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ON WEAKLY β-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

Sanjay Tahiliani

Abstract. As a generalization of β-continuous functions, we introduce and study several properties of weakly β-continuous functions in bitopological spaces and we obtain its several characterizations .

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1. Introduction

The notion of β-open sets due to Mashhour et al.[1] or semi-preopen sets due to Andrijević [2] plays a significant role in general topology. In [1] the concept of β-continuous functions is introduced and further Popa and Noiri[13] studied the concept of weakly β-continuous functions.In 1992,Khedr et al.[10] introduced and studied semi-precontinuity or β-continuity in bitopological spaces .In this paper,we introduce and study the notion of weakly β-continuous functions in bitopological spaces further and investigate the properties of these functions.

2. Preliminaries

Throughout the present paper,(X,τ_1,τ_2)(resp.(X,τ))) denotes a bitopological (resp.topological) space.Let (X,τ) be a topological space and A be a subset of X.The closure and interior of A are denoted by Cl(A) and Int(A) respectively.

Let (X,τ_1,τ_2) be a bitopological space and let A be a subset of X.The closure and interior of A with respect to τ_i are denoted by iCl(A) and iInt(A),respectively,for i=1,2.

Definition 2.1. A subset A of a bitopological space (X,τ_1,τ_2) is said to be

(i).(i,j)-regular open[4] if A=iInt(j(Cl(A)) where $i\neq j$, i,j=1,2.
(ii).(i,j)-regular closed[5] if A=iCl (j(Int(A)) where $i\neq j$, i,j=1,2.
(iii).(i,j)-preopen[7] if $A\subseteq$ iInt(j(Cl(A)) where $i\neq j$, i,j=1,2.

Definition 2.2. A subset A of a bitopological space (X,τ_1,τ_2) is said to be
(τ_i,τ_j)-semi-preopen(briefly (i,j)-semi-preopen)[10] if there exists a (i,j)-preopen set U such that $U\subseteq A\subseteq$jCl(U) or it is said to be (i,j)-β-open if $A\subseteq$jCl(iInt(jCl(A))),where $i\neq j$,i,j=1,2.The complement of (i,j)-semi-preopen
set is said to be (i,j)-semi-preclosed[10] or it is said to be (i,j)-β-closed if iInt(jCl(iInt(A))$\subseteq$A, where $i\neq j$,i,j=1,2.

Lemma 2.1.Let (X,τ_1,τ_2) be a topological space and $\{A_\lambda:\lambda\in\Delta\}$ be a family of subsets of X.Then (i).If A_λ is (i,j)-β-open for each $\lambda\in\Delta$,then $\bigcup_{\lambda\in\Delta} A_\lambda$ is (i,j)-β-open.

(ii). If A_λ is (i,j)-β-closed for each $\lambda\in\Delta$,then $\bigcap_{\lambda\in\Delta} A_\lambda$ is (i,j)-β-closed.

Proof.The proof follows from Theorem 3.2 of [10] .
(ii).This is an immediate consequence of (i).

Definition 2.3.Let A be subset of a bitopological space (X,τ_1,τ_2).
(i).The (i,j) β-closure[10] of A,denoted by (i,j)-βCl(A),is defined to be the intersection of all (i,j)-β-closed sets containing A.
(ii). The (i,j) β-interior of A,denoted by (i,j)-βInt(A),is defined to be the union of all (i,j)-β-open sets contained in A.

Lemma 2.2.Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X.Then
(i).(i,j)-βInt(A) is (i,j)-β-open.
(ii).(i,j)-βCl(A) is (i,j)-β-closed.
(iii).A is (i,j)-β-open if and only if A=(i,j)-βInt(A).
(iv).A is (i,j)-β-closed if and only if A=(i,j)-βCl(A).

Proof.(i) and (ii) are obvious from Lemma 2.1.
(iii) and (iv) are obvious from (i) and (ii).

