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On using the cosmic microwave background shift parameter in tests of models of dark energy

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ABSTRACT

Context. The so-called shift parameter is related to the position of the first acoustic peak in the power spectrum of the temperature anisotropies of the cosmic microwave background (CMB). It is an often used quantity in simple tests of dark energy models. However, the shift parameter is not directly measurable from the cosmic microwave background, and its value is usually derived from the data assuming a spatially flat cosmology with dark matter and a cosmological constant.
Aims. To evaluate the effectiveness of the shift parameter as a constraint on dark energy models.
Methods. We discuss the potential pitfalls in using the shift parameter as a test of non-standard dark energy models.
Results. By comparing to full CMB fits, we show that combining the shift parameter with the position of the first acoustic peak in the CMB power spectrum improves the accuracy of the test considerably.

Key words. cosmology: theory – cosmological parameters

1. Introduction

Comparing cosmological models to current observational data can be cumbersome and computationally intensive. Large multi-dimensional parameter spaces cannot be probed by grid-based methods but more sophisticated approaches are required, for example Monte Carlo Markov Chains (MCMC) (Gamerman & Lopes 2006; Lewis & Bridle 2002). Of the most commonly used cosmological data sets available today, cosmic microwave background (CMB) and large-scale structure (LSS) observations in particular require computational effort in parameter estimation. Given computing time and patience, this is not a problem, at least in principle, when testing models in which the evolution of linear density perturbations is well understood and hence calculable. However, for several of the more imaginative dark energy models the situation is more complicated. In cases where the model is specified by an action, like the DGP model (Dvali et al. 2000), one should in principle be able to set up the equations for linear perturbations, but in practice this has turned out to be difficult and it is only recently that progress in this direction has been made in this particular case (Koyama & Maartens 2006; Koyama 2006; Sawicki et al. 2006; Song et al. 2006). And even though the equations are known, they may be so complicated to treat numerically as to make it practically impossible to explore the parameter space of the model in MCMC. In addition, there are a large number of dark energy models, based on phenomenological considerations, that lack the detail to allow one to proceed with well-defined calculations. Examples of such models include the various proposed modifications of the Friedmann equation, where the model is simply not specified well enough to allow the calculation of the density perturbations (see e.g. Freese & Lewis 2002; Gondolo & Freese 2003). The justification of such models may be questioned, but the state of our understanding of dark energy argues for keeping an open mind. One would like to have some means of testing both groups of models, incorporating as much empirical information as possible, but avoiding the need to calculate the behaviour of density perturbations. In practice, this means restricting the observational tests to those involving the age and distance scale, in particular the luminosity distance-redshift relationship as probed by supernovae of type Ia (SNIa). Important as the supernova data are, they are still not very restrictive if one allows for e.g. non-zero spatial curvature or a time-varying equation of state for dark energy (Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003; Barris et al. 2004; Riess et al. 2004; Astier et al. 2006; Clocchiatti et al. 2006; Wood-Vasey et al. 2007; Miknaitis et al. 2007; Davis et al. 2007).

To tighten up constraints on dark energy models, a common approach is therefore to include additional information about the distance scale from the CMB in the form of the so-called shift parameter (Efstathiou & Bond 1999) that is related to the position of the first acoustic peak in the power spectrum of the temperature anisotropies (Amarzguioui et al. 2006; Barger et al. 2007; Elgarøy & Multamäki 2005; Davis et al. 2007; Fairbairn & Goobar 2006; Fairbairn & Rydbeck 2007; Lazkoz et al. 2006; Nesseris & Perivolaropoulos 2007; Rydbeck et al. 2007; Wang & Mukherjee 2006; Xia et al. 2006; Zhao et al. 2005, 2006).

Recently, after the baryon acoustic oscillations (BAO) where observed in the SDSS Luminous Red Galaxy sample (Eisenstein et al. 2005), it has also become common to include the information about the angular scale of the oscillations (Davis et al. 2007; Wright 2007). What one should bear in mind, however, is that these distance scales are not directly measured quantities, but are derived from the observations by assuming a specific model, usually the flat ΛCDM model or a slight variation thereof. Care needs to be exercised when using these derived quantities to test more exotic dark energy models.

