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On Vector Fitting Methods in Signal/Power Integrity Applications

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***Abstract*—Vector Fitting (VF) has been applied to reformulate traditional system identification techniques by introducing a partial-fraction basis to avoid ill-conditioned calculation in broadband system identifications. Because of the reliable and versatility of VF, many extensions and applications have been proposed, for example, the macromodeling of linear structures in signal/power integrity analyses. In this paper, we discuss the macromodeling framework and some main features in VF in terms of data, algorithms and models. Finally, an alternative P-norm approximation criterion is proposed to enhance the macromodeling process.**

***Index Terms*—Signal/Power Integrity, Vector Fitting, Macromodeling, Tutorial, Approximation**

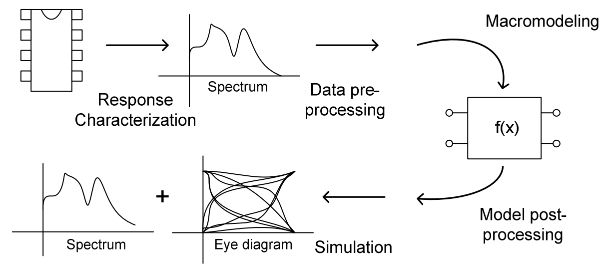

Fig. 1. Common macromodeling flow in signal integrity analyses.

I. Introduction

Vector Fitting (VF) [1] is a numerical technique for sampled response-matching system identification (macromodeling), which involves iterative linear least-squares solves with a partial fraction basis. As opposed to other system identification techniques for broadband (from DC to GHz) system identification, VF avoids ill-conditioned calculation, and therefore works in a more robust and efficient manner. Furthermore, its theoretically-simple and versatile framework can easily incooperate various constraints by introducing a variety of extensions for other areas. VF has also been used in modeling of different electrical systems [1], [2] and extended to different areas, for example, filter design [3]–[5], power network analysis [2], [6] and electromagnetic (EM) simulation [7].

The idea of VF was firstly introduced for transmission line transient modeling in [8]. The underlying idea of VF is to replace the approximated (or initialized) poles with an improved set of poles through implicit weighting (the pole relocation technique), which thereby improves the approximation iteratively. VF approximates an underlying system to a new system using partial fraction basis with real or complex conjugate poles. A number of generalizations and extensions have been proposed for better VF performance and integration with various identification requirements [9]–[24]. VF has been thoroughly discussed in [25], [26]. Its basic implementation is available from [27], whereas its variants have been widely used in industrial electronic design workflows for signal integrity issues [28]–[30].

This paper acts as a tutorial on VF. We first give a brief introduction to the signal/power integrity issues (Section II) and basic formulation of VF (Section III). Then we discuss the applications of VF in system identification (Sections IV, V and VI). Finally an alternative P-norm approximation criterion in VF is proposed for approximation enhancement (Section VII), which is verified through numerical examples (Section VIII).

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II. Macromodeling: System Identification Problem in Signal/Power Integrity

With the increasing operational frequency and decreasing size of integrated circuits (ICs), high-frequency effects, such as signal delay and crosstalk, have become dominant factors limiting system performance in IC design. Accurate and efficient simulation is required to capture the high-frequency behavior of systems, so as to ensure consistent transmissions and reliable ground (and power) distributions in high-speed electronic systems [2], [31]. A common simulation flow is shown in Fig. 1. The sampled structure responses can be obtained by exciting one input port at a time and computing or measuring the responses at the output ports (**Response Characterization**). By approximating the sampled frequency-dependent or time-dependent system response data, a macromodel is generated to replace the original large-order system by a smaller-order one with similar input-output relationship (**Macromodeling**). The macromodel is used to generate spectra and waveforms for signal integrity analysis or coupled with other circuit model blocks (e.g., logic devices) for global simulation (**Simulation**). Peripheral pre-processing and post-processing techniques are used to rectify the macromodel characteristics and enhance the simulation performance.