Lemma 2.3.For any subset A of a bitopological space (X,τ_1,τ_2),$x\in$(i,j)-βCl(A) if and only if $U\cap A\neq \emptyset$ for every (i,j)-β-open set U containing x.

Lemma 2.4.Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X.Then
(i).X~(i,j)-βInt(A)=(i,j)-βCl(X~A).
(ii).X~(i,j)-βCl(A)=(i,j)-βInt(X~A).

Proof.(i).By Lemma 2.2, (i,j)-βCl(A) is (i,j)-β-closed.Then X~(i,j)-βCl(A) is (i,j)-β-open.On the other hand,X~(i,j)-βCl(X~A)$\subseteq$A and hence X~(i,j)- βCl(X~A)$\subseteq$(i,j)-βInt(A). Conversely,let $x\in$(i,j)-βInt(A).Then there exists (i,j)-β-open set G such that $x\in G\subseteq A$.Then X~G is (i,j)-β-closed and X~A$\subseteq$X~G.Since $x\notin$X~G,$x\notin$(i,j)-βCl(X~A) and hence (i,j)- βInt(A)$\subseteq$X~(i,j)-βCl(X~A).Therefore X~(i,j)-βInt(A)= (i,j)-βCl(X~A).

(ii).This follows immediately from (i).

Definition 2.4.Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X.A point x of X is said to be in the (i,j)-θ-closure[8] of A,denoted by (i,j)-Cl_θ(A),if $A\cap$jCl(U) $\neq \emptyset$ for every τ_i-open set U containing x,where i,j=1,2 and i≠j.

A subset A of X is said to be (i,j)-θ-closed if A=(i,j)-Cl_θ(A).A subset A of X is said to be (i,j)-θ-open if X~A is (i,j)-θ-closed.The (i,j)-θ-interior of A,denoted by (i,j)-Int_θ(A),is defined as the union of all (i,j)-θ-open sets contained in A.Hence $x\in$(i,j)-Int_θ(A) if and only if there exists a τ_i-open set U containing x such that $x\in U\subseteq$jCl(U)$\subseteq$A.

Lemma 2.5. For a subset A of a bitopological space (X,τ_1,τ_2),the following properties hold:

(i).$X \sim (i,j)\text{-}Int_\theta(A) = (i,j)\text{-}Cl_\theta(X \sim A)$.
(ii).$X \sim (i,j)\text{-}Cl_\theta(A) = (i,j)\text{-}Int_\theta(X \sim A)$.

Lemma 2.6. [8].Let (X,τ_1,τ_2) be a bitopological space.If U is a τ_j-open set of X,then $(i,j)\text{-}Cl_\theta(U)=iCl(U)$

Definition 2.5.A function $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is said to be (i,j)-β-continuous[10] if $f^{-1}(V)$ is (i,j)-β-open in X for each σ_i-open set V of Y.

Definition 2.6.(i). A function $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is said to be (i,j)- weakly precontinuous[12] if for each $x\in X$ and each σ_i-open set V of Y containing f(x),there exists (i,j)-preopen set U containing x such that $f(U)\subseteq jCl(V)$.

(ii). A function $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is said to be (i,j)-weakly-β-continuous if for each $x\in X$ and each σ_i-open set V of Y containing f(x),there exists (i,j)-β-open set U containing x such that $f(U)\subseteq jCl(V)$.

A function $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is said to be pairwise weakly precontinuous [12] (resp.pairwise weakly β-continuous) if f is weakly (1,2)-precontinuous and weakly (2,1)-precontinuous (resp. if f is weakly (1,2)-β-continuous and weakly (2,1)-β-continuous).

Remark 2.1.Since every (i,j)-preopen set is (i,j)-β-open ([10],Remark 3.1), every (i,j) weakly precontinuous function is (i,j) weakly β-continuous for i,j=1,2 and $i\neq j$.The converse is not true.