Here we consider the shift parameter in more detail by comparing its predictions to those obtained from full CMB fits for different types of cosmological models. We identify the limitations of using such a measure and advocate using a combination of the acoustic peak scale along with the shift parameter as a more accurate probe of the CMB power spectrum. Such a combination is quick and easy to implement and should be included in tests of dark energy models where it is either cumbersome or unknown how to calculate the full CMB and matter power spectrum.

2. Theory

The use of the shift parameter as a probe of dark energy is based on the observation that different models will have an almost identical CMB power spectra if all of the following criteria are satisfied (Efstathiou & Bond 1999): $\omega_c = \Omega_c h^2$ and $\omega_b = \Omega_b h^2$ are equal, primordial fluctuation spectrum is unchanged, and the shift parameter,

$$\mathcal{R} = \frac{\omega_m^{1/2}}{\omega_k^{1/2}} \mathrm{sinn}_k(\omega_k^{1/2} y), \tag{1}$$

where $\mathrm{sinn}(x) = \{\sin(x), x, \sinh(x)\}$ for $k = +1, 0, -1$ respectively, with

$$y = \int_{a_r}^{1} \frac{\mathrm{d}a}{\sqrt{\omega_m a + \omega_k a^2 + \omega_\Lambda a^4 + \omega_Q a^{1-3w}}} \tag{2}$$

is constant. In this original definition of the shift parameter, the universe is considered to be filled with matter (dark and baryonic), $\omega_m = \Omega_m h^2$, ω_b, curvature, $\omega_k = \Omega_k h^2$, cosmological constant $\omega_\Lambda = \Omega_\Lambda h^2$ and a dark energy component $\omega_Q = \Omega_Q h^2$ with a constant equation of state w. The density parameter Ω_i is the ratio of the present-day density of component i to the density of a spatially flat universe, $\rho_c = 3H_0^2/8\pi G$, and h is the dimensionless Hubble constant defined by $H_0 = 100\ h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. Integration is carried out from recombination, a_r, until today $a = 1$.

In a spatially flat universe ($k = 0$), the shift parameter reduces to

$$\mathcal{R} = \sqrt{\Omega_m} \int_0^{z_r} \frac{\mathrm{d}z}{E(z)}, \tag{3}$$

where $E(z) \equiv H(z)/H_0$ and $H(z)$ is the Hubble parameter.

The sound horizon at recombination for three massless neutrinos is given by (Efstathiou & Bond 1999)

$$\begin{aligned} r_s &= \frac{c}{\sqrt{3}H_0} \Omega_m^{-1/2} \int_0^{a_r} \frac{\mathrm{d}a}{\sqrt{(a + a_{eq}))(1 + R(a))}} \\ &\approx \frac{19.8\ \mathrm{Mpc}}{\sqrt{\omega_b \omega_m}} \ln\left(\frac{\sqrt{R(a_r) + R(a_{eq})} + \sqrt{1 + R(a_r)}}{1 + \sqrt{R(a_{eq})}}\right), \end{aligned} \tag{4}$$

where $R(a) = 30496\omega_b a$ and $a_{eq} = 1/(24185\omega_m)$ and the recombination redshift can be calculated by using the fitting formulae (Hu & Sugiyama 1996):

$$\begin{aligned} z_r &= 1048(1 + 0.00124\omega_b^{-0.738})(1 + g_1 \omega_m^{g_2}) \\ g_1 &= 0.0783\omega_b^{-0.238}/(1 + 39.5\omega_b^{0.763}) \\ g_2 &= 0.560/(1 + 21.1\omega_b^{1.81}). \end{aligned} \tag{5}$$

The mth Doppler peak has the comoving wave number (Hu & Sugiyama 1996) $m\pi = k_m r_s(a_r)$, and hence the location of the first peak in multipole space is approximately given by

$$\ell_a \approx \pi \frac{d_A(z_r)}{r_s(a_r)}, \tag{6}$$

where $d_A = c\mathcal{R}/H_0\omega_m^{1/2}$ is the angular diameter distance. Rewriting Eq. (6), we have

$$\ell_a \approx 151\pi\omega_b^{1/2}\mathcal{R}\left(\ln\left(\frac{\sqrt{R(a_r) + R(a_{eq})} + \sqrt{1 + R(a_r)}}{1 + \sqrt{R(a_{eq})}}\right)\right)^{-1}. \tag{7}$$

The location of the first peak is hence a combination of the shift parameter and the size of the sound horizon at recombination, as is expected on physical grounds. Even though the relation between ℓ_a and $\mathcal{R}$ is linear, the two parameters are not degenerate and in fact complement each other well in constraining models, as is shown later. Note that the analytical expressions above were derived for the ΛCDM class of models. For more general models, the relevant quantities should be calculated numerically, although in some cases analytical results have been derived (Grupposo & Finelli 2006).