Generally, for a single-port (one input port and one output port) system, macromodeling techniques intend to fit a linear-time invariant (LTI) system to the desired continuous-time frequency-sampled response $H(s)$ at a set of calculated/sampled points at the input and output ports. The model

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is usually a state-space system or a rational transfer function with a set of basis $\{\phi_n\}$

$$H(s) \approx \frac{N(s)}{D(s)} = \sum_{n=1}^{N} b_n \phi_n(s) \Big/ \sum_{n=1}^{N} \widetilde{b}_n \phi_n(s), \quad (1)$$

where $\widetilde{b}_n, b_n \in \Re$ and N is the macromodel order. The algorithm is usually required to fit hundreds of sampled data points for each port. Therefore, the linear-structure macromodeling can be classified as a large-scale broadband system identification problem. There are many strict constraints in this macromodeling procedure, such as accurate and physically-consistent response approximation, low computation complexity, and numerically-robust computation in the broadband, massive-ports (massive-coupled) and large-order system modeling cases.

In the L_2 sense, the optimal model of a system can be obtained through minimizing the following objective function

$$\min \left\| \frac{N(s)}{D(s)} - H(s) \right\|_2 . \quad (2)$$

However, this is a numerically-sensitive non-linear problem with no prior information about the exact pole and zero locations of the system under identification. The response is usually approximated using Prony's method [32] for a coarse solution or other identification frameworks, such as continuous-time domain Sanathanan-Koerner (SK) iteration [33] or equivalent discrete-time domain counterpart [34], for a finer solution. The objective function of the SK iteration in the tth iteration is

$$\min \left\| \frac{N^{(t)}(s)}{D^{(t-1)}(s)} - \frac{D^{(t)}(s)}{D^{(t-1)}(s)} H(s) \right\|_2 . \quad (3)$$

By arranging the weighting function $\sigma^{(t)}(s) := D^{(t)}(s)/D^{(t-1)}(s)$, the model parameters can be determined using a least-squares solving

$$\underbrace{\frac{N^{(t)}(s)}{D^{(t)}(s)} \frac{D^{(t)}(s)}{D^{(t-1)}(s)}}_{(\sigma H)^{(t)}(s)} - \underbrace{\frac{D^{(t)}(s)}{D^{(t-1)}(s)}}_{\sigma^{(t)}(s)} H(s) \approx 0 . \quad (4)$$

If a monomial power series basis function is used in (4) for broadband macromodeling, i.e., $\phi_n(s) = s^n$, the traditional SK iteration approach will suffer from an ill-conditioned Vandermonde matrix calculation [9]. Therefore, Vector Fitting (VF) is proposed as a robust and simple broadband macromodeling technique, which has been widely applied in practice. In this paper, the discussion of VF is divided into three sections:

1) Data section ($H(s)$): Input data choices (Section IV-A), pre-processing of data (Section IV-B) and model (Section IV-C);
2) Algorithms section ($H(s) \to N(s)/D(s)$): Identification criterion and framework (Section V-A) and numerical implementation (Section V-B);
3) Models section ($N(s)/D(s)$): Post-processing for model physical consistency (Section VI-A) and simulation (Section VI-B).

III. Formulation of Vector Fitting (VF)

In VF, given a set of poles $\{\alpha_n\}$, (1) is approximated using a summation of partial fraction basis and a unity basis with their model parameters $\{c_n\}$ and d,

$$H(s) \approx \frac{N(s)}{D(s)} = \left(\sum_{n=1}^{N} \frac{c_n}{s+\alpha_n} \right) + d. \quad (5)$$

By including the weighting function $\sigma(s)$, (5) is linearized into an iterative separable denominator calculation, namely, for the tth iteration,

$$\underbrace{\left(\sum_{n=1}^{N} \frac{c_n^{(t)}}{s+\alpha_n^{(t)}} \right) + d^{(t)}}_{\sigma H^{(t)}(s)} \approx \underbrace{\left(\left(\sum_{n=1}^{N} \frac{\gamma_n^{(t)}}{s+\alpha_n^{(t)}} \right) + 1 \right)}_{\sigma^{(t)}(s)} H(s) , \quad (6)$$

which falls into the framework of SK iteration (4) [9], [10].