3. Characterizations

Theorem 3.1.For a function $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$,the following properties are equivalent:

(i). f is (i,j)-weakly β-continuous.
(ii). $(i,j)\text{-}\beta Cl(f^{-1}(jInt(iCl(B))))) \subseteq f^{-1}(iCl(B))$ for every subset B of Y.
(iii). $(i,j)\text{-}\beta Cl(f^{-1}(jInt(F))) \subseteq f^{-1}(F)$ for every (i,j)-regular closed set F of Y.
(iv). $(i,j)\text{-}\beta Cl(f^{-1}(Cl(V)) \subseteq f^{-1}(iCl(V))$ for every σ_j-open set V of Y.
(v). $f^{-1}(V) \subseteq (i,j)\text{-}\beta Int(f^{-1}(jCl(V)))$ for every σ_i-open set V of Y.

Proof. (i) →(ii). Let B be any subset of Y.Assume that $x\in X\sim f^{-1}(iCl(B))$.Then $f(x) \in Y\sim iCl(B)$ and so there exists a σ_i-open set V of Y containing f(x) such that $V\cap B= \emptyset$,so $V\cap jInt(iCl(B)))= \emptyset$ and hence $jCl(V)\cap jInt(iCl(B)))= \emptyset$. Therefore,there exists (i,j)-β-open set U containing x such that $f(U) \subseteq jCl(V)$.Hence we have $U\cap f^{-1}(jInt(iCl(B)))= \emptyset$ and $x\in X\sim(i,j)\text{-} \beta Cl(f^{-1}(((jInt(iCl(B))))$ by Lemma 2.3.Thus we obtain $(i,j)\text{-}\beta Cl(f^{-1}(((jInt(iCl(B)))) \subseteq f^{-1}(iCl(B))$.

(ii) →(iii).Let F be any (i,j)-regular closed set of Y.Then F=iCl(jInt(F) and we have $(i,j)\text{-}\beta Cl(f^{-1}((jInt(F)))=(i,j)\text{-}\beta Cl(f^{-1}((jInt(iCl(jInt(F))))) \subseteq f^{-1}(iCl(jInt(F))) = f^{-1}(F)$.

(iii) →(iv).For any σ_j-open set V of Y,Then iCl(V) is (i,j)-regular closed . Then we have (i,j)-βCl(f^{-1}((V))) ⊆ (i,j)-βCl(f^{-1}((jInt(iCl(V)))) ⊆ f^{-1}(iCl(V)).

(iv) →(v).Let V be any σ_i-open set of Y.Then Y~jCl(V) is σ_j-open set in Y and we have (i,j)-βCl(f^{-1} (Y~jCl(V))) ⊆ f^{-1} (iCl(Y~jCl(V))) and hence X~(i,j)-βInt(f^{-1}(jCl(V))) ⊆X~f^{-1} (iInt(jCl(V))) ⊆X~f^{-1}(V).Therefore,we obtain f^{-1}(V) ⊆ (i,j)-βInt(f^{-1}(jCl(V))).

(v)→(i).Let x∈X and let V be a σ_i-open set containing f(x).We have x∈ f^{-1}(V) ⊆(i,j)-βInt(f^{-1}(jCl(V))).Put U=(i,j)-βInt(f^{-1}(jCl(V))).By Lemma 2.2,U is (i,j)-β-open set containing x and f(U) ⊆ jCl(V).This shows that f is (i,j)-weakly β-continuous.

Remark 3.1.Let $\tau = \tau_1 = \tau_2$ and $\sigma=\sigma_1=\sigma_2$ .Then by Theorem 3.1 we obtain the results for a function f:(X,τ)→(Y,σ) established in Theorem 2 of [13] .

Theorem 3.2. For a function f:(X,τ_1,τ_2)→ (Y,σ_1,σ_2),the following properties are equivalent:

(i). f is (i,j)-weakly β-continuous.
(ii). f((i,j)- βCl(A)) ⊆((i,j)-Cl_θ(f(A)) for every subset A of X.
(iii). (i,j)- βCl(f^{-1}(B)) ⊆ f^{-1}((i,j)-Cl_θ(B)) for every subset B of Y.
(iv). (i,j)- βCl(f^{-1}(jInt((i,j)-Cl_θ(B)))) ⊆ f^{-1}((i,j)-Cl_θ(B)) for every subset B of Y.