The best fit value calculated from the WMAP-team provided MCMC chains for the shift parameter in the standard flat ΛCDM model is

$$\mathcal{R} = 1.71^{+0.03}_{-0.03}, \tag{8}$$

which is in good agreement with Wang & Mukherjee (2006). This result is practically equal for the ΛCDM and wCDM models with or without dark energy perturbations. The acoustic peak position as measured by Eq. (7) calculated from the same data is

$$\ell_a = 303.6^{+1.1}_{-1.2}. \tag{9}$$

One should always bear in mind the conditions for the shift parameter to be applicable. If one wants to use the shift parameter as a constraint on a dark energy model, then first of all the distribution of the shift parameter has to be derived from the CMB data. This cannot be done without assuming a model. Since one is only looking for a constraint on the expansion history of the universe, it is easy to forget that one is also making assumptions about the primordial power spectrum of density fluctuations, since these form the basis for calculating the CMB anisotropies. In effect, one is therefore always making implicit assumptions about inflation, even though what one wants to test is the kinematics of the dark energy model under scrutiny.

In order to demonstrate the significance of these underlying assumptions, we show the distributions of the shift parameter in Fig. 1 derived from MCMC chains for the ΛCDM model with four different primordial power spectra: the standard power-law version, power-law with running scalar spectral index, with tensor modes, and with both tensor modes and running scalar spectral index. The distributions for $\mathcal{R}$ are visibly different in the four cases. Therefore, whenever one uses the shift parameter one should be clear about the assumptions made in deriving its distribution from the CMB data. The acoustic scale ℓ_a, also shown in Fig. 1, varies significantly less when changing the assumptions about the primordial power spectrum.

In Fig. 2 we show the distributions for $\mathcal{R}$ and ℓ_a when we assume a power-law primordial power spectrum, but make different assumptions about the matter and energy content of the Universe. Here the distributions are less scattered, and again we note that the acoustic scale exhibits less variation than the

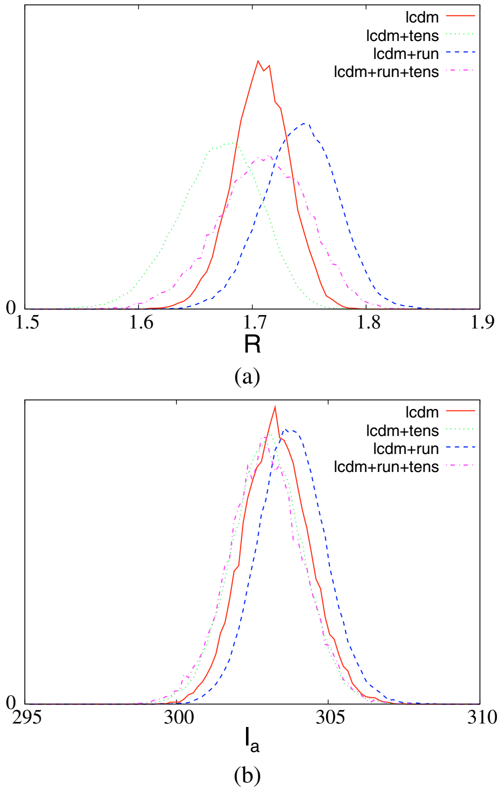

Fig. 1. The distribution of the shift parameter $\mathcal{R}$ **a**) and the acoustic scale ℓ_a **b**) derived from MCMC chains with the WMAP data for four different types of primordial perturbations: power-law with no running scalar spectral index and no tensor modes (red solid line), power-law with tensor modes (green dotted line), running spectral index and no tensor modes (blue dashed line), and both tensor modes and running spectral index (purple dot-dashed line).

shift parameter. As a demonstration of using the shift parameter blindly, we also plot the distribution of $\mathcal{R}$ and ℓ_a when we allow for massive neutrinos. This is clearly wrong since the size of the sound horizon is now changed and hence the basis of using the shift parameter is no longer valid. This is important to take note of, because we know that neutrinos do have a mass that should always be included as a free parameter in cosmological parameter estimation.