In numerical implementation, provided all poles are real and N_s frequency-sampled data points are given, an expression from (6) is formed for each frequency-sampled point s_i, $i = 1, 2, \ldots, N_s$,

$$\mathbf{A}_i \mathbf{x} = b_i, \quad (7)$$

where $b_i = H(s_i)$, $\mathbf{x} = \begin{bmatrix} c_1^{(t)} & \cdots & c_N^{(t)} & d^{(t)} & \gamma_1^{(t)} & \cdots & \gamma_N^{(t)} \end{bmatrix}$, and $\mathbf{A}_i = \begin{bmatrix} \frac{1}{s+\alpha_1^{(t)}} & \cdots & \frac{1}{s+\alpha_n^{(t)}} & 1 & \frac{-H(s_i)}{s+\alpha_1^{(t)}} & \cdots & \frac{-H(s_i)}{s+\alpha_n^{(t)}} \end{bmatrix}$. $\mathbf{x}$ are solved through stacking the row (7) at the N_s sampled points to form an overdetermined linear equations problem,

$$\begin{bmatrix} \mathbf{A}_1^T & \mathbf{A}_2^T & \cdots & \mathbf{A}_{N_s}^T \end{bmatrix}^T \mathbf{x} = \begin{bmatrix} b_1 & b_2 & \cdots & b_{N_s} \end{bmatrix}^T , \quad (8)$$

where it can be solved through normal equations or a QR decomposition. The zeros of $\sigma^{(t)}(s)$ (i.e., the new set of poles $\left\{\alpha_n^{(t+1)}\right\}$) can be calculated as the eigenvalues of the matrix

$$\Psi = \begin{bmatrix} \alpha_1^{(t)} & & & \\ & \alpha_2^{(t)} & & \\ & & \ddots & \\ & & & \alpha_N^{(t)} \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} \gamma_1^{(t)} \\ \gamma_2^{(t)} \\ \vdots \\ \gamma_N^{(t)} \end{bmatrix}^T . \quad (9)$$

If the poles are unstable (i.e., $\Re\left(\left\{\alpha_n^{(t+1)}\right\}\right) > 0$), the poles are flipped against the imaginary axis to the open left half plane for pole stabilization

$$\alpha_n^{(t+1)} := -\alpha_n^{(t+1)}. \quad (10)$$

This is equivalent to cascading an allpass filter $A(s)$ to alter the phase response

$$A(s) = \frac{s+\alpha}{s-\alpha}. \quad (11)$$

The computation is repeated until convergence is achieved, say, $\sigma(s) \approx 1$ and $\left\| \frac{N^{(t)}(s)}{D^{(t)}(s)} - H(s) \right\| \approx 0$, at the N_Tth iterations. Eq. (6) is then reduced to

$$\sum_{n=1}^{N} \frac{c_n^{(N_T)}}{s+\alpha_n^{(N_T)}} + d^{(N_T)} \approx H(s) , \quad (12)$$

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and the residues $\left\{c_n^{(N_T)}\right\}$ and $d^{\{N_T\}}$ can be calculated similarly as in (8). In summary, VF replaces the monomial power series basis by a partial fraction basis, which significantly improves the numerical condition in calculation of (8). The detailed VF formulation is shown in [1], [9], [10]. Pseudocodes are given to summarize the framework of VF:

Algorithm 1 Pseudocodes of Vector Fitting (VF)

```
Find H(z), and assign {α_n^(0)};
repeat
    Calculate {γ_n^(t)} by solving (8) with {α_n^(t)};
    Calculate {α_n^(t+1)} by solving (9) and stabilize the
    unstable poles through (10);
until {α_n^(t)} converges after N_T iterations
Calculate {c_n^(N_T)} and d^(N_T) through (12) with {α_n^(N_T)};
```

IV. Data

Data describe the system response, and are obtained from measurements (e.g., vector network analyzer (VNA)) or EM simulators (e.g., Nexxim [30]). Since data content can affect the properties and quality of the macromodel, different considerations and techniques have been proposed to ensure the input data are maximally informative for identification.