Proof.(i)→(ii).Assume that f is (i,j)-weakly β-continuous. Let A be any subset of X, x∈(i,j)-βCl(A) and V be a σ_i-open set of Y containing f(x).Then there exists (i,j)-β-open set U containing x such that f(U) ⊆ jCl(V).Since x∈(i,j)-βCl(A), by Lemma 2.3,we obtain U∩A≠ Ø and hence Ø≠f(U) ∩f(A) ⊆ jCl(V) ∩f(A).Therefore, we obtain f(x) ∈ (i,j)- Cl_θ(f(A)).

(ii) →(iii).Let B be any subset of Y.Then we have f((i,j)- βCl(f^{-1}(B))) ⊆(i,j)- Cl_θ(f(f^{-1} (B)) ⊆(i,j)- Cl_θ(B) and hence (i,j)- βCl(f^{-1}(B)) ⊆ f^{-1}((i,j)- Cl_θ(B)).

(iii) →(iv).Let B be any subset of Y.Since (i,j)- Cl_θ(B) is σ_i-closed in Y,by Lemma 2.6, (i,j)-βCl(f^{-1}(jInt((i,j)-Cl_θ(B)))) ⊆ f^{-1}((i,j)-Cl_θ(jInt((i,j)-Cl_θ(B))))=f^{-1}(iCl(jInt(((i,j)-Cl_θ(B)))) ⊆ f^{-1}(iCl((i,j)- Cl_θ(B)))= f^{-1}((i,j)-Cl_θ(B)).

(iv) →(i).Let V be any σ_j-open set of Y.Then by Lemma 2.6,V⊆jInt(iCl(V))= jInt((i,j)-Cl_θ(V)) and we have (i,j)-βCl(f^{-1}(V)) ⊆(i,j)-βCl(f^{-1}(jInt((i,j)-Cl_θ(B)))) ⊆ f^{-1}((i,j)-Cl_θ(V))= f^{-1}(iCl(V)).Thus we have (i,j)-βCl(f^{-1}(V)) ⊆ f^{-1}(iCl(V)).It follows from Theorem 3.1 that f is (i,j)-weakly β-continuous.

Remark 3.2. By above Theorem,we obtain the results established in Theorem 4 of [13].

Definition 3.1.A bitopological space (X,τ_1,τ_2) is said to be (i,j)-regular[9] if for each x∈X and each τ_i-open set U containing x,there exists a τ_i-open set V such that x∈V⊆ jCl(V) ⊆U.

Lemma 3.1.[14]. If a bitopological space (X,τ_1,τ_2) is (i,j)-regular,then (i,j)-Cl_θ(F)=F for every τ_i -closed set F.

Theorem 3.3.Let (Y,σ_1,σ_2) be an (i,j)-regular bitopological space.For a function $f:(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$,the following properties are equivalent:

(i).f is (i,j)-β-continuous.
(ii).$f^{-1}((i,j)\text{-}Cl_\theta(B))$ is (i,j)-β-closed in X for every subset B of Y.
(iii). f is (i,j)-weakly β-continuous.
(iv).$f^{-1}(F)$ is (i,j)- β-closed in X for every (i,j)-θ-closed set F of Y.
(v). $f^{-1}(F)$ is (i,j)- β-open in X for every (i,j)-θ-open set V of Y.

Proof.(i)→(ii). Let B be any subset of Y.Since $(i,j)\text{-}Cl_\theta(B)$ is σ_i-closed in Y,it follows from Theorem 5.1 of [10] that $f^{-1}((i,j)\text{-}Cl_\theta(B))$ is (i,j)-β-closed in X.