3. Examples

Using the values of $\mathcal{R}$ and ℓ_a calculated from the MCMC chains for the power-law wCDM model, we can compare the resulting confidence contours with those arising from doing the full CMB fit.

3.1. wCDM model

In Fig. 3 we show the 68%, 95% and 99% confidence levels arising from using the shift parameter and the acoustic peak position for the flat wCDM model. In calculating the confidence limits using the acoustic peak position, we have chosen a flat prior $h = 0.73 \pm 0.03$ and marginalized over h. We have kept the baryon density fixed at the WMAP value $\omega_b = 0.0223$ which we use throughout the paper unless otherwise stated. Comparing this with the probability density plot from the full MCMC chains, e.g., for the wCDM model with no dark energy perturbation shown in th same figure, we see that the shift parameter gives

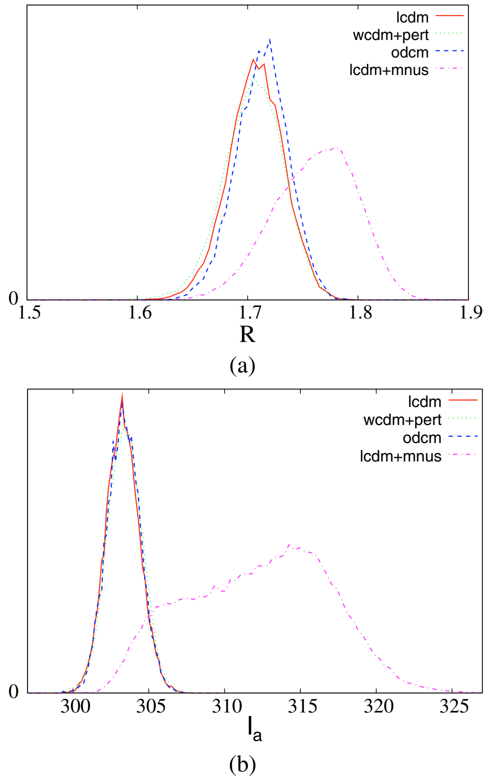

Fig. 2. The distribution of the shift parameter $\mathcal{R}$ **a**) and the acoustic scale ℓ_a **b**) derived from MCMC chains with the WMAP data for four different models: ΛCDM (red solid line), dark energy with constant equation of state and dark energy perturbations (green dotted line), open CDM (blue dashed line), and ΛCDM with massive neutrinos (purple dot-dashed line).

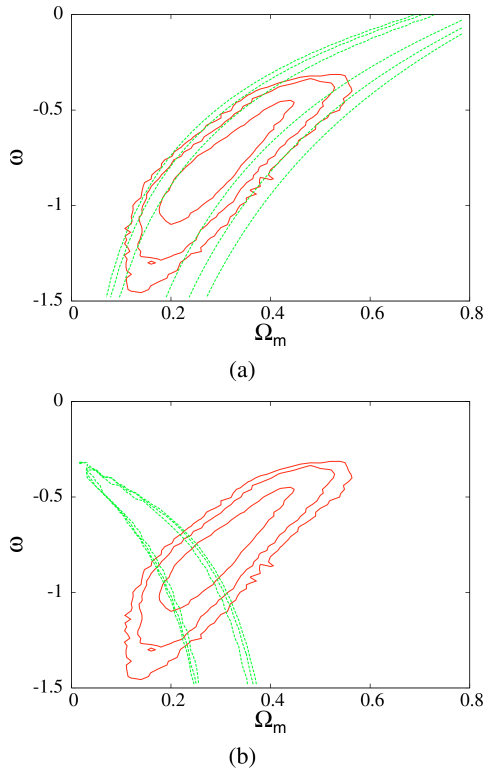

Fig. 3. The 68%, 95% and 99% confidence levels calculated from the shift parameter **a**) and acoustic peak position **b**). In both panels we also show the probability density calculated from the WMAP 3-year data (red solid curves).