A. Input data choices

Continuous-time frequency-sampled data $H(s)$ are used for macromodeling in VF [1], as the frequency-sampled responses capture the high-frequency behaviors of the system. Examples of frequency-sampled data are scattering parameters (S-parameters) for RF objects and admittance parameters (Y-parameters) for interconnects. Alternative data choices, such as frequency response derivative $H'(s)$ [15], phase response $\angle H(s)$ [16] and magnitude response $|H(s)|$ [17], are used for different identification purposes. In practices, frequency-domain macromodeling involves complicated measurements. Truncated time-sampled data (input and output response $X[n]$ and $Y[n]$) are often used, therefore (discrete) time-domain VF have been proposed [13], [14]. Approximation using combination of several classes of data (hybrid-domain approximation) provides extra system information for a more accurate approximation. It has been applied to digital IIR filter approximation [4] and works well in macromodeling process.

B. Pre-processing of data

The system response should correctly describe the system. However, some problems, such as data burst, defects, missing and noise-disturbance, may happen during data collection. Some information may get lost and difficulties and failures in approximation may arise. Therefore, data pre-processing is required to ensure the data are meaningful (e.g., passive and causal, as explained in Section VI-A) to generate a correct macromodel. For example, causality and passivity verification of input data and delay extraction using (generalized) Hilbert transform [35] are developed. Furthermore, causality-constrained data interpolation is developed to generate consistent DC and low-frequency data, which is necessary for simulation but usually not provided in the frequency-sampled data [35].

In addition, a large data set or broadband responses usually have a large variance and may result in ill-conditioned calculation. Pre-filtering techniques, in this scenario, can be used to change the distribution of noise and bias, so as to give a better fitting of important frequency range and a numerically favorable calculation with a small computational cost. An appropriate adaptive or deterministic data selection process and response weighting can also be applied for a better approximation.

C. Pre-processing of model

$A\ priori$ configuration of macromodels should be chosen based on the knowledge of the algorithms (SK iteration) and data for a convenient approximation. For example, an $a\ priori$ model order selection helps generate a minimum size macromodel for efficient simulations with accuracy control. The model order can be selected by applying experimental observation of the frequency response in frequency-sampled data [18], or the Hankel Singular Value (HSV) in (discrete) time-sampled data [14].

V. Algorithms

Given a set of input data, an algorithm is used to determine the model parameters. A good algorithm should have an appropriate identification criterion and should be easy and robust for numerical implementation. We first discuss the algebraical minimization criteria, then the numerical implementation for a numerically favorable model parameters calculation.

A. Identification criterion and framework

The selection of the approximation criteria is important for model approximation. The model should be reliable, obtained within a reasonable computation time, and should admit an exact description of the true system. SK iteration with an L_2-norm prediction error is usually used since it is applicable to different response models. Other criterion extensions are also developed recently for specific applications.

Massive-port macromodeling: VF handles multi-port macromodeling by stacking the system equation matrices of responses of all ports into a single column of over-determined equation for solutions. However, numerical difficulties exist in modeling the systems with a large number of ports (e.g., package parasitic networks and electromagnetic-aware circuits). To model a system with an arbitrary number of ports, a reformation of the VF framework is proposed to approximate the eigenpairs rather than the matrix elements [20]. It gives a more accurate approximation for systems with a large ratio between the largest and smallest eigenvalues.

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Parametric macromodeling: Variabilities in geometry and material properties are generated during the manufacturing process, and become a critical factor in nano-scale high-frequency circuit simulation and design. In order to accurately predict the behavior and reduce the computation time of repeated simulations, a parametric macromodel is used to describe the variational structures

$$H(s,g) \approx \frac{\sum_{n=0}^{N_s}\left(\sum_{p=1}^{P} b_{np}\varphi_p(g)\right)\phi_n(s)}{\sum_{n=0}^{N_s}\left(\sum_{p=1}^{P} \widetilde{b}_{np}\varphi_p(g)\right)\phi_n(s)}, \qquad (13)$$

where $\phi_n(s)$ is the frequency-dependent basis and $\varphi_p(g)$ is the variability-dependent basis with a single variational parameter g and P samples in the variability domain. The variational structures can be described by a macromodel with a polynomial basis or rational function basis [23], [24], [26].