(ii) →(iii). Let B be any subset of Y.Then we have $(i,j)\text{-}\beta Cl(f^{-1}(B)) \subseteq (i,j)\text{-}\beta Cl(f^{-1}((i,j)\text{-}Cl_\theta(B)))= f^{-1}((i,j)\text{-}Cl_\theta(B))$. By Theorem 3.2,f is (i,j)-weakly-β-continuous.

(iii) →(iv). Let F be any (i,j)-θ-closed set of Y.Then by Theorem 3.2,$(i,j)\text{-}\beta Cl(f^{-1}(F)) \subseteq f^{-1}((i,j)\text{-}Cl_\theta(F))= f^{-1}(F)$.Therefore,by Lemma 2.2 , $f^{-1}(F)$ is (i,j)-β-closed in X.

(iv) →(v). Let V be any (i,j)-θ-open set of Y.By (iv),$f^{-1}(Y\sim V)=X\sim f^{-1}(V)$ is (i,j)-β-closed in X and hence $f^{-1}(V)$ is (i,j)-β-open in X.

(v) →(i). Since Y is (i,j)-regular,by Lemma 3.1,$(i,j)\text{-}Cl_\theta(B)=B$ for every σ_i-closed set B of Y and hence σ_i-open set is (i,j)-θ-open.Therefore $f^{-1}(V)$ is (i,j)-β-open for every σ_i-open set V of Y.By Theorem 5.1 of [10], f is (i,j)-β-continuous.

4. Weak β-continuity and β-continuity

Definition 4.1. A function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is said to be (i,j)-weakly* quasi continuous(briefly.w*.q.c)[14] if for every σ_i -open set V of Y,
$f^{-1}(jCl(V)\sim V)$ is biclosed in X.

Theorem 4.1.If a function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is (i,j)-weakly-β-continuous and (i,j)-w*.q.c,then f is (i,j)- β-continuous.

Proof.Let $x\in X$ and V be any σ_i-open set of Y containing f(x).Since f is (i,j)-weakly-β-continuous,there exists an (i,j)-β-open set U of X containing x such that $f(U) \subseteq jCl(V)$.Hence $x\notin f^{-1}(jCl(V)\sim V)$.Therefore,$x\in U\sim f^{-1}(jCl(V)\sim V) =U\cap(X\sim (f^{-1}(jCl(V)\sim V))$.Since U is (i,j)- β-open and $X\sim (f^{-1}(jCl(V)\sim V)$ is biopen,by Theorem 3.3 of [10],$G= U\cap(X\sim (f^{-1}(jCl(V)\sim V))$ is (i,j)-β-open.Then $x\in G$ and $f(G) \subseteq V$.For if $y\in G$,then $f(y) \notin (jCl(V)\sim V)$ and hence $f(y) \in V$.Therefore,f is (i,j)- β-continuous.

Definition 4.2. A function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is said to have a (i,j)-βinteriority condition if $(i,j)\text{-}\beta Int(f^{-1}(jCl(V))) \subseteq f^{-1}(V)$ for every σ_i-open set V of Y.

Theorem 4.2. If a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is (i,j)-weakly β-continuous and satisfies the (i,j)-βinteriority condition,then f is (i,j)-β-continuous.

Proof.Let V be any σ_i-open set of Y.Since f is (i,j)-weakly β-continuous,by Theorem 3.1, $f^{-1}(V) \subseteq$(i,j)- βInt(f^{-1}(jCl(V))).By the (i,j)-βinteriority condition of f,we have (i,j)- βInt(f^{-1}(jCl(V))) $\subseteq f^{-1}(V)$ and hence (i,j)-βInt(f^{-1}(jCl(V))) = $f^{-1}(V)$.By Lemma 2.2 , $f^{-1}(V)$ is (i,j)- β-open in X and thus f is (i,j)-β-continuous.

Definition 4.3. Let (X,τ_1,τ_2) be a bitopological space and let A be a subset of X.The (i,j)-β-frontier of A is defined as follows: (i,j)- βFr(A)=(i,j)-βCl(A) $\cap$(i,j)-βCl(X~A)= (i,j)-βCl(A)~(i,j)-βInt(A).