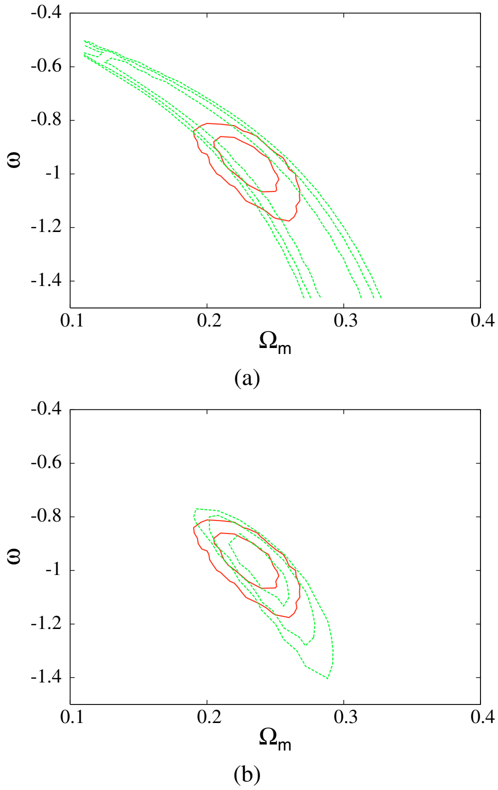

Fig. 4. Comparison of different methods: probability density from the wCDM MCMC chains (solid red lines in both figures), confidence contours (dotted green lines) calculated using ℓ_a **a**) and the combined $\mathcal{R}$, ℓ_a contours **b**).

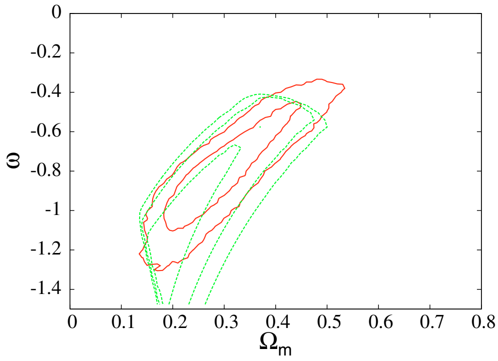

Fig. 5. The 68%, 95% and 99% confidence levels calculated from the combination of the shift parameter and ℓ_a (dotted green lines) and wCDM probability density, $h = 0.5$–1.0.

a good approximation to the CMB data while the acoustic peak position does not. Note that even though ω_b is fixed, $\mathcal{R}$ and ℓ_a contours have fundamentally different shapes, demonstrating the importance of the sound horizon size in calculating ℓ_a.

The fact that the shift parameter approximates the full CMB contours so well for the wCDM model is not surprising since the value of the shift parameter has been calculated from a chain that assumes that the cosmology is of the wCDM type. In other words, we first assume a model and then calculate chains that best fit the data from which we derive a quantity. Following the same prescription one can in fact construct other quantities that also well approximate the full CMB contours, but are not physically motivated.

3.2. Role of the Hubble parameter

In the WMAP chains, the Hubble parameter typically has a fairly large prior, $0.5 < h < 1.0$. If the value of h is constrained by other observations, e.g., the Hubble Key Project reports $h = 0.72 \pm 0.08$ (Freedman et al. 2001), the contours in the (Ω_m, w)-plane look quite different. In Fig. 4, we show the normalized probability density for the wCDM model from the WMAP provided chains (no dark energy perturbations) with a tight h constraint, $h = 0.73 \pm 0.01$, along with the confidence contours calculated by using ℓ_a with and without the shift parameter. The shift parameter is independent of hă and hence the confidence contours are unchanged and shown in Fig. 3. From the figure one can conclude that the combination of $\mathcal{R}$ and ℓ_a appears to be a good approximation to the full CMB data when h is constrained by independent observations. This is further supported when we consider the confidence contours arising from using the $(\mathcal{R}, \ell_a)$ combination with $h = 0.5$–1.0 (flat prior), shown in Fig. 5 along with probability density calculated from the WMAP chains with appropriate cuts. We see that also in this case, the combination of $\mathcal{R}$ and ℓ_a is a reasonably good approximation to the results obtainable from doing the full CMB fit.