B. Numerical Implementation

Due to the nature of iterative calculation, its implementation is usually numerically sensitive. Although VF solves the ill-conditioned calculation by a partial-fraction basis, other problems, such as inappropriate initial guess and noise-contaminated responses, damage the algorithm convergence. Some improvements have been proposed to alleviate these problems.

Initial poles and applied basis: The algorithm gives a set of model parameters (b_n and $\tilde{b}_n$ in (1)) according to the given set of basis ($\phi(s)$), the sampled data and the initial poles. The selected basis affects the conditioning of the system equation matrix in (8) and the accuracy of the solution.

One approach to address this problem is to select an appropriate set of initial poles. The initial poles can be obtained by a simple calculation (e.g., Prony method [32]), or intuitively assigned as a set of weakly-damped initial poles ($\alpha_{1,2} = a \pm j0.01a$) [1]. Another approach is to select a robust basis for calculation, which minimizes the numerical disturbance due to the inappropriate set of poles. Orthonormal basis $\phi_{or\_n}(s)$ [11] and discrete-time domain (z-domain) basis $\phi_{z\_n}(z)$ [3] have been proposed based on this idea, namely,

$$\phi_{or\_n}(s) = \kappa_n\sqrt{2\Re(\alpha_n)}\left(\prod_{j=1}^{n-1}\frac{s-\alpha_j^*}{s+\alpha_j}\right)\frac{1}{s+\alpha_n}, \qquad (14)$$

$$\phi_{z\_n}(z) = \frac{1}{z^{-1}+\alpha_n}, \qquad (15)$$

where κ_n is the normalization coefficient and $*$ denotes complex conjugate. Orthonormal basis, from a mathematical perspective reduces the condition number of the system equation matrix, while the discrete-time basis calculation maps the left Laplace plane to a unit circle plane, and thus improves the numerical condition from a signal-processing perspective. Furthermore, discrete-time domain orthonormal basis is proposed recently for further robustness improvement [12]. Other basis generalizations are also available for different requirements, e.g., modeling the responses with repeated poles [11] and time-sampled data [13], [14].

Macromodeling with noisy signals: Experiences show that the convergence is severely impaired in noise-contaminated signals and biased in the low-frequency region. This is because the unity basis of $\sigma(s)$ in (6) impairs the LS normalization of equation solving. To address this problem, a variable unity basis (γ_0) normalization (16) with an additional relaxed nontriviality condition (17) is adopted for a relaxed least-squares normalization (Relaxed VF) [10], [19],

$$\underbrace{\left(\sum_{n=1}^{N}\frac{c_n^{(t)}}{s+\alpha_n^{(t)}}\right)+d_n^{(t)}}_{(\sigma H)^{(t)}(s)} \approx \underbrace{\left(\left(\sum_{n=1}^{N}\frac{\gamma_n^{(t)}}{s+\alpha_n^{(t)}}\right)+\gamma_0\right)}_{\sigma^{(t)}(s)} H(s), \qquad (16)$$

$$\Re\left(\sum_{k=1}^{N_s}\left(\sum_{n=1}^{N}(\gamma_n\phi_n(s_k))+\gamma_0\right)\right) = N_s+1. \qquad (17)$$

Eq. (17) imposes that the sum of the samples approaches to a nonzero value. This improves the normalization of the transfer function coefficients and the linearization of the iterative SK iteration without affecting the convergence.

Massive-port macromodeling: VF suffers from computational inefficiency when macromodeling massive-port systems due to the unnecessary calculation of c_n in (8) during iterative pole calculation (Step 3). Based on the observation of shared common poles in the macromodel, a QR decomposition is applied to extract the calculation of γ_n of each port response and formulate a compacted calculation [21]. The computational complexity is then reduced from $O\left((P_{in}P_{out}+1)^2 n^2 N_s P_{in}P_{out}\right)$ to $O\left(n^2 N_s P_{in} P_{out}\right)$ for a system with P_{in} input ports and P_{out} output ports, without any lost of accuracy.