Theorem 4.3. The set of all points x of X for which a function $f:(X,\tau_1,\tau_2)\to (Y,\sigma_1,\sigma_2)$ is not (i,j)-weakly β-continuous is identical with the union of the (i,j)-β-frontier of the inverse images of the σ_j-closure of σ_i-open sets of Y containing f(x).

Proof.Let x be a point of X at which f(x) is not (i,j)-weakly β-continuous.Then there exists a σ_i-open set V of Y containing f(x) such that U$\cap$(X~f^{-1}(jCl(V))) ≠ Ø for every (i,j)-β-open set U of X containing x.By Lemma 2.3,x$\in$(i,j)-βCl(X~ f^{-1}(jCl(V))) .Since x$\in$ f^{-1}(jCl(V)),we have x$\in$(i,j)- βCl(f^{-1}(jCl(V))) and hence x$\in$(i,j)- βFr(f^{-1}(jCl(V))).
Conversely,if f is (i,j)-weakly β-continuous at x,then for each σ_i-open set V of Y containing f(x),there exists an (i,j)- β-open set U containing x such that f(U) $\subseteq$jCl(V) and hence x$\in$U$\subseteq$ f^{-1}(jCl(V).Therefore we obtain that x$\in$(i,j)- βInt(f^{-1}(jCl(V)).This contradicts that x$\in$ (i,j)-βFr(f^{-1}(jCl(V)).

Remark 4.1. By above Theorem,we obtain the results established in Theorem 4.7 of [3].

5. Weak β-continuity and almost β-continuity

Definition 5.1.A function $f:(X,\tau_1,\tau_2)\to (Y,\sigma_1,\sigma_2)$ is said to be (i,j)-almost β-continuous if for each x$\in$X and each σ_i-open set V containing f(x),there exists an (i,j)-β-open set U of X containing x such that f(U) $\subseteq$iInt(jCl(V)).

Lemma 5.1. A function $f:(X,\tau_1,\tau_2)\to (Y,\sigma_1,\sigma_2)$ is (i,j)-almost β-continuous if and only if $f^{-1}(V)$ is (i,j)-β-open for each (i,j)-regular open set V of Y.

Definition 5.2.A bitopological space (X,τ_1,τ_2) is said to be (i,j)-almost regular[16] if for each x$\in$X and each (i,j)-regular open set U containing x,there exists an (i,j)-regular open set V of X such that x$\in$V$\subseteq$jCl(V) $\subseteq$U.

Theorem 5.1. Let a bitopological space (Y,σ_1,σ_2) be (i,j)-almost regular.Then a function $f:(X,\tau_1,\tau_2)\to (Y,\sigma_1,\sigma_2)$ is (i,j)-almost β-continuous if and only if it is (i,j)-weakly-β-continuous.

Proof. Necesssity.This is obvious.

Sufficency.Suppose that f is (i,j)-weakly-β-continuous.Let V be any (i,j)-regular open set of Y and x∈ $f^{-1}(V)$.Then we have f(x) ∈V.By the almost (i,j)-regularity of Y,there exists an (i,j)-regular open set V_0 of Y such that f(x) ∈ V_0⊆ jCl(V_0) ⊆V.Since f is (i,j)-weakly-β-continuous,there exists an (i,j)-β-open set U of X containing x such that f(U) ⊆ jCl(V_0) ⊆V.This implies that x∈U⊆ $f^{-1}(V)$.Therefore we have $f^{-1}(V)$ ⊆(i,j)-βInt($f^{-1}(V)$) and hence $f^{-1}(V)$= (i,j)-βInt($f^{-1}(V)$).By Lemma 2.2, $f^{-1}(V)$ is (i,j)-β-open and by Lemma 5.1,f is (i,j)-almost β-continuous.