3.3. Role of baryons

In the previous calculations, we have kept the baryon density fixed at the WMAP 3-year value, $\omega_b = 0.0223$. This value is derived assuming the ΛCDM model and hence when using the $\mathcal{R}$ or ℓ_a to study other cosmologies, one should be somewhat cautious when using this value. A more robust, with respect to changing cosmology, measure of ω_b comes from Big Bang nucleosynthesis (BBN), which gives $0.017 \le \omega_b \le 0.024$ (95% confidence level) with three massless neutrinos. We find that changing the baryon density within the 95% limits from BBN has only a small effect on the results. In particular, when compared to the effect of changing the Hubble parameter, the significance of varying the baryon density within the BBN limits is negligible.

4. Non-standard cosmologies

The shift parameter is particularly useful as a quick measure of how a given cosmological model fits the CMB data. In order to assess the validity of this approach, we compare here the parameter constraints arising from the shift parameter and from doing the full CMB fit on a non-standard model, namely on a general Friedmann equation. Such a model is generalization of the standard Friedmann equation and as such serves as a useful generic no-standard model. We also consider the use of the acoustic peak position as an useful approximation to the full CMB data fit.

4.1. Modified Friedmann equation

Modified Friedmann equations arise, e.g., in alternative theories of gravity. As an example, in the well known DGP model (Dvali et al. 2000), the Friedmann equation on the brane is of the form

$$H^2 \pm \frac{H}{r_c} = \frac{8\pi G}{3}\rho_m, \tag{10}$$

where ρ_m is the matter density on the brane and r_c is the cross-over scale at which gravity starts to feel the effects of the fifth dimension.

Here we will consider modifications to the Friedmann equation with no spatial curvature in the spirit of (Dvali & Turner 2003; Elgarøy & Multamäki 2005). A generalized Friedmann equation can be written as

$$f(H, H_{\rm c}) = H_0^2\Omega_{\rm m}(1+z)^3, \tag{11}$$

where instead of modifying the matter content we consider modifications of gravity by having an arbitrary function f. The critical scale, $H_{\rm c}$, is close to the present Hubble parameter, H_0, and determines when modifications from the standard Friedmann equation start to have an effect. At early times, when $H \gg H_{\rm c}$, we know from BBN constraints that $f(H) \approx H^2$. Keeping this mind and expanding in terms of $H_{\rm c}/H$ gives

$$H^2\left[1 + \sum_{n=1}^{\infty} c_n \left(\frac{H_{\rm c}}{H}\right)^n\right] = H_0^2\Omega_{\rm m}(1+z)^3, \tag{12}$$

from which it is clear that non-standard effects only start to have an effect at late times when $H \sim H_{\rm c}$. Expanding the sum gives

$$H^2\left[1 + c_1\frac{H_{\rm c}}{H} + c_2\left(\frac{H_{\rm c}}{H}\right)^2 + ...\right] = H_0^2\Omega_{\rm m}(1+z)^3. \tag{13}$$

In this form, one can interpret the cosmological constant as a second order correction to the Friedmann equation while the first order correction corresponds to the DGP model. Generally, the nth order correction for a flat universe is hence

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{\rm m}(1+z)^3 + (1-\Omega_{\rm m})\left(\frac{H}{H_0}\right)^{\alpha}, \tag{14}$$

where $\alpha = 2 - n$. The leading correction to the Friedmann equation was previously studied using current CMB, SNIa and LSS data by Elgarøy and Multamäki (2005).

4.2. Confidence contours

The CMB spectrum arising within the context of the modified Friedmann equation, Eq. (14), can be straightforwardly calculated by using, e.g., CMBFAST (Seljak & Zaldarriaga 1996) (see Elgarøy & Multamäki 2005, for a detailed description) and fitted to the WMAP 3 year data. The resulting confidence contours are shown in Fig. 6 along with contours from using the shift parameter and the acoustic peak position ($h = 0.73 \pm 0.03$, flat prior). Again, we find that the combination of the acoustic peak along with the shift parameter gives a reasonable approximation to the full CMB fit.