VI. Models

The macromodel (model) describes the Input-Output (I/O) characteristics of the approximated system, for analysis and coupled simulation with other circuit models. The model should be accurate, physically consistent and of low complexity for simulation. Necessary post-processing techniques are adopted to ensure a correct simulation.

A. Post-processing for a physically consistent model

The macromodel should be physically consistent, i.e., real-valued, stable, passive and causal [36].

Real-valued: Real-valued macromodels do not generate complex-valued responses for real-valued input data. However, the original VF may generate complex-valued macromodels if the complex poles are not restricted to conjugate pairs. Some modifications in (7)-(9) are required to construct a real-valued macromodel, as explained in [1]. Complex-valued computations of (8) are separated into its real and imaginary parts to avoid numerical errors, at the expense of an increased problem size.

Stable: Stable macromodels do not generate response beyond limits for any input signal. An unstable pole can be stabilized through a non-linear pole flipping in (11). The flipping, however, does not affect the norm criterion in (3) and the algorithm convergence.

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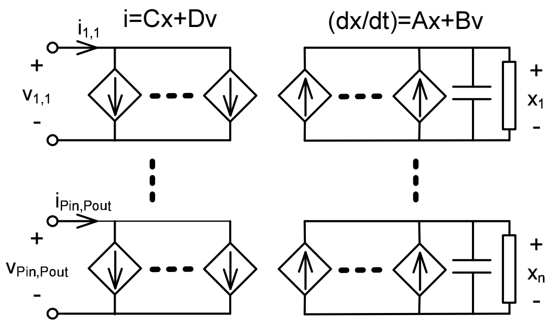

Fig. 2. Equivalent circuit realization of a P_{in}-input-ports and P_{out}-output-ports system ($\mathbf{i} = \mathbf{Cx} + \mathbf{Dv}$ and $\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{Bv}$), formed by the sampled admittance data.

Passive: Passive macromodels do not generate energy, yet VF may generate slightly non-passive macromodels due to numerical errors. Therefore, passivity enforcement through perturbation of model parameters is required to passify the model, and a detailed study is shown in [37].

Causal: Causal macromodels do not generate output signal according to the future input. However, modeling electrically-long structures (i.e., responses with a signal delay) using a purely rational macromodel may suffers from inapplicable fitting and often generates a non-causal model. A reformulated VF is developed [22]. With the D obtained time delays $\{\tau_d\}$, the response can be fitted via

$$H(s) \approx \frac{\sum_{n=0}^{N} \sum_{d=1}^{D} b_{nd}\phi_n(s)\, e^{-s\tau_d}}{\sum_{n=0}^{N} \widetilde{b}_{nd}\phi_n(s)}. \tag{18}$$

B. Post-processing for simulation

The approximant macromodel is used to generate the frequency response, time-domain reflectometry (TDR) waveforms, time-domain transmissometry (TDT) waveforms and eye diagrams for channel analysis, or coupled with other models for overall simulation. Therefore, the models should be fully integrated with simulation tools for efficient analysis. The macromodel can be described by a pole-residue form in Matlab Simulink or Verilog-A description for high-level simulation. The macromodel can also be described as an equivalent circuit in a SPICE netlist for co-simulation with other (non-linear) macromodels [38]. A standard equivalent circuit in Fig. 2 can be generated using differential-equation realization.

VII. P-NORM APPROXIMATION IN VF

To satisfy different macromodeling requirements and give a more realistic description of the system, the approximation framework (3) is extended to a P-norm (L_p) approximation. The minimization framework (3) is generalized to

$$\min \left\| \frac{N^{(t)}(s)}{D^{(t-1)}(s)} - \frac{D^{(t)}(s)}{D^{(t-1)}(s)} H(s) \right\|_p, \tag{19}$$

for which the over-determined equations can be efficiently solved by convex programming. The approximation framework can be generalized to a user-defined norm (e.g., region-dependent norm) approximation or (norm-)constrained approximation to meet different macromodeling requirements. For example, L_∞ (Chebyshev norm) approximation gives a smaller

