method uses a training symbol containing four equal length parts: the first two are identical and the last two are the negative of the first two. Modification 1) can also be applied in the second method and gives robustness to ISI channel. The simulation results show that both proposed methods have significantly smaller estimator variance than [6] under both AWGN channel and ISI channel. The performance of the method in [7] is also included in the comparison as another reference. As an overall performance for both channels, the second proposed method with modification 1) gives the best result.

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