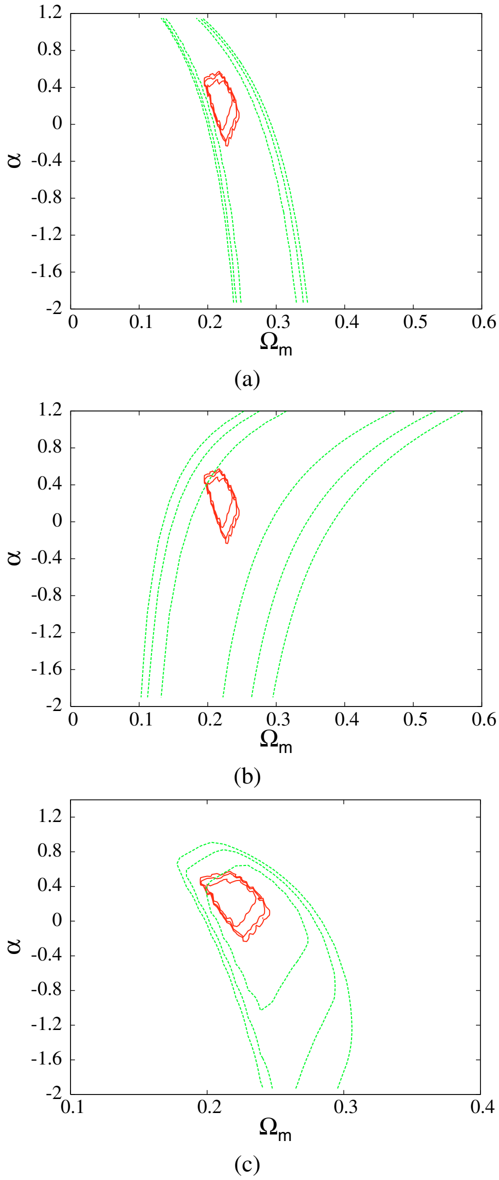

Fig. 6. The 68%, 95% and 99% confidence levels in the $\Omega_{\rm m}$-α plane calculated from the shift parameter **a)** and the acoustic peak position **b)** and their combination **c)** (dotted green lines). The parameter α defines the correction term in the modified Friedmann equation, Eq. (14). We also show the results from fitting to the full WMAP 3yr TT data (red solid lines) in all figures.

5. Conclusions

In this work we have reconsidered the use of the shift parameter as a quick and easily implementable probe of dark energy. We find that while it gives an excellent measure of the CMB spectrum for the ΛCDM model, with $\omega_{\rm b}$ and $\omega_{\rm c}$ fixed, caution should be exercised when using the shift parameter to compare and constrain non-standard cosmological models. Another paper (Wang & Mukherjee 2007) advocating the combination of $\mathcal{R}$ and $\ell_{\rm a}$ as a constraint on dark energy models appeared after we had submitted the present paper. Our results seem to be consistent with theirs, but they put more emphasis on models with curvature, and less on the model-dependence of the shift parameter.

Even when using $\mathcal{R}$ to constrain more standard type models, such as the wCDM model, careful consideration should be given to the value of the Hubble parameter, h. Since the shift parameter is independent of h, but the full CMB spectrum fit is most definitely not, the shift parameter can be misleading when applied blindly.

The shift parameter is a geometrical measure as it measures the size of apparent sound horizon at recombination. Keeping the sound horizon size fixed, different cosmological models lead to different background expansion and hence the shift parameter can be used to compare and constrain different models. However, also the sound horizon size changes when varying cosmological parameters, most notably changing the matter density, $\Omega_{\rm m}$ and the Hubble parameter, h. In addition, massive neutrinos will also have an effect. Hence, in general the shift parameter will not be an accurate substitute for the CMB data and may in principle give misleading results when used to constrain non-standard results.

In addition, the value of the shift parameter used to constrain different dark energy models is derived by first assuming the ΛCDM model, fitting the model to the data and then calculating the value of $\mathcal{R}$. Again, for a general model, using the value obtained in this manner is questionable since for a different model one may expect the shift parameter to be different, while the CMB spectrum can fit well with observations.

In order to enhance the effectiveness of using the shift parameter as a cosmological tool, we have considered adding information from the location of the first CMB peak, ℓ_a. Combining these two easily calculable observables, allows one to encompass information from both the size of the sound horizon at recombination and the angular diameter distance to it. As such, it more effectively constrains the allowed parameter space, including the Hubble parameter that is not fixed when using only the shift parameter. A possible caveat is, again, the fact that the numerical value of both of these parameters is calculated within the ΛCDM framework, but by comparing to different models, we see that the combination proves to be a good and efficient probe of non-standard cosmologies.

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