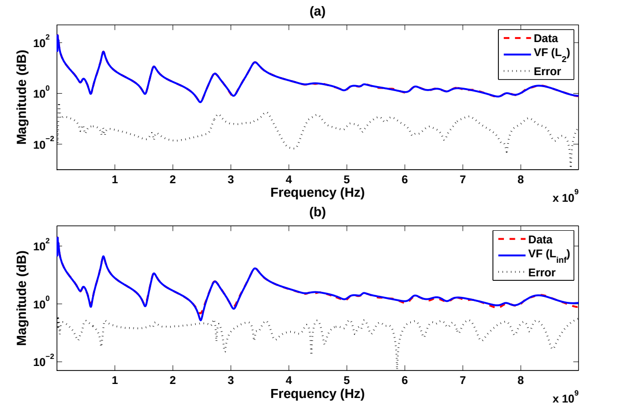

Fig. 3. Magnitude responses of the power distribution network: (a) approximation using L_2 norm, and (b) approximation using L_{inf} (L_∞) norm.

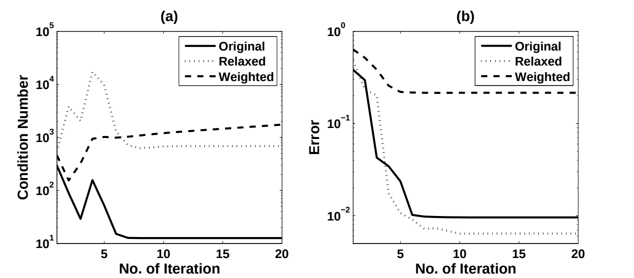

Fig. 4. (a) Condition number of the system equation matrix in (8), and (b) L_2 error of the approximation using original VF (6), relaxed VF (16) and weighted VF.

macromodel for a linear-phase (time-delayed) response, L_2 approximation gives a more accurate macromodel for a noisy response, and L_1 approximation is favorable for system identification with an impulsive-noise-contaminated signal.

VIII. NUMERICAL EXAMPLES

The VF is coded in Matlab m-script files and run in the Matlab 7.5 on a 1GB-RAM 3.4GHz PC. The example arises from a power distribution network of an IC power plane [2], whose admittance responses range from DC to 9GHz. The port response is fitted using relaxed VF [19] with a 35th-order macromodel with 10 iterations (18.28 seconds) and a set of linear-spaced initial poles, which gives 0.0064 L_2 and 0.0022 L_∞ error in fitting. Fig. 3 plots the magnitude-domain responses of the converged approximant. Fig. 4 shows the condition number of the system equation matrix (8) and the L_2 error during iterations. In general, VF converges quickly (≤ 10 iterations), especially for minimum-phase (passive) response. For further analysis of generalizations of VF, we repeat the example using VF without relaxed constraint and relaxed VF with a inverse-magnitude weighting. The quantitative comparison is shown in Fig. 4. It shows that the weighting does not contribute much to the numerical condition, but it affects the convergence. The relaxation may affect the numerical condition of the calculation, but it also significantly improves the accuracy of the approximation. At last, we repeat the example under an SNR of -35dB. In this case, relaxed VF

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converges with 0.0193 L_2 and 0.0014 L_∞ error. This shows the relaxed VF is robust to the noisy response approximation.

The responses are also fitted using L_∞ norm approximation with the same configuration and clean signal, which gives an approximation with 0.0165 L_2 error and 0.0016 L_∞ error. The magnitude-domain response of the converged approximation in Fig. 3 shows that L_∞-norm approximation renders a more accurate low-frequency (near DC) approximation which is important for simulation, and P-norm approximation can be used as an alternative approximation criterion.

IX. Conclusions

By applying a partial fraction basis, Vector Fitting (VF) has demonstrated its numerical robustness in broadband system identification. The good performance and versatile extensibility of VF render it an attractive tool for signal/power integrity analyses. In this paper, different issues related to VF have been discussed for obtaining a good macromodel for simulation. Furthermore, a P-norm approximation criterion is proposed to provide an alternative measure to meet different requirements.

Acknowledgments

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