Remark 5.1. By above theorem,we obtain the result established in Theorem 6 of [11].

Definition 5.3. A bitopological space (X,τ_1,τ_2) is said to be pairwise Hausdroff or pairwise-T_2[9] if for each pair of distinct points x and y of X,there exists τ_i-open set U containing x and a τ_j-open set V containing y such that U∩V= Ø for i≠j,i,j=1,2.

Definition 5.4. A bitopological space (X,τ_1,τ_2) is said to be pairwise β-Hausdroff or pairwise β-T_2 if for each pair of distinct points x and y of X,there exists a (i,j)-β-open set U containing x and a τ_j-open set V containing y such that U∩V= Ø for i≠j,i,j=1,2.

Theorem 5.2.Let (X,τ_1,τ_2) be a bitopological space.If for each pair of distinct points x and y in X,there exists a function f of (X,τ_1,τ_2) into pairwise T_2 bitopological space (Y,σ_1,σ_2) such that

(i).f(x) ≠f(y).
(ii).f is (i,j)-weakly-β-continuous at x,
(iii). f is (j,i)-almost-β-continuous at y,
then (X,τ_1,τ_2) is pairwise β-T_2 .

Proof.Let x and y be a pair of distinct points of X.Since Y is pairwise -T_2,there exists a σ_i-open set U containing f(x) and a σ_j-open set V containing
f(y) such that U∩V= Ø.Since U and V are disjoint,we have jCl(U) ∩jInt(iCl(V))= Ø.Since f is (i,j)-weakly-β-continuous at x,there exists an (i,j)-β-open set U_x of X containing x such that f(U_x) ⊆jCl(U).Since f is (j,i)- almost-β-continuous at y,there exists (j,i)-β-open set U_y of X containing y such that f(U_y) ⊆jInt(iCl(V)).Hence we have U_x ∩ U_y = Ø.This shows that
(X,τ_1,τ_2) is pairwise β-T_2 .

Remark 5.2. By above theorem,we obtain the result established in Theorem 13 of [11].

6.Some Properties

Definition 6.1. A bitopological space (X,τ_1,τ_2) is said to be pairwise Urysohn[6] if for each distinct points x,y of X, there exists τ_i open set U and τ_j open set V such that x∈U, y∈V and jCl(U) ∩iCl(V))= Ø for i≠j, i,j=1,2.

Theorem 6.1. If (Y,σ_1,σ_2) is pairwise Urysohn and f:(X,τ_1,τ_2)→ (Y,σ_1,σ_2) is pairwise weakly-β-continuous injection,then (Y,σ_1,σ_2) is pairwise β-T_2 .

Proof.Let x and y be two distinct points of X.Then f(x) ≠f(y).Since Y is pairwise Urysohn,there exist τ_i-open set U and τ_j-open set V such that f(x) ∈U,f(y) ∈V and jCl(U) ∩iCl(V)= Ø.Hence f^{-1}(jCl(U)) ∩ f^{-1}(iCl(V))=

Ø.Therefore, (i,j)-βInt(f^{-1}(jCl(U))) ∩(j,i)-βInt(f^{-1}(jCl(V))) = Ø.Since f is pairwise weakly β-continuous,by Theorem 3.1 , x∈ f^{-1}(U) ⊆(i,j)-βInt(f^{-1}(jCl(U))) and y∈ f^{-1}(V) ⊆(j,i)-βInt(f^{-1}(iCl(V))).This implies that (X,τ_1,τ_2) is pairwise β-T_2 .

Remark 6.1. By above theorem,we obtain the result established in Theorem 4.4 of [3].

Definition 6.2. A bitopological space (X,τ_1,τ_2) is said to be pairwise connected [15](resp.pairwise β-connected) if it cannot be expressed as the union of two non empty disjoint sets U and V such that U is τ_i open and V is τ_j open(resp.U is (i,j)-β-open and V is (j,i)-β-open).

Theorem 6.2. If a function f:(X,τ_1,τ_2)→ (Y,σ_1,σ_2) is pairwise weakly-β-continuous surjection and (X,τ_1,τ_2) is pairwise β-connected,then (Y,σ_1,σ_2) is pairwise connected.

Proof.Suppose that (Y,σ_1,σ_2) is not pairwise connected.Then there exists a τ_i -open set U and τ_j-open set V such that U≠ Ø,V≠Ø, U∩ V= Ø and U∪V=X.Since f is surjective, f^{-1}(U) and f^{-1}(V) are non empty.Morever f^{-1}(U) ∩ f^{-1}(V)= Ø and f^{-1}(U) ∪ f^{-1}(V)= X.Since f is pairwise weakly β-continuous, by Theorem 3.1,we have f^{-1}(U) ⊆ (i,j)-βInt(f^{-1}(jCl(U)) and f^{-1}(V) ⊆ (j,i)-βInt(f^{-1}(iCl(V))).Since U and V are σ_j-closed and σ_i-closed,respectively,we have f^{-1}(U) ⊆ (i,j)-βInt(f^{-1}(U)) and f^{-1}(V) ⊆ (j,i)-βInt(f^{-1}(V)). Hence f^{-1}(U) = (i,j)-βInt(f^{-1}(U)) and f^{-1}(V) = (j,i)-βInt(f^{-1}(V)). By Lemma 2.2, f^{-1}(U) is (i,j)-β-open and f^{-1}(V) is (j,i)-β-open in (X,τ_1,τ_2).This shows that (X,τ_1,τ_2) is not pairwise connected.

Remark 6.2. By above theorem,we obtain the result established in Theorem 13 of [13].

Definition 6.3. A subset K of a bitopological space (X,τ_1,τ_2) is said to be (i,j)-quasi H-closed relative to X [4] if for each cover {U_α:α∈Δ} of K by τ_i-open sets of X,there exists a finite subset Δ_0 of Δ such that K⊆∪{jCl(U_α): α∈Δ_0 }.

Definition 6.4. A subset K of a bitopological space (X,τ_1,τ_2) is said to be (i,j)-β-compact relative to X if every cover of K by (i,j)-β-open sets of X has a finite subcover.

Theorem 6.3. If f:(X,τ_1,τ_2)→ (Y,σ_1,σ_2) is pairwise weakly-β-continuous and K is (i,j)-β-compact relative to X,then f(K) is (i,j)-quasi H-closed relative to Y.

Proof.Let K be (i,j)-β-compact relative to X and {V_α:α∈Δ} be any cover of f(K) by σ_i-open sets of (Y,σ_1,σ_2). Then f(K) ⊆∪{ V_α:α∈Δ} and so K⊆∪ { f^{-1}(V_α):α∈Δ}.Since f is (i,j) weakly-β-continuous, by Theorem 3.1 ,we have f^{-1}(V_α) ⊆(i,j)-βInt(f^{-1}(jCl(V_α)) for each α∈Δ.Therefore,K⊆∪{(i,j)-βInt(f^{-1}(jCl(V_α))):α∈Δ}.Since K is (i,j)-β-compact relative to X and (i,j)-βInt(f^{-1}(jCl(V_α))) is (i,j)- β-open for each α∈Δ,there exists a finite subset Δ_0 of Δ such that K⊆∪{(i,j)-βInt(f^{-1}(jCl(V_α))): α∈Δ_0}.This implies that f(K)⊆ ∪{f((i,j)-βInt(f^{-1}(jCl(V_α)))): α∈Δ_0}⊆∪{f(f^{-1}(jCl(V_α))) : α∈Δ_0}⊆∪ {jCl(V_α)) : α∈Δ_0}.Hence f(K) is (i,j)-quasi H-closed relative to Y.

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Address

Department of Mathematics,Delhi University,Delhi-110007, India
Postal Address: 250,Double Storey,New Rajinder Nagar,New Delhi-110060, India
E-mail: sanjaytahiliani@yahoo.com